The art of pitching: A theory of lobbying in procurement*

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Abstract

We study lobbying as a seller's ploy to affect the buyer's learning process about the value of a singular good that he wishes to procure. In particular, we argue that a lobbying seller strategically distorts "soft" rather than private information. Our innovation is to model this as the seller "jamming" the buyer's signal – not just by shifting its mean, but – by skewing (increasing the third moment of) its distribution. An unobserved marginal increase in lobbying effort expands demand, thus, unless too expensive, the seller always lobbies, no matter how suspicious the buyer is. Crucially, even when correctly anticipated (in equilibrium), lobbying increases the price elasticity of the buyer's demand. This leads to a lower equilibrium price and increased efficiency. In the (skew-)normal learning model, in equilibrium the seller gains, the buyer loses as a result of lobbying. Nonetheless, the information gleaned during the process keeps the buyer from refusing to engage in it.

Keywords: innovation; learning about valuation; price elasticity; signal jamming

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1 Introduction

This paper is about a key component of selling innovation: lobbying. By employing a novel modelling approach, we gain important insights about how the seller tries to influence the buyer's learning process about his valuation.

As our leading example, consider a manufacturer intent on having its new weapons system included in the US defense budget. It hires a lobbyist – often a retired general – whose efforts begin with contacts at the lower levels in the Pentagon or the different Services, in an attempt to gain their endorsement for the system to qualify as an Unfunded Requirement.[1] These Points of Contact (PoC) need to be convinced to be in favor of the system on offer. Indeed, their take on the system is the "signal" that Congress observes. Every (new) system is a set of functionalities whose value in different battle scenarios is difficult to foresee, precisely what the PoC is expected to assess. Thus, Congress actually *learns about its own preferences* through the PoC's interaction with the lobbyist. The lobbyist's role is not to provide technical specifications or test results – that would be better done by the manufacturer's engineers. Instead, the lobbyist's primary responsibility is to *spin a compelling narrative* about why the system is precisely what the Pentagon needs – and therefore that it should have a high willingness to pay for it.[2]

The broad lines of this story are hardly special to the arms industry. Indeed, as Nownes (2006) puts it, despite many differences in the processes by which novel goods and services are purchased, "... there is one constant in the procurement process: lobbying." The above is a representative example of the common situation where a seasoned supplier comes up with a new product and the buyer is unclear how it fits his needs. This, however, is not the only procurement set-up where lobbying can play an important role. A second type of application is when the supplier itself is new. Lobbying then is about giving a favorable

[1] See Kambrod, 2007, for a detailed step-by-step account of what this process entails and how lobbying plays out in each of these steps.

[2] Being briefed by a retired four-star general with experience both in the field and in the industry, and who has been paid to familiarize himself with these functionalities will impress the PoC more than a Powerpoint presentation by a technician. The size of the lobbying industry inside the Beltway is a clear sign that this more costly way to engage the PoC results in a higher probability of success.

impression to an investor mainly about the human capital involved in a project. For an example of this type, consider a start-up pitching to a Venture Capitalist (VC). They include all the predictably relevant information in a "deck" that they send to the VC. However, the VC decides whether to invest only after several interviews, where his/her primary concern is to evaluate the team, over and above the project itself as, in the start-up world, adaptability is key for survival. A better prepared team will be able to give a better impression.

Finally, the setting for lobbying is often a combination of the previous two: the buyer needs to figure out both the quality/dependability of the supplier and his evaluation of the good. A typical example of this type is the modern-day version of a door-to-door sales person. Say, someone peddling state-of-the-art CT scan equipment to hospitals. The doctors involved know from the prospectus the specifications, but have some uncertainty about how much better a diagnostic tool it would be than their existing equipment. They also have their doubts about the maintenance (or software update) service provided by the company represented by the lobbyist (the quality of which the lobbyist herself might ignore as well). The *narrative* provided by her can potentially bias the doctors towards a more optimistic belief about both.

Based on our examples, we propose that lobbying in procurement should be construed as a case of *signal jamming*, in the tradition of Holmström's (1982/1999) model of career concerns:[3] the lobbyist manipulates the information acquisition process of the buyer. We argue that the standard model in this literature, which assumes that the signal jammer biases the signal by shifting its distribution in her favor, would be incomplete. Indeed, if this were all there was to lobbying, in equilibrium the buyer would simply subtract the (expected) bias from the signal received, and thus, such a model would predict that lobbying has no effect on the price or the probability of purchase. Modeling the effects of lobbying as simply the introduction of a reverse-engineerable bias would imply that ending up with good impressions after removing the bias, is as informative as getting bad vibes. We claim that this is not the case. Getting a bad impression despite the best selling efforts should be considered very informative. Certainly much more than favorable

[3] See also Fudenberg and Tirole (1986) and Dewatripont et al. (1999).

impressions, pushing which is what good salesmanship is all about. In our view, being aware of his own susceptibility to influence, the buyer would think it more likely that his assessment is overblown the higher this posterior turns out to be. That is, if after processing the signal, he forms a high estimate of the product's value, this fact should be a sign that the lobbyist is likely to have been more successful than if his estimate were lower – even in equilibrium. A relative version of "too good to be true". To capture this, buyer needs to (be expected to) bias so that he generates a "fatter right tail" of the signal distribution. That is, the "signal" derived from the conversation with the lobbyist should be considered – not only biased but also – *skewed*.

In order to investigate the consequences for the terms and probability of trade of taking into account this inference by the buyer, we analyze a simple model capturing the above characteristics. Buyer[4] (he) is uncertain about his valuation of a good/service on offer. He procures an informative signal that Seller (she) can manipulate (bias and skew). Seller privately chooses her level of manipulation and names her price. Having observed the price and the signal (but not the extent of manipulation),[5] Buyer decides whether to buy from Seller or not. Adopting the standard probability model employed in many areas of management science (and economics), we assume that, absent manipulation, the signal is joint-normally distributed with the true value. In this setup, lobbying as we understand it is naturally modeled by generalizing to the skew-normal family of distributions.[6]

Note that – in order to get at the heart of the problem – we strip our model of other phenomena that we do not consider intrinsic to lobbying, although they may accompany it. First, we assume that during the process of lobbying Seller has no informational advantage to count on: all the measurable aspects must be revealed during the due diligence phase. In short, we do not consider the (strategic) transmission of asymmetric information an intrinsic part of lobbying, and consequently do not employ a signalling

[4] We avoid a large number of the article "the" by giving Buyer and Seller proper names.

[5] Buyer knows that Seller can "blow smoke" – and might even correctly anticipate how much – but he cannot catch her at it. As is common in the signal-jamming literature, Seller's inability to commit plays a crucial role in our analysis.

[6] See Azzalini (1985). This interesting probability model, to our knowledge, has never before been used in economics or management science.

model.[7] Second, we assume away possible corruption or rent seeking behavior. While, sadly, these are frequently occurring phenomena, we need to understand how lobbying works in their absence.

Building on this insight, we derive its immediate consequence: lobbying not only biases the buyer's estimate, but also makes the noise-to-signal ratio increasing in the realization of that signal. Thus, (the anticipation of) lobbying increases the responsiveness of demand to price when the signal is positive, that is, when the buyer is inclined to buy. This higher elasticity results in lower equilibrium prices. Lobbying still happens in equilibrium: whatever the conjectures of the buyer, the probability of sale is higher when the (unobserved) intensity of lobbying increases. Using our skew-normal probability model, we can also prove that Seller benefits, while Buyer loses because of lobbying. Nonetheless, Buyer will rather put up with lobbying than base his decision on the prior. That is, rather than forego the possibility of gleaning information from talking to the lobbyist.

In the next section we briefly review the most relevant literature. In Section 3 we present our model and state our hypotheses. Section 4 introduces the skew-normal formulation, while Section 5 presents our results. Section 6 contains some concluding remarks.

2 A brief literature review

Our research question is closely related to the literature on advertising. While the fact that we have a single buyer as opposed to a continuum of consumers is mathematically inconsequential, we open up the black box of how the buyer reacts to 'advertising' by modelling his belief updating process (in the presence of uncertainty about the advertising effort). It is customary to distinguish between persuasive and informative advertising.[8] In the first case, sellers' advertising efforts affect the preferences of buyers. In this tradition, Bloch

[7] Thus, "lying" is not part of our model, since it would presuppose the existence of private information. Lying is different from embellishing the message, as commercials do when they surround a product with images that have nothing to do with its characteristics.

