Private Credit: Risks and Benefits of a Maturity Wall*

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Abstract

A maturity wall occurs in private credit funds when the fund reaches its maturity date, where it can no longer roll over its loans. Unlike banks, which are not bound by a maturity wall, private credit funds can better incentivize borrowers, albeit at the cost of inefficient liquidation. Using a model, we show that private credit not only expands access to credit but also takes business away from banks. By stealing business, it removes riskier loans from banks' balance sheets. At the aggregate level, expected payoff increases but tail events become more severe due to the potential for excessive liquidation by private creditors.

Keywords: private credit, maturity wall, incentives, inefficient liquidation, bank loans, bank balance sheets, systemic risk

JEL Classification: G21, G23, G28

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I Introduction

Private credit has grown rapidly in recent years, from virtually non-existent in the early 2000's to roughly $1.6 trillion in assets under management globally in 2023. In its 2024 Global Financial Stability Report, the International Monetary Fund suggests the beneficiaries are firms that otherwise would be excluded from bank loans and cautions that the flow of credit to such firms may contribute to the amplification of negative shocks to the economy. We develop a model of private credit that allows us to study how private credit loans and bank loans interact. We focus on a key feature of private credit funds that they are closed-end funds with a limited lifetime,[1] and therefore are barred from renewing loans. This contrasts with banks that have long horizons and thus can roll over loans indefinitely if necessary. We use the model to address several questions. Does private credit extend loans that banks would not? Does private credit directly compete with banks? How are private credit loans different from bank loans? How does private credit affect the riskiness of bank balance sheets? Does private credit lending increase aggregate risk?

We develop a three-period model in which the borrower can choose between bank and private credit financing. The cash flow in the intermediate period is uncertain and depends on the borrower's effort choice. With bank financing, a low cash flow in the interim period can lead to liquidation or renegotiation. If the long-term cash flow is high enough, banks roll over the loan by writing off some of the debt, and thus allow the borrower to survive. In contrast, private credit funds cannot do this because of their limited lifetime. In the model, we capture the maturity wall by assuming private credit funds have to liquidate all their investments in the interim period, which prevents them from rolling over debt — we relax this assumption later. This is inefficient because some portion of the long-term cash flow is lost and puts

[1] Most private credit funds are structured as limited partnerships with 5 to 10 years of lifetime (Board of Governors of the Federal Reserve System, 2024). About 20% of the US market is organized in the form of business development corporations, most of which are perpetual (International Monetary Fund, 2024).

them at a disadvantage relative to bank financing.

There is a benefit to having a maturity wall. When taking loans from private credit funds, borrowers are fully aware that in case of bad interim performance the loan will not be rolled over and the project is terminated. This threat of termination à la Bolton and Scharfstein (1990) is made credible ex post by the limited lifetime of private credit funds and induces the borrower to exert high effort leading up to the interim period. Given that banks have long-term financing that extends beyond the interim period, they cannot commit to be tough ex post if the project is doing poorly. This creates the possibility that some borrowers benefit from the tough and uncompromising stance of private credit funds given that otherwise they cannot commit to exerting high effort. We thus show that private credit funds also have an advantage over banks.[2]

The model produces the following taxonomy of borrowers. Borrowers that are very profitable or have low agency costs will have no problem committing to high effort and getting bank financing; private credit would be too expensive for these borrowers. At the other extreme, borrowers with severe agency issues and weaker long-term profitability cannot get bank loans but instead may be able to get financing from private credit funds. This "democratization of credit" expands economic activity, but at a cost. We show that when shocks between borrowers are correlated, the presence of private credit generates large economic downturns because of the inefficient liquidation of the projects they finance. In general, these private credit loans have higher interest rates than high quality bank loans to compensate for larger losses in downturns.

There are also other borrowers facing severe agency issues that are profitable enough to be funded by a bank, but not profitable enough to be able to commit to high effort given the low powered incentives provided by banks. These loans are riskier bank loans due to the low effort of the borrower. Private credit funds, on the other hand, can incentivize high effort and make these loans safer, which results in

[2]See Ellias and de Fontenay (2025) for a discussion of other advantages and disadvantages of private credit over bank financing.

"business stealing". The borrowers that switch from bank funding to private credit funding face a lower probability of failure with private credit and thus have lower interest rates; they would face higher interest rates had they opted for a bank loan. As such, "business stealing" by private credit takes away riskier loans from banks' balance sheets and can have a stabilizing effect in the banking sector, at the cost of smaller bank balance sheets. However, in case of failure these private credit loans fare worse than they would have as bank loans due to the inefficient liquidation. Switching from bank funding to private credit funding increases the expected payoff of these borrowers at the cost of a larger drop in payoff in a tail aggregate event.

At the core of our results is the limited lifetime of the private credit fund; it is a commitment device for private credit funds not to roll over debt when the borrower is not performing well. This commitment acts as an ex-ante incentive for the borrower to exert high effort making loans by private credit funds possible when bank loans are not, and in some cases even leading to loans that are more attractive than bank loans, resulting in "democratization of credit" and "business stealing" , respectively. We show that the optimal contract involves probabilistic termination in case of low cash flow in order to balance the disciplining effect and the cost of liquidation. Since probabilistic termination is hard to commit to in practice, one can think of limited lifetime funds as a second-best solution. Thus our model highlights the value of finite lifetimes in funding vehicles more generally, and helps explain why finite lifetime funds are so pervasive, e.g., venture capital, private equity, and private credit.

We extend the model to the case in which private credit funds that are being wound down can sell their loans instead of liquidating the project. A new private credit fund or a bank can buy the loan, in effect rolling it over. Our main results hold if there are sufficient information frictions between the private credit fund being wound down and the potential new lender. High information frictions are likely to exist for borrowers who cannot produce intermediate cash flows as the potential new lenders would be suspicious of taking over the loans from such borrowers.[3] In another extension, we allow for a difference in financing costs between banks and private credit funds. We show that an increase in bank financing costs – e.g., due to more stringent bank capital regulation – increases the prevalence of private credit loans as banks retreat from financing some borrowers.

The model predicts that private credit funds a mix of borrowers; some that could not get a bank loan and others that would have been able to secure a bank loan but still prefer private credit due to its cheaper financing. Nevertheless, under reasonable assumptions about banks' loan portfolios, the model predicts average interest rates charged by banks are lower than those charged by private credit funds. Business development corporations are generally organized as perpetual entities and thus do not benefit from the commitment value of a maturity wall. Thus, they should be expected to behave more like banks and renegotiate loans and extend their maturity rather than liquidate them. These corporations, and their loan contracts, can serve as a control group when studying the benefits of a maturity wall. Our model is also broadly consistent with the observation that private credit funding started growing as banks' financing costs increased after the Global Financial Crisis. We discuss further empirical implications in the main text.

Our model implies that the emergence of private credit funds has implications for banks' loan portfolios and aggregate risk more generally. Due to business stealing, the emergence of private credit funds shrinks banks' balance sheets, but also moves some of larger potential loan losses away from banks' and into private credit funds. Consequently, banks look less risky across the business cycle with the introduction of private credit, despite the fact that in the model agents do not care about risk. The reduced risk in bank portfolios comes at the cost of lower profitability in aggregate expansions.

Due to democratization of credit and business stealing, lending activity from pri-

[3] Especially, given that private credit is associated with fewer disclosure requirements (Ellias and de Fontenay 2025) that can hurt the firm when it is time to roll over its debt.

vate credit funds increases expected aggregate output. This increase in aggregate output comes at the cost of more severe tail events due to inefficient liquidations in the private credit sector. Nonetheless, we show that the maturity wall that can lead to massive inefficient liquidation of private credit-funded borrowers is an essential part of private credit's business success. Thus, policy interventions should avoid saving borrowers ex post because this undermines the ex ante incentives provided via the limited lifetime of private credit funds.

Section II presents an overview of relevant literature. In the rest of the paper, we present our theoretical model and main results in Section III, and consider the optimal contract in Section IV. We discuss the assumptions and extensions in Section V, and empirical and policy implications in Section VI. Section VII concludes. Proofs not included in the text are relegated to the Mathematical Appendix.

II Related literature

Previous work has shown that the threat of termination can incentivize the borrower (Stiglitz and Weiss, 1983; Bolton and Scharfstein, 1990). In Berglöf and von Thadden (1994), the presence of short-term creditors strengthens the bargaining position of lenders in a model of non-verifiable income. Bolton and Scharfstein (1996) argues the number of creditors can be used to balance the benefit of incentives and the cost of inefficient liquidation. In Donaldson, Piacentino and Thakor (2021), non-banks' higher cost of funding allows them to credibly commit to terminate projects with low cash flows in order to induce borrower effort. Thus, their model implies private credit funding disappears as the difference in financing costs decreases, while our model implies the opposite. Their focus is on competition between banks and non-banks (including private equity funds), while our focus is on the financial stability and aggregate risk implications of the advance of private credit funds.

Chemmanur and Fulghieri (1994) shows banks have an incentive to be flexible in loan renegotiation. Indeed, Roberts and Sufi (2009) confirm that a majority of bank loans are renegotiated and their maturity extended; such practice can lead to evergreening and zombie lending (Hu and Varas, 2021; Faria-e-Castro, Paul and Sánchez, 2024). Our paper is also related to the literature on the costs and benefits of rollover risk (Calomiris and Kahn, 1991; Flannery, 1994; Acharya, Gale and Yorulmazer, 2011; He and Xiong, 2012).