[8] Complementary advertising, where consumers derive utility from the fact that the good they have bought is widely advertised, is not relevant for our case.

and Manceau (1999) and Chen et al. (2009) study a model where advertising changes – via an exogenous "consumer response function" – the distribution of buyer's "location" in a Hotelling interval. The effects on profitability, prices, etc., depend on the shape of the function. Instead, we model lobbying effort as a (biased) informational process that does not change Buyer's preferences, but his information, so that questions like the effect of lobbying on the buyer's (expected) payoff can be posed. In that sense, our approach is more closely related to the literature on informative advertising. A branch of this literature, in particular, studies advertising as a process of informing consumers about horizontally differentiated products' fit to their preferences. Leading examples are Anderson and Renault (2009) and, even closer to our setting, Lewis and Sappington (1994). Advertisements convey information to consumers, but this information is unbiased. Lobbying, we postulate, shares both persuasive and informative aspects: it conveys information without changing preferences, but it does so in a biased way. In fact, the phenomenon of lobbying may be more related to the advertising of experience goods, a case the other two branches of the literature on advertising typically ignore. Indeed, we postulate that lobbying may affect the buyer's before-purchase conjecture of (relative) quality. Ever since the seminal work of Nelson (1974), the literature on advertising of experience goods[9] has focussed on advertising as an instrument for signaling quality (for a recent example, see Chen et al. 2024).[10] Perhaps our closest predecessor in this literature is Grunewald and Kräkel's (2017) study of advertising incentives arising from price competition, without assuming asymmetric information and therefore signalling. Apart from other less important differences – vertical differentiation and asymmetry –, their information model is a traditional signal-jamming one where effort shifts the signal observed by the buyers with no further effect on its distribution.[11] Thus, in our context, any intended bias would be filtered out in equilibrium, and price (elasticity) would not be affected by advertising.

[9] See Renault (2016) for a recent survey of this and other advertising literatures.

[10] That is, an action by an informed player that will convey information to another player who observes that action. We depart on both accounts, since we do not assume any information advantage by the sellers and assume that the buyer does not observe (the intensity or even existence of) bias in lobbying. Again, for the phenomenon of lobbying, we claim these to be more accurate assumptions.

[11] They also assume that effort has an information advertising effect: it makes it more likely that the ad reaches a consumer. For lobbying in procurement, this is not relevant.

In the general signal-jamming literature in the papers that are closest to ours, the analysis is based on the strategic communication of private information, what we assume away in this analysis in order to zero in on the intricacies of lobbying proper. As examples, see Dellarocas (2006), who looks at signal jamming by competing firms in the context of Internet opinion forums, or Ayra et al. (2010) who look at discretionary disclosure of proprietary information. In terms of other moments of the distribution being affected, we have only found examples with the variance: see Drugov and Troya-Martínez (2019). Of course, the variance is a symmetric moment, so it does not serve our purpose.

There is a large literature that models lobbying as a rent-seeking contest.[12] In a contest, competitors exert (costly) effort to improve their probability of success in appropriating a prize, which may or may not depend on the competitors' efforts. In that literature, the mapping from effort vectors to the probability of success and the value of the prize is treated as a black-box, and information is usually assumed to be symmetric and complete. The latter is obviously a disadvantage for the study of lobbying as a process of information generation/transmission. Another commonly noted shortcoming of this approach is the lack of micro-foundations for that mapping, even when information is assumed asymmetric. The literature has produced some attempts to provide micro-foundations for (the most commonly used of) these mappings, for example, most related to our problem, Lagerlöf (2007) and Skaperdas and Vaidya, (2012). Yet the approach is too rigid to constitute a promising avenue. Instead, we begin with information and pricing micro-foundations and let the "success" and "prize" mappings be an endogenous consequence of agents' decisions.

Another strand of the literature on lobbying, focused on political influence, has studied the interplay between voters, special interest, and political parties around the choice of policies (see, for example Grossman and Helpman, 1996) under complete information. Of particular interest for us are the papers that consider incomplete information (see for example, Bennedsen and Feldmann, 2006, and Dahm and Porteiro, 2008). However, these papers do not consider biasing the information, rather whether or not there are incentives for obtaining and transmitting it. Of course, they also consider a game of

[12] See Corchón and Serena (2018) for a recent survey of the literature on contests.

campaign contributions instead of procurement. A related literature outwith political economy explores the provision of verifiable information to a decision maker (see for example Brocas and Carrillo, 2007, Henry, 2009, and Henry and Ottaviani, 2019). In these papers, when the effort put into the search for information is unobservable to the decision maker, the sender ends up worse off, unlike in our model.

Finally, Bayesian persuasion has been the workhorse model for truthful biasing of information in many contexts, since the seminal article of Kamenica and Gentzkow (2011). In all this literature the principal assumption is that the senders *commit* to a signal generating process, which has to be a coarsening of the true distribution. In our model, the choice of lobbying effort is not observable and as a result we can capture the effects of Buyer's expectation about lobbying on the lobbying that actually happens (and prices).

3 Modelling lobbying in procurement

Consider as a starting point a common setting in information economics: An agent is uncertain about some payoff-relevant parameter, θ, and he wishes to make inferences about its expected value. At the start, his information is described by a probability distribution – the prior – over θ. He can also collect additional information, in form of a signal, $\widehat{\theta}$. The relevance of the signal for predicting θ is modelled by postulating the pair $(\theta, \widehat{\theta})$ as the realization of a random vector with a known distribution (whose marginal on θ is the prior, of course). The agent infers information about θ from the observation of the realized value of $\widehat{\theta}$ via the conditional distribution. For instance, it is common to postulate that $(\theta, \widehat{\theta})$ is bivariate normal, $N(0, \Omega)$. Note that this implies that $\widehat{\theta} - \theta \equiv \varepsilon$, the "noise in the signal", is – normally distributed, independent of θ and – unbiased.

In the procurement context, Buyer is in a similar situation. When he communicates with Seller – or her lobbyist –, he obtains a signal. It is Seller who "sends" this signal by communicating with Buyer, concomitantly shaping his inference – the distribution of the added noise, if you will – by (investing in) lobbying. Thus, our model postulates that Buyer may purchase from Seller some good/service that he values at θ. Neither party observes θ, but its prior distribution is common knowledge. The game starts with Seller (privately) choosing a lobbying intensity $\alpha \in [0, \infty)$, as well as proposing a price, p.[13] The cost of lobbying is $c(\alpha)$, with $c(0) = 0$, $c'(0) = 0$, $c'(.) \geq 0$, $c''(.) \geq 0$. Next, in communicating with Seller, Buyer observes $\widehat{\theta}$, an imperfect signal about θ – whose distribution is affected by the value (unobserved by Buyer) of α. Finally, given his conjecture about α, denoted by $\widetilde{\alpha}$,[14] Buyer decides whether to buy for p, leading to an expected payoff of $E_{\widetilde{\alpha}}[\theta|\widehat{\theta}] - p$, or to stay with the *status quo*, normalized to 0. Both parties are assumed to be risk neutral. Since this is a dynamic game of asymmetric information, our solution concept is Perfect Bayesian Equilibrium (PBE).[15]

Note that Buyer obtains his signal by communicating with Seller. While he cannot prevent her from lobbying while communicating, of course, he could always choose not to communicate with her at all (we assume that the prior includes all the information that Buyer needs from Seller that are not subject to distortion) and learn nothing. That is, the prior (a summary of all that Buyer can learn without giving the opportunity to Seller to lobby) would be the posterior: Seller could not charge a price above the prior mean, Buyer would be indifferent between the status-quo and Seller's product, obtaining an expected payoff of zero. He does not prefer this option, as the signal contains valuable information, that lobbying cannot erase.[16]

Note that we assume that Seller has to name her price without observing the realization

[13] We give all the bargaining power to Seller. There would be no qualitative changes if she only had a probability $\gamma < 1$ of making the offer, and Buyer made the offer with the remaining probability.

[14] Of course, in equilibrium $\widetilde{\alpha} = \alpha$.