There is a growing empirical literature that looks at private credit or non-bank lending more generally. Cai and Haque (2024) shows that the average maturity of private credit loans is five years; given the lifetime of the funds, this suggests the existence of a maturity wall. Block, Jang, Kaplan and Schulze (2024) presents a survey of private credit fund managers. They show that private credit is extended both to companies able to get a bank loan and to those that cannot get bank financing, implying both "democratization of credit" and "business stealing" are empirically relevant. Chernenko, Erel and Prilmeier (2022) shows that non-banks lend to riskier companies. The International Monetary Fund (2024) shows that interest rates charged on private credit loans are higher than for leveraged loans, and average annual credit losses are higher than for bank loans. Erel, Flanagan and Weisbach (2024) shows that private credit fund returns are equity-like and risk-adjusted do not outperform after fund management fees.

Chernenko, Ialenti and Scharfstein (2025) shows that business development companies are well capitalized, suggesting that their success is not due to regulatory arbitrage. Haque, Mayer and Stefanescu (2025) shows that many private credit borrowers also borrow from traditional banks. Boyarchenko and Elias (2024) argues that non-bank financing depends on the supply of bank financing. Acharya, Cetorelli and Tuckman (2024) is the only paper to deal with the financial stability implications of private credit expansion and argues that banks and non-banks risks are interwoven.

There is also a literature on the structure of private capital funds and the interactions between private equity and private credit. Previous work on private equity fund structure focused on pre-commitment of capital and capital calls (Axelson, Strömberg and Weisbach, 2009; Maurin, Robinson and Strömberg, 2023), while our focus is on the limited lifetime of the funds. Consistent with our paper, Kastiel and Nili (2024) argue that continuation funds in private equity are fraught with conflicts of interest and thus not popular among investors. In some cases, the borrower has private equity and private credit from the same sponsor firm (Buchner, Lopez-de Silanes and Schwienbacher, 2022; Haque, Jang and Mayer, 2023; Davydiuk, Erel, Jiang and Marchuk, 2024). Jang (2024) shows that private credit funds are more likely to renegotiate their loans (during the fund's lifetime) than banks, but usually only if the private equity fund that owns the firm injects equity.

III Model

We first outline the model and then solve it for a single borrower; first, with only bank financing available and then when private credit financing is also possible. We then discuss the interaction between private credit and bank financing in case of many different types of borrowers.

III.1 Model setup

There are three time periods $t = 0, 1, 2$. The timeline of the model is depicted in Figure 1. There is a borrower endowed with a project at $t = 0$ that requires one unit of investment. This initial investment must be borrowed from either a bank or a private credit fund. All agents are risk-neutral and there is no discounting. Both banks and private credit funds have sufficient resources and operate in competitive markets and thus break even. The only difference between banks and private credit funds is that banks live until period $t = 2$ while private credit funds have a limited lifetime and exit at $t = 1$. The borrower maximizes the payout from the project, and

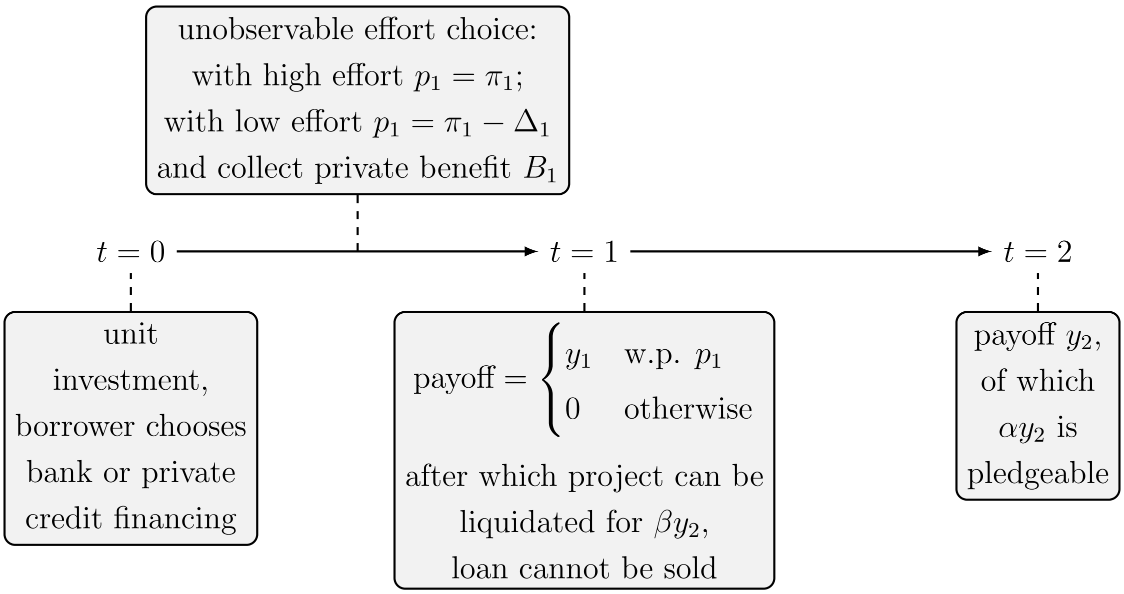

Figure 1: Timeline of the model.

has an outside option worth 0 that can be taken at any time.

With probability p_1 the project pays off y_1 at $t = 1$ and 0 otherwise.[4] At $t = 2$, the project pays off y_2 for sure. While y_1 is fully pledgeable, only a fraction $\alpha \in (0, 1)$ of y_2 is pledgeable. As in Holmström and Tirole (1998), moral hazard affects output at $t = 1$ as the borrower's effort is unobservable, hence non-contractable. The probability of success is $p_1 = \pi_1$ if the borrower exerts effort. If the borrower does not exert effort (i.e., shirks), the probability of success drops to $p_1 = \pi_1 - \Delta_1$, but the borrower receives private benefit B_1. The project can be liquidated at $t = 1$ for $\beta y_2 \geq 0$.[5]

We make the following parametric assumptions.

Assumption 1. $\pi_1 y_1 + (1 - \pi_1)\beta y_2 > 1$

Assumption 2. $\Delta_1 y_1 > B_1$

Assumption 3. $0 < \beta < \alpha$

[4]The cash flow y_1 does not necessarily have to come from the project, it may represent cash flow from refinancing e.g., with a bank loan or a newly established private credit fund. We explore this possibility in Section V.

[5]This means any liquidation procedure that does not leave the borrower in place to collect future income.

Assumption 1 ensures that the project can pay off enough in first period to be financed with a one period loan. Assumption 2 means shirking is inefficient, and Assumption 3 states that liquidation is inefficient.

III.2 Model solution with bank financing only

In this section, we solve for the subgame perfect equilibrium of the model with bank financing only. Denote the promised, combined repayments by the borrower at $t = 1$ and $t = 2$ as R_B, where the subscript 'B' indicates bank financing. R_B is determined in equilibrium. Note that any repayment R_B exceeding $y_1 + \alpha y_2$ cannot be enforced and thus it is always weakly dominant for the lender to demand repayment

$$R_B \leq y_1 + \alpha y_2. \tag{1}$$

We refer to this as the contract feasibility constraint.

In case of success at time $t = 1$ at most y_1 can be repaid of the loan and the additional repayment promised for $t = 2$ is thus $R_{B,s} = \max(0, R_B - y_1)$, with the subscript 's' denoting success. In case of failure, which we denote with the subscript 'f', nothing can be repaid at $t = 1$ since there are no cash flows available. In this case, the loan might have to be renegotiated and the maximum the borrower can pledge to repay at $t = 2$ is αy_2. This yields promised repayment of $R_{B,f} = \min(\alpha y_2, R_B)$ for $t = 2$. Note that if $\alpha y_2 < R_B$ part of the loan is forgiven and maturity is extended if the project fails to produce any cash flow at $t = 1$.

We solve the model backwards. First we analyze the bank's refinancing decision at $t = 1$ in case the borrower cannot fully repay its loan. Note that if the bank liquidates the project it gets βy_2 while if it allows the project to continue, the maximum it can get from the borrower at $t = 2$ is αy_2. Since $\beta < \alpha$ by Assumption 3, it is never optimal for the bank to shut down the project ex post at $t = 1$. Thus the following Lemma holds.

Lemma 1. Bank rollover decision. *Even if the borrower still owes the bank after the first period cash flow, the bank never liquidates the project, and forgives any remaining loan repayment exceeding the pledgeable income αy_2.*

Now we turn to the loan contract at $t = 0$. Consider first the borrower's problem. Since $R_B \leq y_1 + \alpha y_2$, the loan is fully repaid by $t = 2$ if the project is successful at $t = 1$, and the borrower's payoff is $y_1 + y_2 - R_B \geq 0$. If the project is fails (i.e., it yields no cash flow in the intermediate period), it is not liquidated and yields payoff of $y_2 - R_{B,f} \geq 0$ to the borrower, where $R_{B,f}$ is the repayment due after renegotiation. A feasible contract thus ensures the participation constraint of the borrower is met at every possible state. The contract may either incentivize the borrower to exert high effort or not. The borrower exerts high effort if and only if the incentive constraint (rewritten here in a simplified way) is met

$$y_1 - R_B + R_{B,f} \geq \frac{B_1}{\Delta_1}, \tag{2}$$

otherwise the borrower optimally shirks.

We now turn to the bank's problem. Banks are competitive and break even in expectation,

$$p_1 R_B + (1 - p_1) R_{B,f} = 1. \tag{3}$$

From this break-even condition the interest rate charged by the bank is

$$R_B = \begin{cases} 1 & \text{if } \alpha y_2 \geq 1 \\ \frac{1-(1-p_1)\alpha y_2}{p_1} & \text{if } \alpha y_2 < 1 \end{cases} \tag{4}$$

so that $R_B > 1$ whenever $\alpha y_2 < 1$.

In a subgame perfect equilibrium, the bank offers the borrower a feasible contract $\{R_B, R_{B,s}, R_{B,f}\}$ at time $t = 0$ that satisfies (1) and guarantees financing of one

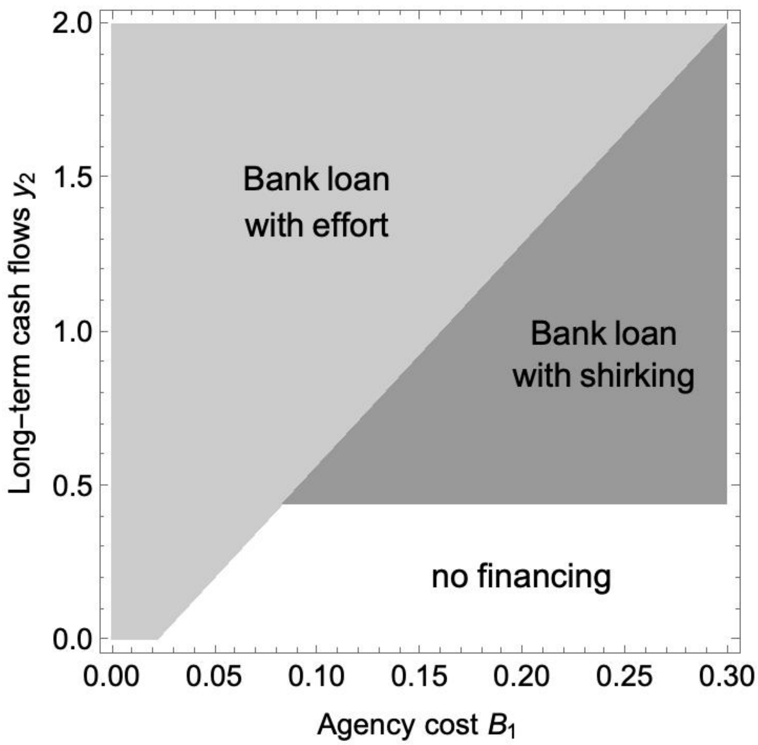

Figure 2: Financing regions with bank financing only.

Notes. The y-axis depicts long-term project cash flows y_2 and the x-axis depicts agency cost B_1. Parameters: $\Delta_1 = 0.25, \pi_1 = 0.9, y_1 = 1.2, \beta = 0.3, \alpha = 0.5$. The region of $y_2 \geq \frac{1}{\alpha}$, in which bank loans are riskless, is excluded from the graph.

unit of investment. Given the contract, the borrower exerts effort if and only if the incentive constraint (2) holds and otherwise shirks. The bank breaks even in expectation satisfying equation (3) conditional on the borrower's effort choice. We formalize the equilibrium solution in a proposition below and use Figure 2 to illustrate the financing contracts offered in equilibrium for different parameter values.

Figure 2 shows the different financing regions in the (B_1, y_2)-space for a set of parameter values. In the upper left area of the graph where long-run cash-flows y_2 are high relative to agency B_1, the bank finances the project, the borrower exerts effort, and the loan has low risk. For moderate values of agency B_1 relative to long-run cash-flows y_2, the bank finances the project, the borrower shirks, and the loan is high risk. If the agency B_1 is sufficiently high relative to long-run cash-flows y_2, then there is no feasible contract that guarantees that the bank breaks even and the project cannot be financed by the bank. Since it is always optimal for the bank to renegotiate the contract at time $t = 1$, the bank cannot incentivize the borrower to exert effort and thus the project is not profitable enough.

We now formalize the equilibrium solution.

Proposition 1. Bank financing only.

Denote the following boundaries on long-term cash flows:

a) $y_2 \geq \bar{y}_2^{B,I} = \frac{1-\pi_1\left(y_1-\frac{B_1}{\Delta_1}\right)}{\alpha}$ *induces borrower effort;*

b) $y_2 \geq \bar{y}_2^{B,P} = \frac{1-(\pi_1-\Delta_1)y_1}{\alpha}$ *so contract is feasible allowing the bank to break even if the borrower shirks.*

When only bank financing is available, the projects is financed with:

i) Risk-free loan if $y_2 \geq \frac{1}{\alpha}$*: the borrower exerts effort and the project is financed by a riskless loan with promised repayment* $R_B = 1$*;*

ii) Low-risk loan if $y_2 \in \left[\bar{y}_2^{B,I}, \frac{1}{\alpha}\right)$*: the borrower exerts effort and the project is financed by a low risk loan with promised repayment* $R_B = \frac{1-(1-\pi_1)\alpha y_2}{\pi_1}$ *that is renegotiated with probability* $1-\pi_1$ *in the interim period;*

iii) High-risk loan if $y_2 \in \left[\bar{y}_2^{B,P}, \bar{y}_2^{B,I}\right)$*: the borrower shirks and the project is financed by a high risk loan with promised repayment* $R_B = \frac{1-(1-\pi_1+\Delta_1)\alpha y_2}{\pi_1-\Delta_1}$ *that is renegotiated with probability* $1-\pi_1+\Delta_1$ *in the interim period.*

If $y_2 < \min\left(\bar{y}_2^{B,I}, \bar{y}_2^{B,P}\right)$*, the project is not financed by the bank.*

III.3 Model solution with bank and private credit financing

This section introduces private credit funding. Denote the original repayment agreed upon to the private credit fund as R_{PC}. Private credit funds only care about getting as much repayment as possible in the interim period, without any concern for foregone long-run cash flows. Thus, a feasible private credit contract satisfies

$$R_{PC} \leq y_1 + \beta y_2. \tag{5}$$

Again, we solve the model backwards, starting with the interim period. Private credit funds exit in the interim period and thus have to liquidate the project if it does not yield enough cash-flow by then. This leads to the following lemma.

Lemma 2. Private credit liquidation decision. *If the borrower still owes the private credit fund after the first period cash flow, then the private credit fund liquidates the project.*

Turn now to the contract terms at time $t = 0$. Consider the problem of the borrower. If the project is successful and yields enough cash flow to pay the lender back, $R_{PC} \leq y_1$ (which we show is always the case in equilibrium), the borrower is not liquidated and gets $y_1 - R_{PC} + y_2 \geq 0$ net of the payment to the private creditor. If the project fails, it is liquidated and the borrower gets 0. A feasible contract by a private credit fund thus guarantees the participation constraint of the borrower holds at every state.

To determine the borrower's incentive compatibility constraint, note that the borrower exerts effort with private credit financing if and only if the incentive constraint (rewritten here in a simplified way) is met

$$y_1 - R_{PC} + y_2 \geq \frac{B_1}{\Delta_1}, \tag{6}$$

otherwise the borrower shirks.

Private credit funds are competitive and break even in expectation

$$p_1 R_{PC} + (1 - p_1)\beta y_2 = 1. \tag{7}$$

The break even condition yields the interest rate charged by the private credit fund

$$R_{PC} = \frac{1 - (1 - p_1)\beta y_2}{p_1}. \tag{8}$$

When the borrower exerts high effort, the above (8) combined with Assumption 1 ensures that $R_{PC} \leq y_1$ in equilibrium. We now show that high effort is always optimal if private credit financing is chosen in equilibrium.

In a subgame perfect equilibrium, at time $t = 0$, bank offers a feasible contract $\{R_B, R_{B,s}, R_{B,f}\}$ that satisfies (1), and private credit offers a feasible contract $\{R_{PC}, \mathbb{1}_{Liq}\}$ (where $\mathbb{1}_{Liq}$ denotes the liquidation decision) that satisfies (5), and each contract guarantees financing of one unit of investment. Under the bank contract, the borrower exerts effort if and only if the incentive constraint (2) holds and otherwise shirks; while under the private credit contract, the borrower exerts effort if and only if the incentive constraint (6) holds and otherwise shirks. Bank and private credit funds break even in expectation, given the borrower's effort choice in each case.

When both bank and private credit financing are available, the borrower may choose to borrow from one or the other. If the borrower chooses the same effort with bank and private credit financing, then bank financing is preferred by the borrower since it avoids wasteful liquidation, the cost of which is eventually borne by the borrower through higher interest rates. This yields the following Lemma.

Lemma 3. *Private credit financing in equilibrium always has to induce high effort, otherwise it would not be chosen by the borrower.*

Thus, the only case in which the choice of the borrower is not obvious is if it has to choose between a high-risk bank loan that does not induce effort and a private credit loan that does. In that case, the expected payoff to the borrower if it chooses the bank loan is $B_1 + (\pi_1 - \Delta_1)y_1 + y_2 - 1$ since the borrower shirks and receives B_1, the project continues until the end, and the lender is paid back 1 in expectation. If instead the borrower chooses private credit, then the borrower's expected payoff is $\pi_1(y_1 + y_2) + (1 - \pi_1)\beta y_2 - 1$, since the borrower exerts effort but the project is only continued in good times and liquidated in bad times while the lender receives 1 in expectation. In summary, private credit is preferred to bank financing if and only if

$$y_2 \leq \frac{\Delta_1 y_1 - B_1}{(1 - \pi_1)(1 - \beta)}. \tag{9}$$

The ability to attract a borrower to private credit depends on how large long-term cash flows are relative to the size of the moral hazard distortion. Larger long-term cash flows make bank financing more appealing, whereas larger agency distortions make private credit financing more appealing.