[15] Since he does not observe α, Buyer's expected payoff depends on the conjecture, $\widetilde{\alpha}$, that the observables – p and $\widehat{\theta}$ – lead him to make with respect to α. In equilibrium the conjecture coincides with Seller's choice. However, when Buyer realizes that play has veered off the equilibrium path – that is, when the price p surprises him – PBE imposes no restrictions on his conjectures. In principle, the mapping from out-of-equilibrium prices to these conjectures (that may also depend on $\widehat{\theta}$) could lead to multiple equilibria. Nonetheless, we do not consider this potential signaling role of pricing important for the problem at hand. Thus, we restrict attention to the unique equilibrium with passive beliefs, where Buyer does not change his equilibrium conjecture about α no matter what price and signal he observes: he considers unexpected prices to be a mistake. In fact, as we will see, the optimal price (absent Buyer's inferences from such price) is indeed independent of α (and depends only on $\widetilde{\alpha}$).

[16] To see this, note that, for any given p, he could decide to buy only for a signal sufficiently high so that expected profits are positive.

of the signal. This is realistic, as the lobbyist cannot tell how effective she has been (also, often the price is not set by the person who lobbies but her boss).[17]

Even at this level of generality, we may formulate our main hypotheses. First, *H1 conjecturing more intense lobbying (higher* $\widetilde{\alpha}$*) should result in Buyer discounting the signal* $\widehat{\theta}$ *by more,* and so *H2 it should lead to a lower WTP* $\left(E_{\widetilde{\alpha}}[\theta|\widehat{\theta}]\right)$. That would imply a higher threshold signal, above which Buyer buys from Seller (for any given price). Similarly, *H3 the higher is* $\widetilde{\alpha}$*, the less sensitive should Buyer's WTP be to an increase in the realized signal,* $\widehat{\theta}$. If these two conditions are met (and signal distributions are log-concave, a standard, commonly satisfied technical assumption), then *H4 a higher* $\widetilde{\alpha}$ *results in a more elastic Buyer's demand, and so, ceteris paribus a lower equilibrium price.* Nevertheless, since the skew is moving mass to the right of the price in the signal distribution, *H5 for any given price and Buyer's conjecture, a higher "true"* α *should increase the probability of sale,* and so *H6 lobbying should be expected to occur in equilibrium.* Then, conjecturing that the quantity effect of α on the price elasticity of demand does not overturn the effect of $\widetilde{\alpha}$ and the larger responsiveness of demand to price, in equilibrium – where $\alpha = \widetilde{\alpha} > 0$ – *H7 we should expect a higher probability of sale*, while *H8 the price should be lower* with more lobbying (say, as cost of lobbying decreases). Then we should expect that, for low enough cost of lobbying *H9 expected gains from trade are higher due to lobbying*,[18] as the deadweight loss is also reduced: a lower price reduces the deadweight loss related to Seller's market power.

In the next section, we confirm these hypotheses using a probability model that is a simple (skewed) variation of the standard Gaussian learning model, nesting it when lobbying is absent.

[17] If she did observe the signal, she could calculate Buyer's conditional expectation and price accordingly, so her incentive would be to maximize Buyer's expected valuation. As we have discussed in the Introduction, in equilibrium this is a fruitless endeavor, so our model would not be able to explain the prevalence of lobbying.

[18] Without further distributional assumptions we do not have a clear prediction about expected profits. Though clearly gross profits of at least one firm should increase.

4 The Normal/Skew-Normal formulation

We assume that the prior over θ is a standard normal distribution, and the signal, $\widehat{\theta}$, is the sum of the true realization and an independently distributed noise term, $\varepsilon(\alpha)$, a skew-normal (SN) random variable, $SN(0, w, \alpha)$.[19] That is, we are assuming that $\varepsilon(\alpha)$ has density

$$f_\varepsilon(x) = \frac{2}{w}\phi\left(\frac{x}{w}\right)\Phi\left(\alpha\frac{x}{w}\right),$$

where $\phi(.)$ and $\Phi(.)$ are the density and distribution functions, respectively, of a standard normal random variable. Note that, when $\alpha = 0$ and $w = 1$, we recover the normal distribution: $f_\varepsilon(x) \equiv \phi(x)$.

We will use the shape parameter, α, which can be thought of as a measure of the skewness of the distribution[20], as the intensity of lobbying.

We wish to isolate the effect of the increased skewness, without a concomitant change in the "quality" of the signal. To this effect, we set the scale parameter, w, so that the resulting variance

$$w^2\left(1 - \frac{2}{\pi}\frac{\alpha^2}{1+\alpha^2}\right)$$

stays constant. We choose the constant variance to be 1, to maintain continuity with the standard normal at $\alpha = 0$. That is, we set the scale parameter at[21]

$$w(\alpha) = \sqrt{\frac{1+\alpha^2}{1+\left(1-\frac{2}{\pi}\right)\alpha^2}}.$$

Note that a positive α biases the signal as well. Indeed, the expected value of the skewed noise is

$$E[\varepsilon(\alpha)] = \sqrt{\frac{2}{\pi}}\frac{\alpha w(\alpha)}{\sqrt{1+\alpha^2}} = \sqrt{\frac{2}{\pi\alpha^{-2}+\pi-2}}, \tag{1}$$

increasing in $\alpha \in [0, \infty)$, from 0 to $\sqrt{\frac{2}{\pi-2}} \approx 1.3236$. As this phenomenon is highly realistic – and as discussed in the Introduction, strategically irrelevant – we do not eliminate it by an α-dependent location parameter.

[19] Recall that the three parameters of the skew-normal distribution are the location, the scale and the shape parameters, respectively.

[20] It does not coincide with it, but actual skewness is monotonically increasing in the shape parameter.

[21] Note that $w(\alpha)$ is increasing in $\alpha \in [0, \infty)$, from 1 to $\sqrt{\frac{1}{1-\frac{2}{\pi}}} \approx 1.6589$.

Figure 1 below shows the probability density function of the noise distribution, $SN\left(0, \sqrt{\frac{1+\alpha^2}{1+\left(1-\frac{2}{\pi}\right)\alpha^2}}, \alpha\right)$, for different values of α. Note that $\alpha = \infty$ we would have the half-Normal distribution.

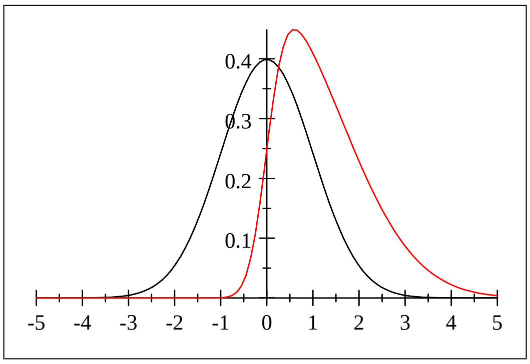

Figure 1: The noise distribution for $\alpha = 0$ and 5.

As the prior of θ is $N(0,1)$ and the noise, $\varepsilon(\alpha)$, follows a $SN(0, w(\alpha), \alpha)$, – Proposition 2.3 in Azzalini (2013) – the signal, $\widehat{\theta}(\alpha) = \theta + \varepsilon(\alpha)$, follows a $SN\left(0, \sqrt{1+w^2(\alpha)}, g(\alpha)\sqrt{1+w^2(\alpha)}\right)$, with

$$g(\alpha)\sqrt{1+w^2(\alpha)} = \frac{\alpha w(\alpha)}{\sqrt{1+\alpha^2+w^2(\alpha)}} = \sqrt{\frac{1}{2\alpha^{-2}+1-\frac{2}{\pi}}}. \tag{2}$$

As $g(\alpha)\sqrt{1+w^2(\alpha)}$ is increasing in α, from 0 to $\sqrt{\frac{\pi}{\pi-2}}$, we can interpret α as a measure of the skewness of signal as well.

5 Results

We are now ready to start with the analysis of the model. We first investigate how Buyer's conjecture about Seller's lobbying effort, $\widetilde{\alpha}$, affects the outcome. This is not just relevant as a step towards calculating equilibrium, but it also reveals the consequences of incorrect conjectures, helping us understand how much firms – both suppliers and procurers – should be concerned about the possibility of these.