Before formally stating the equilibrium solution in a proposition, we use Figure 3 to illustrate the financing regions that arise in equilibrium in (B_1, y_2)-space for different parameter values. There are five regions in general: First, the top left indicates projects that are always financed by banks with high effort due to high y_2 even when private credit financing is available. Second, in the bottom right area no financing is available from either source due to high agency B_1 and low payoffs y_2. Third, the triangle on the bottom, nested between the two areas above, is the "democratization of credit" , projects that could not get bank financing due to low y_2 and low effort, but can get private credit financing because the threat of ex-post liquidation provides an ex-ante incentive to work hard. Fourth, the area in the middle identifies the "business stealing" projects that can be bank-financed with high-risk loans, but where private credit financing becomes more advantageous to the borrower because of the incentives to exert effort that it entails. Fifth, the right-most area, above the no-financing area, are projects that are always bank financed – even if

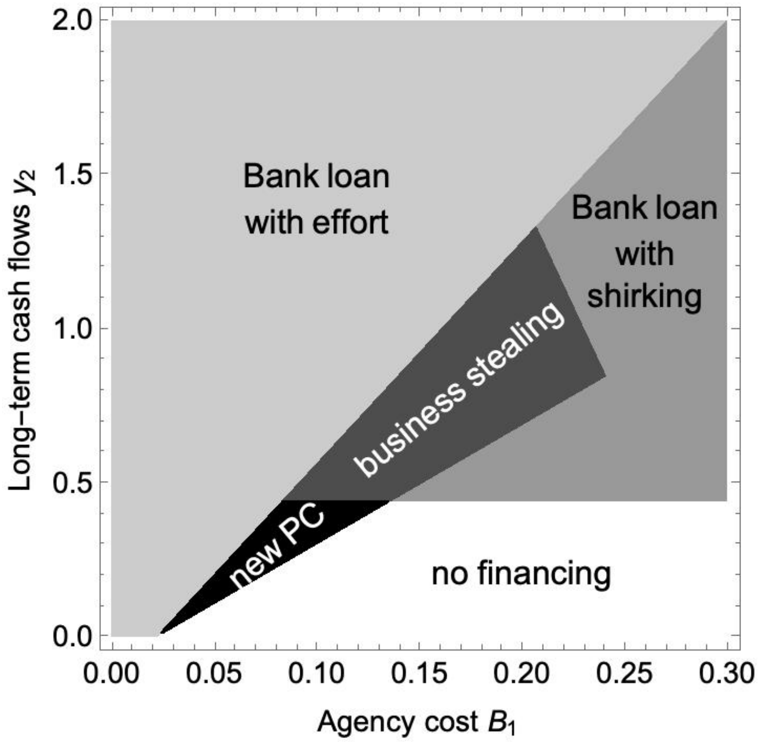

Figure 3: Financing regions if both bank and private credit are available.

Notes. The y-axis depicts long-term project cash flows y_2 and the x-axis depicts agency cost B_1. Parameters: $\Delta_1 = 0.25, \pi_1 = 0.9, y_1 = 1.2, \beta = 0.3, \alpha = 0.5$. The region of $y_2 \geq \frac{1}{\alpha}$, in which bank loans are riskless, is excluded from the graph.

private credit is available – but no effort is exerted. Proposition 2 presents the details.

Proposition 2. Bank and private credit financing.

Denote the following boundaries:

a) $y_2 \geq \bar{y}_2^{PC,I} = \frac{1-\pi_1\left(y_1-\frac{B_1}{\Delta_1}\right)}{\pi_1+(1-\pi_1)\beta}$ *which induces effort with PC financing*

b) $y_2 \geq \bar{y}_2^{switch} = \frac{\Delta_1 y_1 - B_1}{(1-\pi_1)(1-\beta)}$ *for which borrower prefers high-risk bank financing over private credit financing.*

When both bank and private credit financing are available, the project is financed with:

i) *Risk-free bank loan* *if* $y_2 \geq \frac{1}{\alpha}$*: the borrower exerts effort and the project is financed by a riskless bank loan with promised repayment* $R_B = 1$*;*

ii) *Low-risk bank loan if* $y_2 \in \left[\bar{y}_2^{B,I}, \frac{1}{\alpha}\right)$*: the borrower exerts effort and the project is financed by a low-risk bank loan with promised repayment* $R_B = \frac{1-(1-\pi_1)\alpha y_2}{\pi_1}$ *that is renegotiated with probability* $1-\pi_1$ *in the interim period;*

iii) *High-risk bank loan if* $y_2 \in \left[\bar{y}_2^{B,P}, \bar{y}_2^{PC,I}\right)$ *or* $y_2 \in \left[\max\left(\bar{y}_2^{B,P}, \bar{y}_2^{switch}\right), \bar{y}_2^{B,I}\right)$*: the borrower shirks and the project is financed by a high-risk bank loan with promised repayment* $R_B = \frac{1-(1-\pi_1+\Delta_1)\alpha y_2}{\pi_1-\Delta_1}$ *that is renegotiated with probability* $1-\pi_1+\Delta_1$ *in the interim period;*

iv) *Low-risk private credit loan if* $y_2 \geq \bar{y}_2^{PC,I}$ *and* $y_2 < \min\left(\bar{y}_2^{B,I}, \bar{y}_2^{switch}\right)$*: the borrower exerts effort and the project is financed by a low-risk private credit loan with promised repayment* $R_{PC} = \frac{1-(1-\pi_1)\beta y_2}{\pi_1}$*, and is liquidated with probability* $1-\pi_1$.

If $y_2 < \min\left(\bar{y}_2^{PC,I}, \bar{y}_2^{B,P}\right)$*, the project is not financed neither by the bank, nor by the private credit fund.*

Case iv in the proposition identifies the projects where private credit financing is chosen by the borrower. It consist of two distinct parts. We refer to the upper part ($\bar{y}_2^{PC,I} \leq y_2$) as the "business stealing" region. The projects in this region could be financed by banks, but bank financing cannot provide the incentives for the borrower to exert high effort. Hence, if financed by bank loans, these projects would face a high probability of becoming non-performing and of having to be renegotiated. In contrast, by taking private credit financing, the borrower exerts effort thus increasing the probability of success. We refer to the lower part ($y_2 < \bar{y}_2^{B,P}$) of case iv as the "democratization of credit" . These are projects that private credit finances but banks would not be willing to finance. Banks cannot break even with these projects, but private credit funds break even because their financing provides incentives for the borrower to exert effort.

Although Figure 3 shows that all the above regions exist for certain parameters, Proposition 2 does not establish the conditions under which private credit financing emerges. The following Lemma gives a necessary and sufficient condition under which private credit financing emerges in equilibrium.

Lemma 4. Existence of private credit financing. *There exist projects with some* (B_1, y_2) *parameters that are financed by private credit if and only if* $\alpha < \pi_1 + \beta(1 - \pi_1)$.

Since by assumption $\pi_1 + \beta(1 - \pi_1) < 1$, the above lemma highlights that at least some of the long-term payoff must be non-pledgeable in order for private credit to emerge. Thus, unless almost all long-term payoff y_2 is pledgeable (α sufficiently to one), private credit emerges in equilibrium. Since π_1 is the probability of success of low-risk projects (i.e., with high effort), it is reasonable to think of π_1 as being close to one. Accordingly, the above lemma states that $\alpha < \pi_1$ ensures the emergence of private credit, thus if π_1 is close to one, the above lemma is not very restrictive on the set of α's for which private credit emerges.

III.4 Consequences of introducing private credit financing

In this section, we compare the equilibrium with only bank financing to the equilibrium with both bank and private credit financing. Up to now, we concentrated on one single borrower. From here on, we assume there are a continuum of borrowers distributed over the admissible parameter range.

First, we analyze the effect of business stealing. If there is a non-zero mass of projects to be financed in the "business stealing" region, then the introduction of private credit funds decreases the amount of loans made by banks, decreasing bank balance sheet. "Business stealing" involves bank loans for which the borrower does not exert effort with bank financing but does so with private credit financing. Thus, "business stealing" decreases the expected prevalence of non-performing loans (that need to be renegotiated) on bank balance sheet.[6] At the same time, the expected

[6]This statement is only true in a weak sense, since if the entire mass of borrowers is composed

aggregate payoff of projects increases due to "business stealing" as the projects have a higher expected payoff under private credit financing. In addition, if the "democratization of credit" region has a non-zero measure of borrowers in equilibrium, then expected aggregate payoff increases also due to more projects being financed. We formalize the above insights in the following proposition, which holds for any distribution of borrowers in the admissible range of parameters.

Proposition 3. Bank balance sheet and aggregate output. *Compared with the equilibrium with only bank finance, the introduction of private credit (weakly):*

i) decreases bank balance sheets;

ii) decreases the expected share of non-performing loans on bank balance sheets;

iii) increases expected aggregate payoff.

Statement ii in the proposition has a direct transposition in terms of interest rates; the introduction of private credit funding decreases the expected share of (low effort) high-interest rate loans on bank balance sheets. However, for a general distribution of borrowers, we cannot say whether interest rates charged by banks on average decrease with the introduction of private credit. This is because banks that lend primarily to low quality borrowers may find themselves with a reduced pool of borrowers composed with the worst of the borrowers (lower y_2) after losing business to private credit. We formalize this insight later in Proposition 4.

Two properties are worth emphasizing. First, with the introduction of private credit, banks may offer a larger fraction of their loans at lower rates, this is not a consequence of the need for banks to remain competitive, but rather that private credit takes lower quality business (where bank interest rates are higher) away from banks.

of projects for which no effort is exerted, then the expected likelihood of non-performing loans does not change with the introduction of private credit financing.

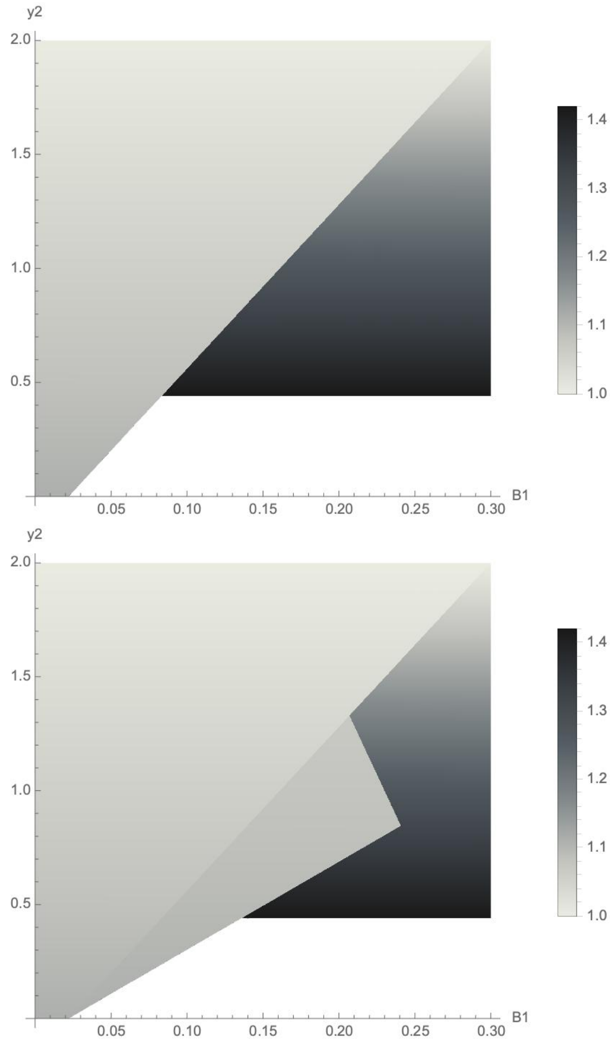

Figure 4: Equilibrium promised loan repayment

Notes. Top panel: only bank financing. Bottom panel: with both bank and private credit financing. The y-axis depicts long-term project cash flows y_2 and the x-axis depicts agency cost B_1. Parameters: $\Delta_1 = 0.25, \pi_1 = 0.9, y_1 = 1.2, \beta = 0.3, \alpha = 0.5$. The region of $y_2 \geq \frac{1}{\alpha}$, in which bank loans are riskless, is excluded from the graph.

Second, it is not possible to sign the difference in average interest rates between bank and private credit financing. To see this later point, consider Figure 4. The figure shows the promised repayments (interest rates) for the equilibrium loans with and without private credit financing, respectively the bottom and top panels, using the same parameters as in Figure 3. The bottom panel shows the wide dispersion of bank interest rates relative to interest rates charged by private credit funds. Thus, depending on the distribution of borrowers in the parameter space, the difference in average interest rates in bank and private credit financing can be positive or negative.

III.5 Aggregate shocks

To analyze aggregate risk and payoffs in different states of the world, we introduce a simple structure of correlation between individual project outcomes indicated in Assumption 4 below. We assume projects are fully correlated, but the main insights generally hold for a positive correlation between project outcomes. To further simplify the analysis, we assume borrowers are distributed in the parameter space over a line such that only long-term cash flow y_2 varies and the distribution of borrowers along y_2 cuts through the most relevant regions in Figure 3.

Assumption 4. Fully correlated projects. *Fix all parameters of the projects expect for y_2. Define three aggregate states in which project success does not directly depend on the choice of financing but only on effort choice:*

i) Expansion state happens with probability $\pi_1 - \Delta_1$ and all projects succeed;

ii) Intermediate recession/expansion state happens with probability Δ_1 and all projects with high effort succeed while all projects with low effort fail;

iii) Severe tail event state happens with probability $1 - \pi_1$ and all projects fail.

The above assumption simplifies the analysis while capturing realistic features of business cycles, where expansions are characterized by high productivity across a large fraction of the economy, whereas recessions, and especially large left-tail events, are characterized by low productivity across most sectors of activity. The assumption also allows for intermediate recessions/expansions where borrowers exerting high effort succeed but those that shirk fail.

We discuss aggregate risk and payoffs in expansions and recessions. Note that with the assumption of risk-neutral banks and private credit funds, whether shocks are aggregate or idiosyncratic does not affect the terms of the contract that these lenders give to borrowers. Again, we compare the equilibrium in which private credit is also available with the one in which it is not.

As in Subsection III.4, the implications of introducing private credit financing for aggregate payoffs in different states of the world depend on the distribution of borrowers, i.e., the distribution of the types of loans held by banks before the introduction of private credit). If banks hold only loans with high long-term payoffs y_2 where the borrower exerts effort, $y_2 \geq \bar{y}_2^{B,I}$, introducing private credit has no effect on the financial system as neither business stealing, nor "democratization of credit" arises.

However, if prior to the introduction of private credit funds are also willing to lend to lower quality borrowers, $y_2 \geq \bar{y}_2^{PC,I}$, banks' balance sheet improves as higher risk bank loans move to private credit funding when it becomes available. Since "business stealing" moves high-interest loans from the banking sector to private credit funds, promised interest payments to banks decrease. But promised interest payments materialize in the good aggregate state, which implies that banks' payoff, and thus profit, in expansions decreases once private credit is available to borrowers. At the same time, since the borrowers that are poached from banks are the ones that are less profitable –with low y_2 and thus low liquidation value– in a bad state the average value of non-performing bank loans is higher. Thus, the lower bank profits during aggregate upturns are offset by smaller bank losses during aggregate downturns in the equilibrium with private credit compared to the equilibrium with bank financing only. The downside is that private credit funds now hold all of these loans, and being subject to a maturity wall, they cause excessive liquidation in a severe tail event. In summary, private credit has a positive effect on the economy by reducing the risk of banks that have lower quality loans in their portfolios that are most or all poached by private credit funds by moving aggregate loan losses to the private credit sector and reducing loan losses in the banking sector, but has a negative effect on the economy by forcing excessive liquidation in large tail events.

A somewhat counterintuitive result arises for banks that have a large concentration of high-risk loans (i.e., $y_2 \leq \bar{y}_2^{B,I}$) in their portfolios and charge high interest rates. When borrowers are mostly lower quality, with lower long-term payoffs y_2, private credit funds steal the better part of these worse loans and leave the rest with banks. This means that these banks' loan losses (as a percentage of their assets) increase after the introduction of private credit. We formalize the above results in Proposition 4.

Proposition 4. Aggregate risk. *The introduction of private credit:*

i) Has no effect on banks if all projects have very high long-term payoffs $y_2 \geq \bar{y}_2^{B,I}$*;*

ii) If all projects have relatively high long-term payoffs $y_2 \geq \bar{y}_2^{PC,I}$*, then it (weakly)*

- *a) decreases average interest rate charged on bank loans, and the average interest on bank loans is lower than that charged by private credit;*
- *b) decreases banks' returns on loans in an expansion, but also decreases banks' loan losses in a severe tail event;*
- *c) increases aggregate loan losses in a severe tail event, with losses (weakly) concentrated in private credit loans.*

iii) If all projects have very low long-term payoffs $y_2 \leq \bar{y}_2^{B,I}$*, then it (weakly)*

- *a) increases average interest rate charged on bank loans*
- *b) increases banks' returns on loans in an expansion, but also increases banks' loan losses in a severe tail event;*
- *c) increases aggregate loan losses in a severe tail event.*

Table 1 shows a numerical example with two sets of parameters both corresponding to case ii in the proposition above, which we believe is the most relevant case. The examples use a uniform distribution of projects over y_2. The top panel sets the agency problem to moderately high $B_1 = 0.1$. In this example banks' balance sheet is 1.56 with bank financing only, and 0.44 of borrowers do not get any financing. Banks' balance sheet drops to 1.44 with the introduction of private credit, and overall financing reaches 1.70 borrowers (1.44+0.26). With the introduction of private credit, banks' loan losses in a severe tail event are mitigated to −36%, from −39% when only bank financing is available. Overall private credit funds have much better returns (profits) in expansions, 9.7% compared to banks' 4%. However, the maturity wall is felt in the tail event where private credit does much worse with loan losses up to −87% compared to banks' −36%. Overall, the aggregate payoff of all projects increases from 3.78 to 3.99 in an expansion due to private credit introduction and this is large enough to just about offset the higher loan losses in the tail event and the aggregate payoff is virtually unchanged dropping from 1.903 to 1.899.

In the bottom panel, we look at the case where the agency problem is very high $B_1 = 0.2$. This means that many high-risk loans remain on the balance sheet of banks even in the presence of private credit. It also means that relatively more projects end up with low effort so that the intermediate aggregate state becomes a recession. In this case, while private credit introduction mitigates banks' loan losses in the tail event from −39% to −32%, and especially in the intermediate recession state from −30% to −17%, in the expansion state banks charge higher interest rates of 11% than private credit funds, which charge 7.8%. In this parametrization banks are more profitable than private credit funds in expansions, even though private credit funds fare much worse in a tail event losing −70% of their loan value. Here, all improvement in aggregate payoffs comes from moderate recessions becoming less severe due to high incentives provided by private credit funds in loans stolen from banks.

$B_1 = 0.1$	Bank only			Private Credit and Bank		
	expansion	recession	tail	expansion	recession	tail
Bank balance sheet	1.56			1.44		
Bank loan return	6.8%	−2.1%	−39%	4%	4%	−36%
Private credit balance sheet	-			0.26		
Private credit loan return	-	-	-	9.7%	9.7%	−87%
Aggregate payoff	3.78	3.63	1.903	3.99	3.99	1.899

$B_1 = 0.2$	Bank only			Private Credit and Bank		
	expansion	recession	tail	expansion	recession	tail
Bank balance sheet	1.56			0.97		
Bank loan return	17%	−30%	−39%	11%	−17%	−32%
Private credit balance sheet	-			0.59		
Private credit loan return	-	-	-	7.8%	7.8%	−70%
Aggregate payoff	3.78	2.77	1.903	3.78	3.48	1.61

Table 1: Outcomes by states with fully correlated projects.