The key variable of interest is Buyer's posterior expectation of his valuation, conditional on the signal observed, based on his conjecture (whether correct or not) about the lobbying intensity chosen by Seller.

Proposition 1 *Buyer's posterior expectation of his value is given by*

$$E_{\widetilde{\alpha}}\left[\theta|\,\widehat{\theta}\right] = \frac{\widehat{\theta}}{1+w^2(\widetilde{\alpha})} - g(\widetilde{\alpha})H\left(g(\widetilde{\alpha})\widehat{\theta}\right), \tag{3}$$

where $H(x) \equiv \frac{\phi(x)}{\Phi(x)}$ *is the reverse hazard rate of the standard normal distribution.*

Based on Proposition 1, we can now start confirming our hypotheses. We state them as lemmas, and the ones not established in the text, we prove in the Appendix.

Towards confirming our first hypothesis it is instructive to start with the counterfactual. If the noise were a $N(\mu, 1)$ random variable (and therefore the signal were biased, by μ, but not skewed), then the posterior, $E\left[\theta|\,\widehat{\theta}\right]$, would be a convex, linear combination of the "debiased" signal, $\widehat{\theta} - \mu$, and the expected value of the prior, 0:

$$\frac{1}{2}\left(\widehat{\theta} - \mu\right) + \frac{1}{2}(0).$$

Thus, $E\left[\theta|\,\widehat{\theta}\right]$ would be an affine function of $\widehat{\theta}$, with $\frac{E\left[\theta|\widehat{\theta}\right]}{d\widehat{\theta}}$ constant and equal to the weight put on the debiased signal. Interpreting the weight on the signal as a discounting factor, we can observe that a higher signal realization would not change how Buyer discounted the signal: his suspicions that "he has been had" would be unaltered.[22]

When lobbying not only shifts the distribution of the signal but also skews its distribution, things look different. In order to write the posterior as a weighted average of signal and prior mean, let $\underline{\widehat{\theta}} > 0$ be the solution to $E_{\widetilde{\alpha}}\left[\theta|\,\widehat{\theta}\right] = 0$,[23] and observe that, by Proposition 1,

$$E_{\widetilde{\alpha}}\left[\theta|\,\widehat{\theta}\right] = \omega(\widehat{\theta})(\widehat{\theta} - \underline{\widehat{\theta}}) + (1 - \omega(\widehat{\theta}))0,$$

[22] As a result, Buyer could completely neutralize the bias and his demand, and thus the price set by Seller, would be the same as in the absence of lobbying. However, Seller would still have an incentive to lobby unobserved, so in equilibrium lobbying would occur, but Seller would lose as a result (the cost of lobbying). This is hardly an accurate description of reality.

[23] We will argue below that the solution exists and is unique.

where

$$\omega(\widehat{\theta}) = \frac{1}{1+w^2(\widetilde{\alpha})} - g(\widetilde{\alpha})\frac{H(g(\widetilde{\alpha})\widehat{\theta}) - H(g(\widetilde{\alpha})\underline{\widehat{\theta}})}{\widehat{\theta} - \underline{\widehat{\theta}}}$$

is the weight on the debiased value of the signal.[24] Using (3) and the fact that $H''(.) > 0$ for $\widehat{\theta} \geq 0$, we can easily see that $\omega'(\widehat{\theta}) = \frac{d^2 E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta}\right]}{d\widehat{\theta}^2} < 0$.[25] Consequently,

Lemma 1 ***(H1)*** *When $\widetilde{\alpha} > 0$, the higher is its realization, $\widehat{\theta} \geq 0$, the more Buyer discounts the signal.*

That is, as hypothesized, the higher is its realization, the lower weight Buyer puts on the (debiased) signal, as he finds it more likely that it is a result of Seller's lobbying – noise.

Our next result shows that the increasing skepticism as signals get higher has a direct effect on how Buyer varies his posterior with $\widetilde{\alpha}$. After all, the result of lobbying is that higher signal realizations become more likely.

Lemma 2 ***(H2)*** *For every $\widehat{\theta} \geq 0$, $E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta}\right]$ is decreasing in $\widetilde{\alpha}$.*

Thus, as we expected, when Buyer conjectures a higher lobbying effort, he interprets the same (non-negative) signal in a more pessimistic way.

Next, we identify Buyer's demand function. By (3), for any $\widetilde{\alpha} \geq 0$, $E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta} = 0\right] < 0$, while, by Lemma 1, $E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta}\right]$ grows monotonically in $\widehat{\theta}$, and it has no upper limit (as $\lim_{x\to\infty} H(x) = 0$). Therefore,

$$E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta}\right] = p \tag{4}$$

has a unique solution in $\widehat{\theta} \in (0,\infty)$ for any $p \geq 0$. Denote that solution by $\widehat{\theta}^*(p,\widetilde{\alpha})$. Then, Buyer will purchase at price p if and only if the signal is above the threshold $\widehat{\theta}^*(p,\widetilde{\alpha})$. A straightforward corollary of Lemma 2 is that this threshold signal increases with $\widetilde{\alpha}$: if Buyer expects higher lobbying effort, he requires a higher signal to be willing to buy at any given price.

[24] It can be shown, using the fact that $H'(x) \in (-1,0)$ for $x \geq 0$,that the weight is in $[0,1]$ for $\widehat{\theta} \geq 0$.

[25] The result is only proven for non-negative signals, but Buyer would never buy with a negative signal anyway, since it would lead to a negative posterior.

Let us now turn to Seller's problem. Till this point we only needed to know $\widetilde{\alpha}$, Buyer's conjecture about α. When we take Seller's point of view, we need to include the actual α – that is, her choice variable – and her conjecture about $\widetilde{\alpha}$. For the sake of transparency, we assume that this second-order belief is always correct, as it must be in equilibrium.

By the preceding discussion the demand function Seller is faced with is

$$D(p;\alpha,\widetilde{\alpha}) = 1 - F_{\widehat{\theta},\alpha}\left(\widehat{\theta}^*(p,\widetilde{\alpha})\right),$$

where $F_{\widehat{\theta},\alpha}$ denotes the CDF of the signal given α.[26] The probability of sale depends on the price, actual lobbying intensity α, and conjectured lobbying intensity $\widetilde{\alpha}$. Note that $\widetilde{\alpha}$ determines the threshold signal, while α determines the signal distribution. Of course, in equilibrium the two must coincide.

It is immediate from the fact that the threshold signal is increasing in $\widetilde{\alpha}$ that Seller faces a lower probability of sale for any given strategy chosen by her (α, p) as $\widetilde{\alpha}$ increases.

Let us have a close look at the price elasticity of the demand function. The – absolute value of – the price elasticity of demand is

$$\epsilon = \frac{f_{\widehat{\theta},\alpha}(\widehat{\theta}^*(p,\widetilde{\alpha}))}{1 - F_{\widehat{\theta},\alpha}(\widehat{\theta}^*(p,\widetilde{\alpha}))}\frac{\partial\widehat{\theta}^*(p,\widetilde{\alpha})}{\partial p}p. \tag{5}$$

Log-concavity of $f_{\widehat{\theta},\alpha}$ (see Azzalini, 1985) and monotonicity of $\widehat{\theta}^*(p,\widetilde{\alpha})$ (Lemma 2) guarantee that the first term is increasing in $\widetilde{\alpha}$. Thus, if and only if $\frac{\partial^2\widehat{\theta}^*(p,\widetilde{\alpha})}{\partial p\partial\widetilde{\alpha}} > 0$, that is, if $E_{\widetilde{\alpha}}\left[\theta|\,\widehat{\theta}\right]$ responds more to changes in $\widehat{\theta}$ the higher $\widetilde{\alpha}$, then ϵ is increasing in $\widetilde{\alpha}$. That is, she faces a more elastic demand at each price, and so the optimal price – implicitly given by $\epsilon(p^*) = 1$ – is the lower, the higher is $\widetilde{\alpha}$. Thus, by showing that $\frac{\partial\widehat{\theta}^*(p,\widetilde{\alpha})}{\partial p}$ is indeed increasing in $\widetilde{\alpha}$, we obtain the next proposition.

Lemma 3 ***(H3, H4)*** *For any level of α, the optimal price is a decreasing function of $\widetilde{\alpha}$: $\frac{\partial p^*(\alpha,\widetilde{\alpha})}{\partial\widetilde{\alpha}} < 0$.*

[26] From (11) in the proof of Proposition 1, this can be written as

$$\int_{\widehat{\theta}^*(p,\widetilde{\alpha})}^{\infty} \frac{2}{\sqrt{1+w(\alpha)^2}}\phi\left(\frac{y}{\sqrt{1+w(\alpha)^2}}\right)\Phi\left(g\left(\alpha\right)y\right)dy.$$

Finally, let us turn to the direct effect of distorting the signal.

Lemma 4 ***(H5)****(Actual) lobbying by Seller increases the probability of sale for any given $p > 0$ (and Buyer conjecture, $\widetilde{\alpha}$): $D(p; \alpha, \widetilde{\alpha})$ is increasing in $\alpha \geq 0$.*

As Seller's profits are

$$\Pi(\alpha, p; \widetilde{\alpha}) = D(p; \alpha, \widetilde{\alpha})p - c(\alpha), \tag{6}$$

she always benefits from some lobbying,[27] as long as it is not observed by Buyer. Let us state this as a lemma.

Lemma 5 ***(H6)*** *Seller lobbies, $\alpha > 0$, for any conjecture of Buyer (and any chosen price).*

We turn to the equilibrium analysis next.

5.1 Equilibrium

The equilibrium $\left(\alpha^*, p^*; \widehat{\theta}^*(p, \alpha^*)\right)$, where we have already used the fact that Buyer's conjecture about α is correct and does not depend on p, is calculated in two steps. First, using Proposition 1 and (4), we identify Buyer's purchasing strategy, $\widehat{\theta}^*(p, \alpha^*)$, for each possible α^*:

$$\frac{\widehat{\theta}^*(p, \alpha^*)}{1 + w^2(\alpha^*)} - g(\alpha^*)H\left(g(\alpha^*)\widehat{\theta}^*(p, \alpha^*)\right) = p. \tag{7}$$

Next, using $\widehat{\theta}^*(p, \alpha^*)$, we solve for the optimal price, using $\epsilon(p^*) = 1$ and (5):

$$\frac{f_{\widehat{\theta}, \alpha^*}\left(\widehat{\theta}^*(p^*, \alpha^*)\right)}{1 - F_{\widehat{\theta}, \alpha^*}\left(\widehat{\theta}^*(p^*, \alpha^*)\right)} \frac{\partial \widehat{\theta}^*}{\partial p}(p^*)p^* = 1. \tag{8}$$

Finally, we determine the equilibrium level of lobbying, α^*, by maximizing (6)

$$\frac{\partial\left(1 - F_{\widehat{\theta}, \alpha^*}\left(\widehat{\theta}^*(p^*, \alpha^*)\right)\right)}{\partial \alpha^*} p^* = c'(\alpha^*). \tag{9}$$

[27] Recall that $c(0) = 0$ and $c\prime(0) = 0$. Without these assumption the result would hold for "low enough" lobbying cost.

We already know from Lemma 5 that this equation has a positive solution.[28] Without positing a specific cost function, we cannot solve for α^*. However, note that α^* varies with the cost function in the standard way,[29] providing no further insights. Therefore, to concentrate on the novel effects, we simply take the solution $\alpha^* > 0$ of (9) as a parameter in the rest of the paper. A higher value of this parameter is to be interpreted as the implicit assumption of a "less steep" cost function.

We have solved (7) and (8) for a range of values of α^* and report these solutions in Figure 2a and 2b.

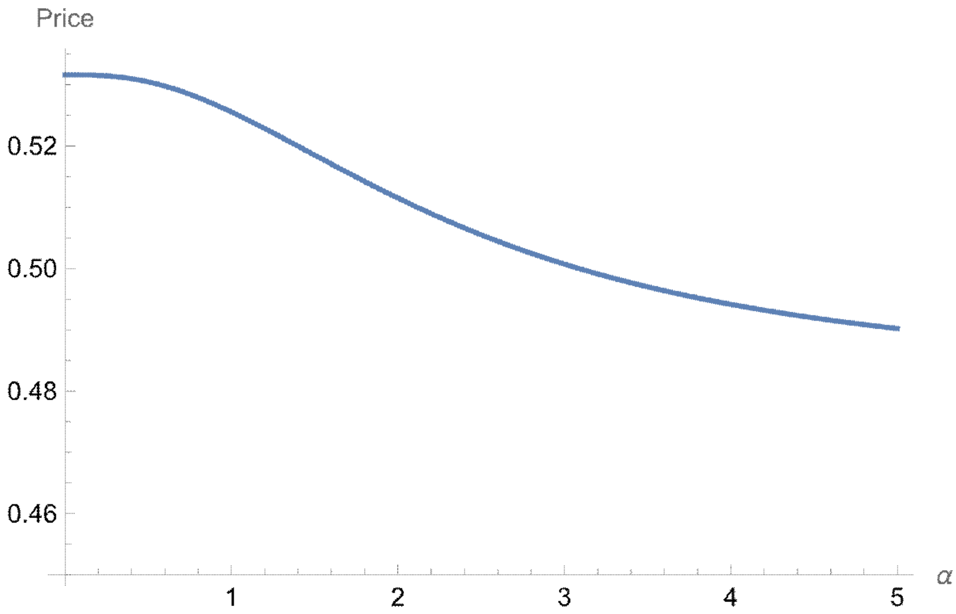

Figure 2a: Price as a function of α^*

[28]It is possible that this first-order condition has no solution (for example if $c(x) \equiv 0$). In that case, α^* is ∞. Note that this is not 'crazy': the limiting values of all the relevant parameters are finite. To avoid the infinite value, we could define $\beta = z - (z^{-1} + \alpha)^{-1}$, what is monotonically increasing from 0 to z as α grows from 0 to ∞, to measure lobbying effort, but we see no need for it.

[29]That is, if we multiply $c(.)$ by $t > (<)1$ the solution in α decreases (increases).

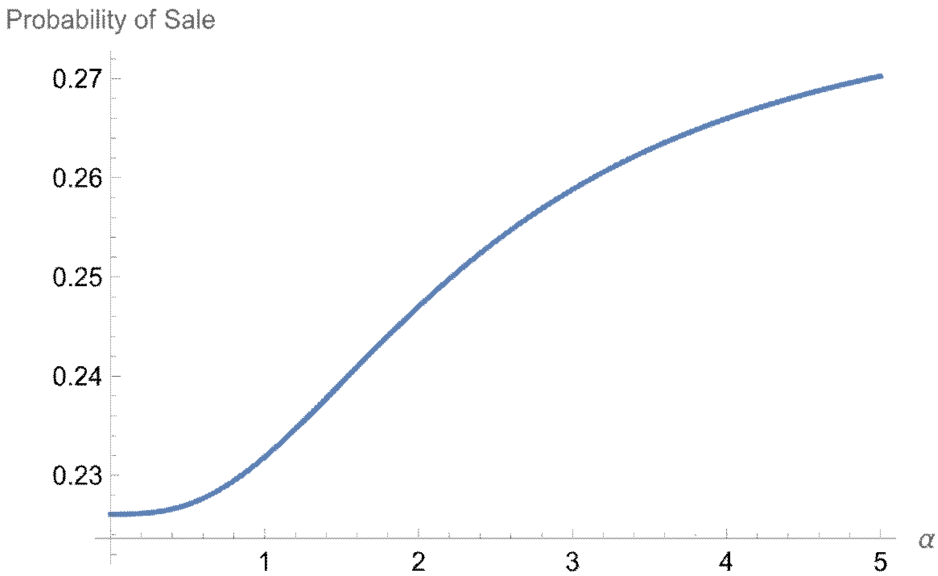

Figure 2b: Probability of sale as a function of α^*

As we can see, confirming **H7** and **H8**, the equilibrium price is decreasing, and the probability of sale (PoS) is increasing in α^*. That is, despite the increase in demand that higher actual α induces for each price (and so the possible reduction in demand elasticity), the effect of a (simultaneous) increase in conjectured $\widetilde{\alpha}$ dominates, and as a consequence equilibrium price is decreasing in α^*. As the equilibrium level of lobbying is inversely related to the cost of lobbying (c.f. Lemma 4) an increase in the cost of lobbying would increase price and decrease PoS.