Notes. Top panel, $B_1 = 0.1$, bottom panel $B_1 = 0.2$. A meausure 2 of borrowers are uniformly distributed over $y_2 \in [0, 2]$. Other parameters: $\Delta_1 = 0.25$, $\pi_1 = 0.9$, $y_1 = 1.2$, $\beta = 0.3$, $\alpha = 0.5$.

IV Optimal contract

Up to now, we consider two types of loans: bank financing, which is long-term and cannot commit to termination of the project, and private credit financing, which is short-term and always terminates the project in case of low cash flow in the intermediate period. In this section, we explore the optimal financing contract offered by a lender that can commit to any repayment plan.[7]

Assume the project payoffs and agency issues are the same as in Section III. In designing the optimal contract, the lender can, at $t = 0$, commit to a liquidation probability at time $t = 1$ that depends on the realized cash flow à la Bolton and

[7]A standard result is that the optimal contract has a convertibility component; the lender takes over the firm to capture the long-term cash flows instead of liquidating the firm. However, in our model, the lack of pledgeability of y_2 is akin to the inalienability of long-term cash flows from the borrower.

Scharfstein (1990). Denote the liquidation probability in case of low cash flow at $t = 1$ as $\gamma \in [0, 1]$.[8] Also denote total repayment as R_h in case of high intermediate cash flow (y_1) and R_l in case of low intermediate cash flow (0) in case the project is continued. In case of liquidation, we assume the lender gets all cash flow βy_2, as this is weakly optimal. The optimal contract prescribes the state-contingent repayment $\{R_h, R_l, \gamma\}$ and an effort level. The latter is equivalent to choosing $p_1 \in \{\pi_1 - \Delta_1, \pi_1\}$ where the choice of $p_1 = \pi_1$ corresponds to high effort. The borrower's incentive constraint becomes:

$$\begin{aligned} &\pi_1(y_1 + y_2 - R_h) + (1 - \pi_1)(1 - \gamma)(y_2 - R_l) \geq \\ &\qquad (\pi_1 - \Delta_1)(y_1 + y_2 - R_h) + (1 - \pi_1 + \Delta_1)(1 - \gamma)(y_2 - R_l) + B_1. \end{aligned} \tag{10}$$

Since lenders are competitive, the optimal contract maximizes the expected payoff of the borrower subject to the lender's break-even condition. The other constraints are the feasibility of state contingent repayment, and the borrower's incentive constraint if high effort is chosen (rewriting equation (10)). Formally,

$$\max_{R_h, R_l, \gamma, p_1} p_1(y_1 + y_2) + (1 - p_1)\{(1 - \gamma)y_2 + \gamma\beta y_2\} - 1 + \mathbb{1}_{p_1 = \pi_1 - \Delta_1} B_1, \tag{11}$$

subject to:

$$p_1 R_h + (1 - p_1)\{\gamma\beta y_2 + (1 - \gamma)R_l\} = 1,$$

$$R_h \leq y_1 + \alpha y_2,$$

$$R_l \leq \alpha y_2,$$

$$\mathbb{1}_{p_1 = \pi_1} \left(y_1 + \gamma y_2 - R_h + (1 - \gamma)R_l - \frac{B_1}{\Delta_1} \right) \geq 0.$$

[8] It is obvious that the lender does not want to liquidate in case of high cash flows at $t = 1$ since that decreases expected payoffs and only tightens the incentive constraint.

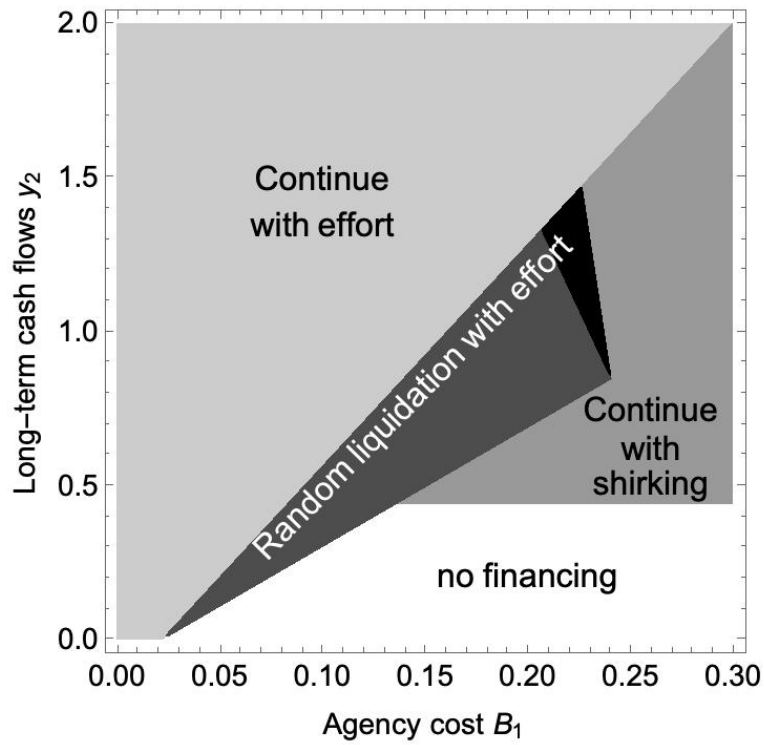

Figure 5: Financing regions with optimal contracting.

Notes. The y-axis depicts long-term project cash flows y_2 and the x-axis depicts agency cost B_1. Parameters: $\Delta_1 = 0.25, \pi_1 = 0.9, y_1 = 1.2, \beta = 0.3, \alpha = 0.5$.

We depict the optimal contract graphically in Figure 5 and leave a statement of the proposition and its proof to the appendix. Comparing Figures 3 and 5, bank financing emerges as the optimal contract in the top left portion of the figure. These are high-quality borrowers that choose to exert effort and are presented with low interest rate financing. Bank financing is also optimal for borrowers represented in the figure by the "Continue with shirking" area. The area identified as "Random liquidation with effort" encompasses all of the private credit funding area identified in Figure 3 plus another triangle at its end on the right colored in black. Inside this area, the probability of liquidation is decreasing in y_2, all else equal, because liquidation is both more inefficient at higher values of y_2 and less necessary to incentive the borrower.[9]

[9]Contracts at the top portion of this area have $\gamma = 0$ — as in bank financing — and contracts at the bottom of the area have $\gamma = 1$ — as in private credit financing.

These contracts dominate private credit because they induce effort by the borrower at lower interest rates. Random liquidation also incentivizes high effort when neither private credit nor bank financing can. These projects are identified by the triangle colored in black in the figure; long-term cash flows are sufficiently high relative to the agency cost that it is optimal to offer a lower interest rate to what banks can offer and to randomize on the liquidation decision.

We have shown that private credit arises in the region in which probabilistic termination is part of the optimal contract with commitment. In the absence of commitment on the lenders' side, limited lifetime funds emerge as a second-best solution to incentivize the borrowers.

V Discussion of assumptions and extensions

In this section, we discuss some of our modeling assumptions and introduce some extensions to the model. In Section V.1 we discuss the maturity of bank loans and how these affect our results. In Section V.2 we provide a micro-foundation for the assumption that private credit funds need to liquidate their investments that do not produce cash flow. We show that one can think of y_1 as credible promises of later cash flows in Section V.3, thus it does not have to fully materialize by $t = 1$. In Section V.4 we discuss the assumption of limited pledgeability of y_2, i.e., for $\alpha < 1$. In Section V.5 we discuss an extension in which the cost of private credit and bank financing differ.

V.1 Loan maturity

In the model, private credit loans must be repaid by $t = 1$, but the bank loan repayment R_B can be repaid over two periods; we assume that whatever cannot be paid back of the bank loan at $t = 1$ is paid at $t = 2$, potentially after renegotiation. So the longer horizon of banks allows them to extend longer maturity loans. We argue that this is not critical for our main results and derive some additional implications.

The repayment demanded by the bank from a borrower that can commit to effort is $R_B = \frac{1-(1-\pi_1)\alpha y_2}{\pi_1} < y_1$ (see Propositions 1 and 2) where the inequality follows from Assumptions 1 and 3. Thus if borrowers can commit to effort when borrowing from a bank, they can pay back the loan fully at $t = 1$ in case of a good outcome and high intermediate cash flows of y_1. However, if a borrower gets a bank loan and cannot commit to high effort, then it faces a high interest rate to compensate the bank for the higher probability of default. In fact R_B could be so high that the borrower cannot pay the loan back fully in period $t = 1$ even in case of success because $R_B > y_1$. In effect these loans are long-term loans ex ante, not only ex post due to renegotiation and maturity extension in the case of a bad outcome at $t = 1$.

The feature that bank loans can be of long maturity even in case of success (and the absence of renegotiation) begs the question of whether this is the main driver of our results as opposed to the maturity wall of private credit funds. As it turns out, the longer maturity of bank loans does not affect the availability of private credit financing. It is true that longer maturity helps in generating "business stealing" (as opposed to "democratization of credit") but the ability of the bank to offer a long term loan is not crucial. If banks were barred from offering longer (two period) maturity loans ex-ante $R_B = \frac{1-(1-\pi_1+\Delta_1)\alpha y_2}{\pi_1-\Delta_1} \leq y_1$ would need to hold. Contract feasibility in the absence of borrower effort is stricter than in Propositions 1 and 2, yielding $\frac{1-y_1(\pi_1-\Delta_1)}{(1-\pi_1+\Delta_1)\alpha} \leq y_2$. Suppose we require that long term cash flows are below this feasibility threshold. In the parametrization of Figure 3, this threshold value of feasibility lies below the highest y_2 in the (B_1, y_2)-space where private credit is chosen in equilibrium. Thus, "business stealing" is present even if we do not allow for two-period bank loans ex ante. In general, the higher y_1, the less binding the one-period loan constraint becomes and the more "business stealing" happens.