Expected Buyer's surplus can also be directly calculated (numerically). As Figure 3a documents, despite the reduction in price, Buyer is worse off with lobbying. Interestingly, this is not due to a less efficient decision of Buyer. Indeed – as Figure 3b shows – confirming **H9**, total surplus (WTP) is actually increasing in α^*. That is, Buyer chooses more efficiently with higher α^*. Buyer's problem is that this improved choice is too expensive, from his point of view: too often, small increases in fit (small θ versus 0) come at a steep price (high p versus 0).

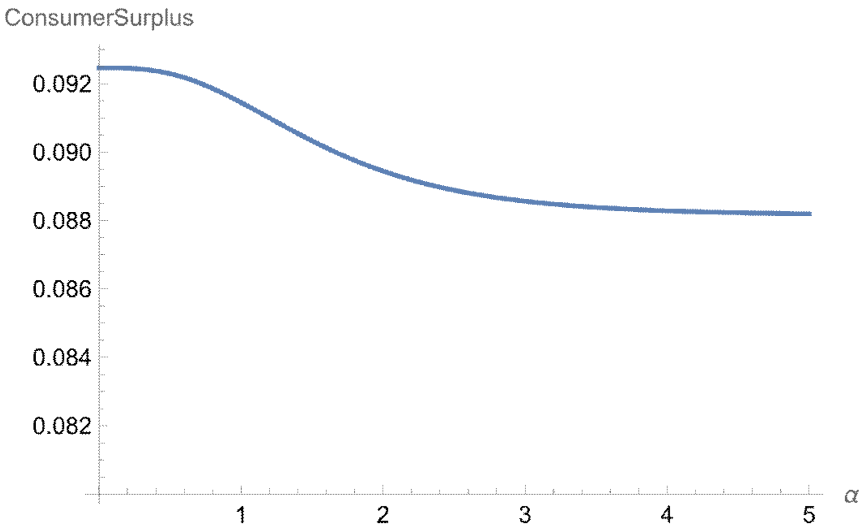

Figure 3a: Buyer's profit as a function of α^*

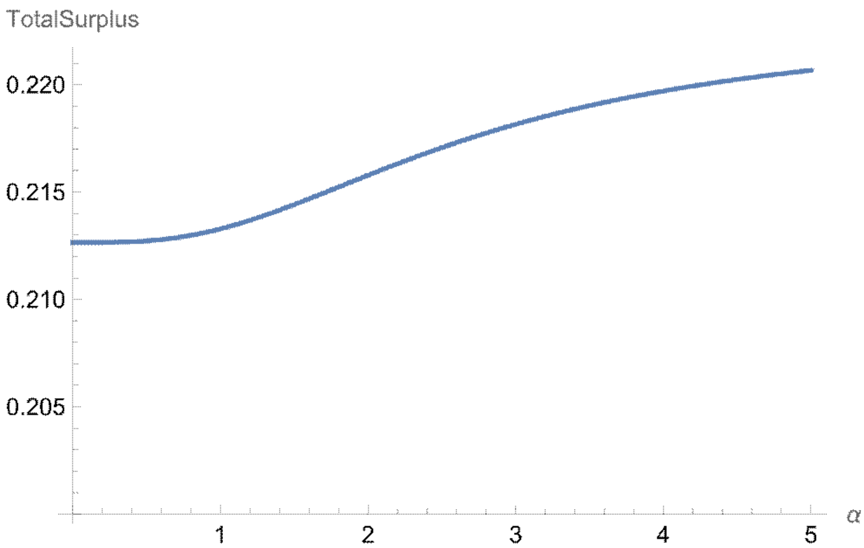

Figure 3b: WTP as a function of α^*

The fact that total surplus increases with α^* is hardly surprising. Indeed, recall that we have all but shut down any change in the quality of the signal coming from α. Indeed, if Seller's price coincided with her marginal cost (zero) the total surplus would be nearly independent of α, as we are keeping the precision (variance) of the signal constant. (For the range of α^* in Figures 2 and 3, this surplus would grow from .282 to .288, Thus, around 2%.) The only real source of (second best, as θ is not observed) inefficiency is market power: Seller charges a price above marginal cost. Other things equal, this deadweight loss – Buyer stays with the status-quo when Seller's product is better in expectation, but not as much as to justify incurring the price – is obviously lower, the lower is the price.

In order to study Seller's expected profits without a specific cost function, we assume that the cost of lobbying is zero, providing an upper bound on profits. As we can see in Figure 4, despite the reduction in price, Seller's profits increase with the level of equilibrium lobbying: the higher probability of sale more than compensates the lower price.

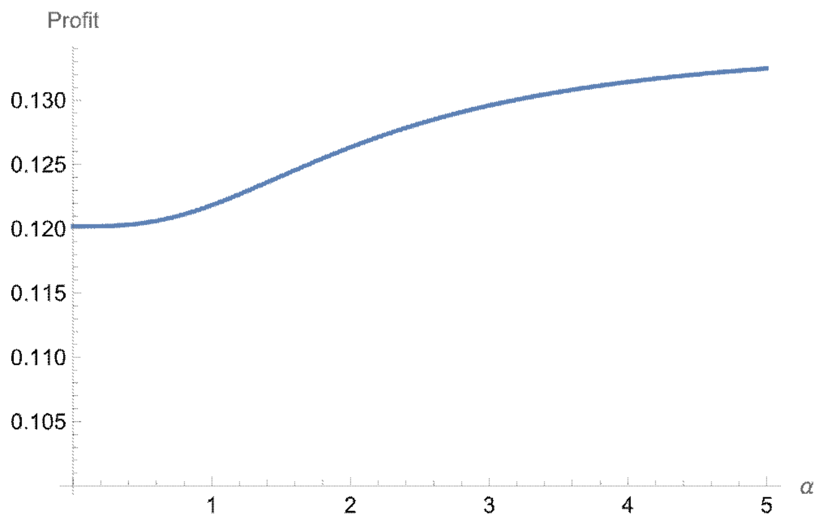

Figure 4: Seller's profit as a function of α^*

Clearly, this qualitative feature continues to hold for a "low enough" cost function. However, for high enough costs, we might end up with a Prisoners' Dilemma situation, where Seller lobbies not because it is profitable, but because Buyer expects him to.[30]

6 Concluding remarks

We have presented – to our knowledge the first – tractable model of procurement lobbying as a signal-jamming process, and argued that in this context the nature of the signal interference should include skewing Buyer's information. We have isolated lobbying in this setup from other related phenomena discussed in the literature.

[30] Of course, this cannot happen in our main example, as the presence of a general is noticeable even if he is in plain clothes.

Our analysis highlights the importance of lobbying for the purpose of supplying a novel good/service of as yet uncertain value. Lobbying casts light on the virtues of the new product. As light may be cast in different angles, lobbyists will choose the most favorable one. Of course, this is anticipated by Buyer, who would then rightly "discount" any favorable impression obtained. Our fundamental contribution is to highlight how subtle the correct discounting and its consequences are. At the basis of it all, a higher signal obtained in communicating with the lobbyist should be more suspect to Seller. The natural implication is a difference-in-difference result: the stronger the lobbying (conjectured or observed), the weaker the weight that Buyer should assign to "better" news, and so the higher the price elasticity of his demand. Thus, when a supplier thinks that their trading partner conjectures that they are lobbying hard, they best respond by lowering the price. At the same time, since an – unanticipated – increase in lobbying increases the likelihood of trade for any price the supplier might choose, she should engage in it (until its marginal cost is too high). Of course, then the trading partner should expect that to be the case, closing the circle.

A less direct consequence, that follows from our analysis of the skew-normal model, is that in equilibrium (at least, gross of lobbying cost) Seller gains, Buyer loses as the result of lobbying. That is, Seller benefits more from the increased probability of trade than what she loses due to the lower price. In Buyer's case, despite the lower price, the lower expected valuation leads to lower expected profits. In sum, Seller gains more than what Buyer loses: for all its potential for manipulation, when rightly assessed, lobbying still serves the purpose of conveying information and, moreover, as it reduces (the price, and so) the deadweight loss associated with any market power of Seller, it increases the probability of trade when trade is efficient.

Our model, or variations of it, may be used to analyze a number of additional questions related to lobbying in procurement. For instance, it is common to associate lobbying with a certain measure of capture. A straightforward way to modelling capture (of an agent still subject to, perhaps, more naive or less expert public) is to assume out of equilibrium (lower than equilibrium) conjectures about the intensity of lobbying (our $\widetilde{\alpha}$). Our analysis provides a first approach to the effects that may be expected from such combination of skewness and capture.

Asymmetric information, and so the design of tools to deal with it, may be invited back now that the bias of (the process of elucidating) information has been analyzed. Misrepresentation and embellishment are different concepts, but they obviously can co-exist.

Finally, a model of competition between multiple sellers for the contract, both in price and in lobbying, including an intriguing discussion of the ways lobbying by competitors might interact, could be built on the framework we have provided as well.

We believe that, by opening these avenues, this paper offers a first step in the direction of better understanding the role that a pervasive phenomenon in procurement, lobbying, plays in the ways that it organizes trade.

Appendix

Proof of Proposition 1. *Buyer's posterior expectation is*

$$E_{\widetilde{\alpha}}\left[\theta|\,\widehat{\theta}\right] = \frac{\widehat{\theta}}{1+w^2(\widetilde{\alpha})} - g(\widetilde{\alpha})H\left(g(\widetilde{\alpha})\widehat{\theta}\right),$$

where $H(x) \equiv \frac{\phi(x)}{\Phi(x)}$ *is the reverse hazard rate of the standard normal distribution.*

We write this proof for arbitrary α, independently of whose belief it is. By Bayes' Rule, $\theta|\,\widehat{\theta}$ is a random variable with density

$$f\left(\theta|\,\widehat{\theta}\right) = \frac{f_{\widehat{\theta}}\left(\widehat{\theta}\middle|\,\theta\right)f(\theta)}{f_{\widehat{\theta}}(\widehat{\theta})}. \tag{10}$$

Recall that $f_{\widehat{\theta}}\left(\widehat{\theta}\middle|\,\theta\right) = f_{\varepsilon}(\widehat{\theta}-\theta)$, and so

$$f_{\widehat{\theta}}\left(\widehat{\theta}\middle|\,\theta\right) = \frac{2}{w(\alpha)}\phi\left(\frac{\widehat{\theta}-\theta}{w(\alpha)}\right)\Phi\left(\frac{\alpha\left(\widehat{\theta}-\theta\right)}{w(\alpha)}\right).$$

We can obtain $f_{\widehat{\theta}}(\widehat{\theta})$ from Proposition 2.3 in Azzalini (2013), since the prior is standard normal. Thus, the signal distribution, $\widehat{\theta} = \theta + \varepsilon$, is also skew-normal, with parameters $(0,\sqrt{1+w(\alpha)^2},g(\alpha)\sqrt{1+w(\alpha)^2})$. Therefore its density is

$$f_{\widehat{\theta}}(\widehat{\theta}) = \frac{2}{\sqrt{1+w^2(\alpha)}}\phi\left(\frac{\widehat{\theta}}{\sqrt{1+w^2(\alpha)}}\right)\Phi\left(g(\alpha)\widehat{\theta}\right). \tag{11}$$

Thus, from (10),

$$f\left(\theta|\,\widehat{\theta}\right) = \frac{\frac{2}{w(\alpha)}\phi\left(\frac{\widehat{\theta}-\theta}{w(\alpha)}\right)\phi(\theta)\,\Phi\left(\frac{\alpha\left(\widehat{\theta}-\theta\right)}{w(\alpha)}\right)}{\frac{2}{\sqrt{1+w^2(\alpha)}}\phi\left(\frac{\widehat{\theta}}{\sqrt{1+w^2(\alpha)}}\right)\Phi\left(g(\alpha)\widehat{\theta}\right)}. \tag{12}$$

The expression (12) simplifies to

$$\frac{1}{\lambda_2}\phi\left(\frac{\theta-\widehat{\theta}\frac{1}{1+w^2(\alpha)}}{\lambda_2}\right)\frac{\Phi\left(\frac{\alpha\left(\widehat{\theta}-\theta\right)}{w(\alpha)}\right)}{\Phi\left(g(\alpha)\widehat{\theta}\right)},$$

where $\lambda_2 = \frac{w(\alpha)}{\sqrt{1+w^2(\alpha)}}$. Thus, $\theta|\widehat{\theta}$ follows an Extended Skew-Normal distribution (see Azzalini,1985, page 177), that is,

$$f\left(\theta|\widehat{\theta}\right) = \phi\left(\lambda_2^{-1}(\theta-\lambda_1)\right)\frac{\Phi(\lambda\cdot\lambda_2^{-1}(\theta-\lambda_1)+\xi)}{\lambda_2\Phi\left(\xi(1+\lambda^2)^{-\frac{1}{2}}\right)}$$

with parameters $\lambda_1 = \widehat{\theta}\frac{1}{1+w^2(\alpha)}$, $\lambda_2 = \frac{w(\alpha)}{\sqrt{1+w^2(\alpha)}}$, $\lambda = \frac{-\alpha}{\sqrt{1+w^2(\alpha)}}$ and $\xi = \alpha\widehat{\theta}\frac{w(\alpha)}{1+w^2(\alpha)}$.

The moment generating function of this random variable (Arnold and Beaver, 2000) is

$$M(t) = e^{t\left(\lambda_1+\frac{\lambda_2^2}{2}t\right)}\Phi\left(\frac{\lambda t\lambda_2+\xi}{(1+\lambda^2)^{\frac{1}{2}}}\right)\frac{1}{\Phi\left(\xi(1+\lambda^2)^{-\frac{1}{2}}\right)},$$

and so,

$$E_\alpha\left[\theta|\widehat{\theta}\right] = M'(0) = \lambda_1+\lambda\cdot\lambda_2(1+\lambda^2)^{-\frac{1}{2}}\frac{\phi\left(\xi(1+\lambda^2)^{-\frac{1}{2}}\right)}{\Phi(\xi(1+\lambda^2)^{-\frac{1}{2}})}.$$

Substituting in, we obtain the result. ■

Proof of Lemma 2. *For every $\widehat{\theta}\geq 0$, $E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta}\right]$ is decreasing in $\widetilde{\alpha}$ and, therefore, for every p, $\widehat{\theta}^*(p,\widetilde{\alpha})$ is increasing in $\widetilde{\alpha}$.*

Substituting for $w = \sqrt{\frac{1+\widetilde{\alpha}^2}{1+\left(1-\frac{2}{\pi}\right)\widetilde{\alpha}^2}}$ in (3),

$$E_{\widetilde{\alpha}}\left[\theta|\widehat{\theta}\right] = \frac{\widehat{\theta}\left(\pi+(\pi-2)\,\widetilde{\alpha}^2\right)}{2\left(\pi+(\pi-1)\,\widetilde{\alpha}^2\right)} - gH\left(g\widehat{\theta}\right).$$

Consequently,

$$\frac{dE\left[\theta|\widehat{\theta}\right]}{d\widetilde{\alpha}} = -\frac{\widetilde{\alpha}\pi g\widehat{\theta}}{\left(\pi+(\pi-1)\,\widetilde{\alpha}^2\right)^2 g} - \frac{dg}{d\widetilde{\alpha}}\left(g\widehat{\theta}H'\left(g\widehat{\theta}\right)+H\left(g\widehat{\theta}\right)\right).$$

Note that, $g\widehat{\theta}$ can take any value in $[0,\infty)$ as $\widehat{\theta}$ ranges from 0 to ∞ and $g\left(\widetilde{\alpha}\right)\geq 0$ for all $\widetilde{\alpha}\geq 0$. Thus, it is sufficient to show that for any $y\in[0,\infty)$,

$$\frac{\widetilde{\alpha}\pi y}{\left(\pi+(\pi-1)\,\widetilde{\alpha}^2\right)^2 g} + \frac{dg}{d\widetilde{\alpha}}\left(yH'\left(y\right)+H\left(y\right)\right) > 0. \tag{13}$$