V.2 Forced liquidation versus asset sales

In the baseline model, we assume that private credit funds cannot sell the loan to a new lender at $t = 1$ if the intermediate outcome is bad and no cash flows materialize ($y_1 = 0$). Thus while banks that can roll over the debt can get the pledgeable income αy_2, private credit funds instead have to liquidate for $\beta y_2 < \alpha y_2$. It is crucial for our results that the private credit funds cannot sell the loan to banks for the pledgeable income αy_2. This assumption can be microfounded in several different ways. We show two examples involving different aspects of asymmetric information: one involving monitoring and due diligence costs, the other using heterogeneous projects.

Monitoring and due diligence costs. Why would liquidation yield better payoffs to the private credit fund than selling the loan to someone else with longer horizons? The reason could be that the private credit fund already has a very good understanding of the project and knows how to best sell it off in parts while the new lender would need a lot of investment to be able to run the project efficiently. When the ownership of loan is transferred, the new owner needs to perform due diligence if the loan's value is not public information. Due diligence is likely to be substantial for a project that has not produced the intermediate cash flow it promised and the financing is in need of renegotiation. This cost might entail understanding the borrower's project but also the additional costs of monitoring the borrower in the future with less knowledge about the project. Assume the buyer of the loan needs to pay an additional cost of $c > 0$ beyond the purchase price of the loan. If the cost is large enough $c > (\alpha - \beta)y_2$, this implies $\alpha y_2 - c < \beta y_2$ and the private credit fund will indeed choose to liquidate the project instead of selling the loan.

Heterogeneous projects. Another aspect of asymmetric information that might inhibit the transfer of a troubled loan is that the private credit fund as the owner knows the value of the loan much better than the potential buyer. The potential buyer only learns the true quality of the project over time after buying it. This is a standard lemons problem. Assume the exact amount of the pledgeable income in

$t = 2$ is random and uniformly distributed over the interval $[0, 2\alpha y_2]$. The true value is exactly known to the private credit fund. The potential buyer only knows the distribution but cannot tell what the pledgeable income is. Thus the pledgeable cash flow is αy_2 in expectation but the realization depends on which project is bought. Selling all loans is not an equilibrium if the liquidation value of the best loan is larger than the average price paid for the loans $\beta 2 y_2 > \alpha y_2$, thus if $\beta > \alpha/2$. Thus, only the subset of loans with lower pledgeable income would possibly be sold. But, the mean pledgeable income of the remaining loans is now lower, implying that a subset of these loans again would not be sold. In this case the equilibrium of selling any subset of loans unravels and the unique equilibrium is one in which no loans are sold, thus all are liquidated, just like in a lemons model. Note that the cutoff for β could be substantially lower than $\alpha/2$ if the distribution over the quality of the loans is such that there are many bad quality loans and a few really high quality ones.

V.3 Intermediate cash flow

In the baseline model Assumption 1 implies there is enough intermediate cash flow to repay the loan from y_1 if it materializes, i.e., $y_1 \geq R_{PC}$. This might seem like a strong assumption, especially that the intermediate cash flow y_1 (e.g., in the next five years) is similar in magnitude as all the other cash flows y_2 that come after that. Here we show that one can easily interpret y_1 as pledgeable cash flow sometime between $t = 1$ and $t = 2$ so that "success" in $t = 1$ means the project can be refinanced.

Assume the cash flow of y_1 materializes only at $t = 1+$ but there is news at $t = 1$ about whether this cash-flow will happen. With probability p_1 the news is good and the cash flow happens with certainty, with probability $1 - p_1$ the news is bad and there is no cash flow at $t = 1+$. Like in Section V.2 we assume that if the loan is refinanced by another party, there is an extra cost to be paid by the new financier which is c_H if the news at $t = 1$ is good and c_L if the news at $t = 1$ is bad with $c_L \geq c_H \geq 0$ and $c_L > 0$. One can easily find a cost in the bad state that is high enough $c_L > (\alpha - \beta)y_2$ such that the private credit fund does not find it optimal to sell the loan in the bad state because $\alpha y_2 - c_L < \beta y_2$ and instead liquidates the project. Also costs in the good state can be low enough $c_H < y_1 + (\alpha - \beta)y_2$ such that the private credit fund finds it optimal to sell the loan in the good state instead of liquidation because $y_1 + \alpha y_2 - c_H > \beta y_2$. In this extension, conditions for R_{PC} and borrower effort do not change, only the payoffs to the borrower as eventually the cost c_H has to be paid in equilibrium. In fact if one sets $c_H = 0$, the model solution does not change. This is reasonable if one thinks of good news as a credible signal about the project having well established future high cash flows.

V.4 Non-pledgeable income

In the baseline model, only α fraction of the $t = 2$ income is pledgeable. Lemma 4 implies that this is also a crucial assumption as $\alpha < 1$ is needed for private credit financing to emerge in equilibrium. Here we provide a simple microfoundation for this assumption by introducing effort choice from $t = 1$ to $t = 2$, similarly to the effort choice made from $t = 0$ to $t = 1$ in the baseline model.

Assume the payoff at $t = 2$ with high effort (from $t = 1$ to $t = 2$) is y_2 with probability 1 and zero otherwise. Without effort the probability of success drops to $1 - \Delta_2$ and private benefit of $B_2 > 0$ is collected. We assume effort is efficient, thus $B_2 < \Delta_2 y_2$. Any promised repayment R from $t = 1$ to $t = 2$ has to satisfy the incentive compatibility constraint in the interim period $y_2 - R \geq \frac{B_2}{\Delta_2}$. This constraint on the promised repayment can be rewritten as $R \leq \alpha y_2$ with $\alpha = 1 - \frac{B_2}{y_2 \Delta_2} < 1$; only the fraction α of the expected cash flow y_2 can be pledged if the lender wants to induce effort. Note that the lender can get $(1 - \Delta_2)y_2$ without inducing effort thus more pledgeable income can be attained with effort than without if and only if $1 - \Delta_2 < 1 - \frac{B_2}{y_2 \Delta_2}$ which holds for $\sqrt{B_2} < \Delta_2 \sqrt{y_2}$; this is a tighter condition than the condition that requires effort to be efficient. Under this tighter condition, the pledgeable income can indeed be written as an $\alpha < 1$ fraction of the expected cash-flow y_2, the exact simplification we use in the baseline model.

V.5 Different cost of bank and private credit financing

The model assumes that neither banks nor private credit funds discount future cash flows. One can easily introduce differences in discounting and thus in expected returns: denote $\rho_B \geq 0$ the discount rate of banks and $\rho_{PC} \geq 0$ the discount rate for private credit funds from $t = 0$ to $t = 1$. For simplicity assume banks still do not discount cash flows from $t = 1$ to $t = 2$. The main insights can be discussed without presenting the solution of the whole model.

As ρ_B increases, the bank's break even condition (3) implies that R_B increases for risky loans as $R_{B,f} = \alpha y_2$ is capped. Since the constraints with private credit financing are not affected, this implies the borrowers' incentive compatibility constraint (2) is more binding, thus the diagonal line separating bank financing and private credit financing in Figure 3 moves up and left, thus expanding the area with private credit financing. Furthermore, private credit financing becomes cheaper compared to risky bank financing and thus the upper right line on Figure 3 separating high-risk bank financing from private credit financing (defined originally by (9)) also moves up and right, generating more business stealing. Thus if bank financing becomes more costly, private credit expands taking over previous bank loans. This model extension describes well the recent expansion of private credit at the expense of bank financing as the costs of bank financing increased relative to private credit financing. These conclusions hold even if private credit loans have a higher expected return and the increase in bank expected returns merely decreases the wedge in expected costs.

VI Empirical and Policy Implications

VI.1 Empirical Implications and Evidence

Here, we discuss the model's testable predictions, and where there already exists empirical evidence, we discuss how to interpret the evidence in light of our model.

Types of private credit lenders. Our paper suggests that any empirical analysis must consider separately limited partnerships that usually have limited life time and business development companies that are typically perpetual. This implication has a bearing on the datasets used in empirical analysis as data about private credit are limited and different data sources have different focus. Fund-level data have good coverage of private credit funds structured both as limited partnerships and business development companies, whereas loan-level data are scarcer with more readily available data being from business development corporations due to regulatory requirements in the US, especially when they are publicly traded. This means loans made by business development companies are overrepresented in loan-level data.

Mix of private credit borrowers. Proposition 2 and Lemma 4 imply that private credit financing consists of both new loans a bank would not grant (democratization of credit) and loans that displace bank financing (business stealing). The survey of Block et al. (2024) includes both US and European private creditors; of US respondents only 40% are business development companies. Consistent with our model, they show that private credit fund managers report that about half of their deals would not have gotten a bank loan, while the other half would have but the borrower still chose private credit. Our model has more nuanced implications showing that private credit loans that replace bank lending are in general made to more profitable companies with more severe agency issues (higher y_2 and B_1) compared to private credit loans that banks would not make. Our model also implies that loans that are stolen by private credit from banks become safer. This might explain why in loan level data, Cai and Haque (2024) find that private credit loans have lower probability of default than comparable bank loans.[10]

Interest rates. Proposition 4 implies that if banks' loan portfolios mostly consist of higher quality loans (case ii), then in equilibrium banks charge lower average interest rates than private credit funds. Indeed, the International Monetary Fund (2024) shows that the average interest rate charged on private credit loans is higher than that for leveraged loans. Chernenko et al. (2022) shows that the interest rate is higher for private credit loans than bank loans even after controlling for observable borrower characteristics. Proposition 4 also has implications for the cross-section of banks. Banks with a high quality loan portfolio — charging low interest rates on average — experience a reduction of risk in their loan portfolio and also lower profitability in expansions due to business stealing. On the other hand, banks that have a high share of high-risk loans — and thus charge high interest rates on average — experience the opposite when private credit is also available.