Substituting in (from (2)) for $g = \widetilde{\alpha}\sqrt{\frac{1+\left(1-\frac{2}{\pi}\right)\widetilde{\alpha}^2}{\left(2+\left(1-\frac{2}{\pi}\right)\widetilde{\alpha}^2\right)\left(2+\left(2-\frac{2}{\pi}\right)\widetilde{\alpha}^2\right)}}$, we need

$$y + \frac{4\pi^2\widetilde{\alpha}^2-7\pi\widetilde{\alpha}^4-8\pi\widetilde{\alpha}^2+2\pi^2\widetilde{\alpha}^4+2\pi^2+6\widetilde{\alpha}^4}{2\pi\left(2+\left(1-\frac{2}{\pi}\right)\widetilde{\alpha}^2\right)^2}\left(yH'\left(y\right)+H\left(y\right)\right) > 0.$$

Since the coefficient is clearly positive, if $yH'(y)+H(y)\geq 0$ the inequality is satisfied. If not then, since the coefficient is easily seen to be increasing in $\widetilde{\alpha}^2$, the worst case is when $\widetilde{\alpha}$ is at it highest. Since the upper limit of the coefficient is $\frac{2\pi^2-7\pi+6}{2\pi\left(1-\frac{2}{\pi}\right)^2}=\pi\frac{2\pi-3}{2\pi-4}$, it is sufficient to show that

$$y+\pi\frac{2\pi-3}{2\pi-4}\left(yH'(y)+H(y)\right)>0. \tag{14}$$

We first show, that $yH'(y)+H(y)$ is increasing for $y>\sqrt{3}$. Let $\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ and define $z(y)=1+\operatorname{erf}\left(\frac{y}{\sqrt{2}}\right)$. Note that

$$H'(y)=-\frac{e^{-\frac{1}{2}y^2}\left(2e^{-\frac{1}{2}y^2}+\sqrt{2}\sqrt{\pi}yz(y)\right)}{\pi z^2(y)},$$

and

$$\frac{d\left(H'(y)y\right)}{dy}=\frac{\sqrt{2}\pi ye^{-\frac{1}{2}y^2}\left(y^2-2\right)z^2(y)+2\sqrt{\pi}\left(3y^2-1\right)e^{-y^2}z(y)+4\sqrt{2}ye^{-\frac{3}{2}y^2}}{\pi^{\frac{3}{2}}z^3(y)}.$$

Then $H'(y)+\frac{d(H'(y)y)}{dy}$ becomes

$$\frac{\sqrt{2}\pi ye^{-\frac{1}{2}y^2}\left(y^2-3\right)z^2(y)+2\sqrt{\pi}\left(3y^2-2\right)e^{-y^2}z(y)+4\sqrt{2}ye^{-\frac{3}{2}y^2}}{\pi^{\frac{3}{2}}z^3(y)},$$

what is positive for $y>\sqrt{3}$. For $y\in[0,\sqrt{3}]$, we plot the left-hand side of (14) in Figure A1:

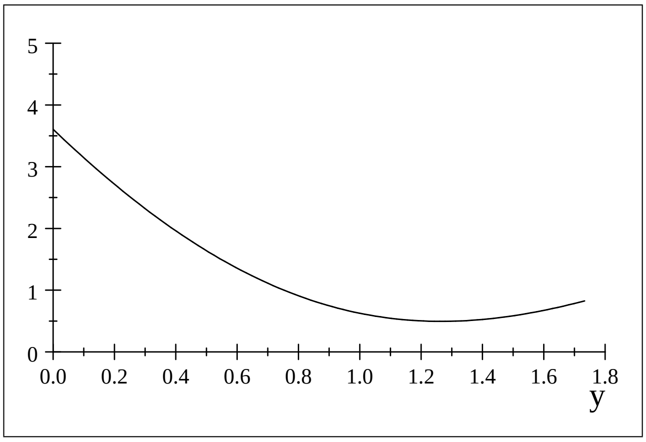

Figure A1: The left-hand side of (14)

The figure shows that the Lemma holds true for $y\in[0,\sqrt{3}]$, and since at $\sqrt{3}$ the expression is positive, by the above discussion it holds for $y>\sqrt{3}$ as well. ■

Proof of Lemma 3 . *The optimal price is a decreasing function of* $\widetilde{\alpha}$*:* $\frac{\partial p^*(\alpha,\widetilde{\alpha})}{\partial \widetilde{\alpha}} < 0$.

From (4) and Proposition 1

$$\frac{\partial \widehat{\theta}^*(p,\widetilde{\alpha})}{\partial p} = \left(\frac{dE_{\widetilde{\alpha}}\left[\theta | \widehat{\theta}\right]}{d\widehat{\theta}} \right)^{-1} \Bigg|_{\widehat{\theta}=\widehat{\theta}^*(p,\widetilde{\alpha})} = \left(\frac{1}{1+w^2(\widetilde{\alpha})} - g(\widetilde{\alpha})^2 H'\left(g(\widetilde{\alpha})\widehat{\theta}\right) \right)^{-1} \Bigg|_{\widehat{\theta}=\widehat{\theta}^*(p,\widetilde{\alpha})}.$$

Note that $H(x)$ is convex:

$$\begin{aligned} \frac{dH'(x)}{dx} &= -H'(x)(x+H(x)) - H(x)(1+H'(x)) \\ &= H(x)\left[(x+H(x))^2 + H(x)(x+H(x)) - H(x)\right] \end{aligned}$$

what is always positive. Since $w(\widetilde{\alpha}), g(\widetilde{\alpha})$ and $\widehat{\theta}^*(p,\widetilde{\alpha})$ also increase with $\widetilde{\alpha}$, we conclude that the price elasticity is increasing in $\widetilde{\alpha}$. Finally, since in monopoly $\epsilon = 1$ for all $\widetilde{\alpha}$, we conclude that p optimum is decreasing in $\widetilde{\alpha}$. ■

Proof of Lemma 4. $D(p;\alpha,\widetilde{\alpha})$ *is increasing in* α.

Letting $x = \sqrt{1+w^2(\alpha)} \in \left[\sqrt{2}, \sqrt{\frac{2(\pi-1)}{\pi-2}}\right]$, and taking derivatives with respect to it

$$\begin{aligned} \frac{dD(p;\alpha,\widetilde{\alpha})}{dx} &= 2\int_{\widehat{\theta}^*(p,\widetilde{\alpha})}^{\infty} \frac{1}{x}\phi\left(\frac{y}{x}\right)\frac{1}{x}\left[-1+\frac{y^2}{x^2}\right]\Phi(gy)\,dy \\ &\quad +2\int_{\widehat{\theta}^*(p,\widetilde{\alpha})}^{\infty} \frac{1}{x}\phi\left(\frac{y}{x}\right)\frac{dg}{dx}\phi(gy)\,dy. \end{aligned}$$

The second line is non-negative. To see this, we write g as a function of x:

$$g = \frac{\sqrt{\pi}\sqrt{x^2-1}\sqrt{x^2-2}}{x\sqrt{-2\pi+3\pi x^2-\pi x^4-2x^2+2x^4}},$$

the derivative of which is

$$\frac{\sqrt{\pi}}{x^2}\frac{(x^2-1)^{\frac{3}{2}}}{\sqrt{x^2-2}\,(3\pi x^2-2\pi-\pi x^4-2x^2+2x^4)^{\frac{3}{2}}}\left(4\pi-4\pi x^2+\pi x^4+8x^2-2x^4\right),$$

what is positive since, $x > \sqrt{2}$, the numerator is concave and at the extremes it is positive (otherwise g would not have been well defined), and

$$4\pi-4\pi x^2+\pi x^4+8x^2-2x^4 = 4\pi+(\pi-2)x^2\left(x^2-4\right) \geq 4\pi-4(\pi-2) = 8,$$

since $x^2(x^2-4)$ reaches its minimum at $x^2 = 2$.

The term in square brackets is positive for any value of y above x and negative otherwise. Thus, if $\theta^* > x$ then the first line is also positive. Otherwise, as $\Phi(gy)$ is increasing in y (since $g > 0$ and independent of y), the first line is more than

$$\frac{\Phi(gx)}{x^2} \int_0^\infty \phi\left(\frac{y}{x}\right)\left(-1 + \frac{y^2}{x^2}\right) dy = \frac{\Phi(gx)}{x\sqrt{2\pi}} \int_0^\infty e^{-.5z^2}\left(-1 + z^2\right) dz = 0.$$

Observing that $\frac{dx}{d\alpha} > 0$ completes the proof. ■

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