Bank versus private credit borrowers. Our model also has implications on which type of borrowers are likely to opt for private credit loans. Proposition 2 shows that private credit loans are in general made to companies with lower long-term profitability and higher agency issues. Lemma 4 also highlights that private credit is more prevalent when pledgeability of cash-flows is lower. This later implication is in line with the finding of Cai and Haque (2024) that "more than half of all value-weighted private credit is provided to borrowers in sectors with relatively low collateralizable or tangible assets such as software, financial services or healthcare services" (see also International Monetary Fund 2024). Cai and Haque (2024) also show that conditional on default, private credit loans have lower recovery rate, which is in line with Proposition 2 that the maturity wall in private credit leads to inefficient liquidation of projects with relatively low long-term cash flow, and thus low liquidation value compared to bank loans.

[10]Note that the definition of default might be different for bank loans and private credit loans.

Growth in private credit. Section V.5 shows that the growth in the private credit market can be associated with the increase in bank regulation that took place since the great financial crisis and the consequent increase in bank funding costs.

VI.2 Policy Implications

In this section, we discuss implications for banks and the overall economy from the presence of private credit.

Bank balance sheets, profit and risk. Proposition 3 shows that private credit steals loans from banks and thus reduces banks' balance sheet. These poached loans are among the riskier bank loans. Thus, the average default risk on banks' balance sheets decreases for banks which had relatively high quality loans before the expansion in private credit. According to Proposition 4 (case ii), this has the effect of creating bank outcomes that are less dispersed over the business cycle: bank returns and profitability are not as high in expansions but losses are also reduced in recessions when private credit is present.

Aggregate output and risk. Proposition 2 implies that expected aggregate output increases in the presence of private credit. However, this increase comes from loans with low long-term profitability which are worth little when liquidated. Also the loans stolen from banks are more likely to be liquidated with private credit financing. This implies that aggregate risk increases once private credit is introduced and this risk is concentrated in private credit loans. Indeed, proposition 4 implies that private credit loans should be more sensitive to aggregate risk, especially to the risk of large, tail events. Consistent with this prediction, Erel et al. (2024) shows that private credit fund returns have higher systematic risk.

Ex post bailouts. We show that the maturity wall that can lead to massive inefficient liquidation of private credit-funded borrowers is an essential part of private credit's business success. In case of mass liquidation by private credit funds, saving borrowers ex post — be it by the government or banks sponsoring private credit funds — undermines the ex ante incentives that private credit financing can provide, in effect making private credit closer to bank financing. To make sure vulnerable investors do not lobby the government for ex-post bailouts, policy should focus on limiting access to private credit funds to those investors that can bear large losses in case of a large recession.

Opacity of funds. Our model highlights the benefit of private credit funds committing to not being able roll over loans. In fact Section V.2 shows that opacity is beneficial to funds since it makes the commitment to not roll over loans by selling them more credible. The model also suggests that the success of private credit funds lies in borrowers with relatively high agency frictions. Although private credit funds may be better equipped to deal with these frictions, investors in these funds may remain uninformed and be subject to agency issues themselves. This concern is heightened when the firm is held by a private equity fund that sponsors the private credit fund.[11] Policy should consider these implications when private credit funds are sold to smaller, less sophisticated investors. Finally, policy should consider banks' participation in this market as investors in private credit funds. The opacity of these funds, and the higher-risk loan portfolios of private credit (as predicted by the "democratization of credit" and "business stealing"), suggest that banks are re-entering the high-interest rate loan market with much less control and information about the loan portfolio and hence, more exposed to unexpected left-tail events. The above concern is particularly relevant given that private credit funds are often under the same umbrella company as private equity funds also invested in the same borrower firm.

[11] Buchner et al. (2022) looks at the relationship between the private equity fund that owns the firm's equity and the private credit fund that acts as its lender. They conclude that about 20% of target companies have the same sponsor on both sides.

VII Conclusion

This paper studies the interaction of bank financing and private credit financing. The model's main assumption is that private credit liquidates the borrower if the borrower's project fails and the private credit loan cannot be repaid; the inability to renegotiate at that stage is motivated by the limited lifetime of private credit closed-end funds, which we denote as a maturity wall. In contrast to private credit, banks' longer horizon allows for renegotiation and extension of non-performing bank loans. The inability to renegotiate private credit loans beyond their maturity generates incentives to exert effort at the cost of excessive liquidation when projects fail in the interim period.

We show that introducing private credit may lead to "democratization of credit" — where projects that could not obtain a bank loan, can obtain a loan from a private credit fund — and "business stealing" — where borrowers that could obtain a bank loan, though at a high interest rate and without strong incentives to exert effort, prefer to be financed by a private credit loan at a lower interest rate and exert effort. These features of the equilibrium with private credit financing are intrinsically linked to the maturity wall of private credit funds.

Democratization of credit and "business stealing" by private credit funds are responsible for several new predictions about the distribution of loan losses between banks and private credit funds, the lower dispersion of payoffs for banks, and banks' smaller balance sheets, findings relevant to the understanding of the stability of the banking sector.

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Appendix A: Mathematical Appendix

For Propositions 1 and 2, should the borrower be indifferent between any two loans, we assume for simplicity the following tiebreaker rules: the borrower first prefers to be financed, second it prefers to exert effort, and lastly, it prefers bank financing to private credit.

Proof of Proposition 1: *We start by stating all the relevant constraints and then analyze which constraints need to be satisfied in the given financing regions.*

Bank break-even constraint. Banks are competitive and break even in expectation, which yields equations (3) and (4).

Feasibility of bank contract. The bank offers a contract that is feasible and constraint (1) holds.

Borrower incentive constraint with bank financing. The borrower exerts effort in the first period if and only if

$$\pi_1(y_1+y_2-R_B)+(1-\pi_1)(y_2-R_{B,f}) \geq (\pi_1-\Delta_1)(y_1+y_2-R_B)+(1-\pi_1+\Delta_1)(y_2-R_{B,f})+B_1, \quad (12)$$

otherwise the borrower shirks. Rearranging, we get (2) in the text.

Borrower participation constraint. With an outside opportunity of zero, contract feasibility guarantees that the borrower is always willing to participate as it has to invest nothing and can only get non-negative payoffs thereafter.

Resulting financing regions:

i) Risk-free loan. Note that if $\alpha y_2 \geq 1$, then by (4) $R_B = 1$. In case of failure in the interim period, $R_{B,f} = 1$. The loan can always be fully repaid by $t = 2$ and thus it is risk-free. With $R_B = R_{B,f} = 1$, and Assumption 2, the borrower's incentive constraint (2) is met, and the borrower exerts effort. As the borrower exerts effort, the contract is feasible by Assumptions 1 and 3.

ii) Low-risk loan. If $\alpha y_2 < 1$ the bank loan is no longer risk-free. If (2) holds and the borrower exerts effort, then $p_1 = \pi_1$, and from (4)

$$R_B = \frac{1-(1-\pi_1)\alpha y_2}{\pi_1} = \alpha y_2 + \frac{1-\alpha y_2}{\pi_1} > 1. \quad (13)$$

With probability $1-\pi_1$ some of the loan needs to be forgiven to make sure it does not exceed the pledgeable income, thus $R_{B,f} = \alpha y_2 < 1$. Contract feasibility requires that $R_B \leq y_1+\alpha y_2$, which *again holds by Assumptions 1 and 3. Plugging the values of R_B and $R_{B,f}$ into the incentive constraint (2), in equilibrium the constraint becomes*

$$y_2 \geq \bar{y}_2^{B,I} = \frac{1 - \pi_1 \left(y_1 - \frac{B_1}{\Delta_1} \right)}{\alpha}, \tag{14}$$

where $\bar{y}_2^{B,I} < 1/\alpha$.

iii) *High-risk loan. If $\alpha y_2 < 1$ and (14) does not hold, then the borrower cannot commit to exert effort. If the borrower shirks, the bank can break even by charging*

$$R_B = \frac{1 - (1 - \pi_1 + \Delta_1)\alpha y_2}{\pi_1 - \Delta_1} = \alpha y_2 + \frac{1 - \alpha y_2}{\pi_1 - \Delta_1} > 1. \tag{15}$$

With probability $1 - \pi_1 + \Delta_1$ the contract is renegotiated in the interim period and $R_{B,f} = \alpha y_2$. Plugging the values of R_B and $R_{B,f}$ into the incentive constraint (2), it is possible to show that the constraint does not hold for $y_2 \leq \bar{y}_2^{B,I}$. Hence, any bank-finance project with $y_2 \leq \bar{y}_2^{B,I}$ cannot induce effort from the borrower. Thus, some projects with sufficiently low long-term cash flows, where the borrower shirks, cannot be financed; contract feasibility requires $R_B \leq y_1 + \alpha y_2$, which is equivalent to requiring $y_2 \geq \bar{y}_2^{B,P}$.

If $\bar{y}_2^{B,P} < \bar{y}_2^{B,I}$ and $y_2 < \bar{y}_2^{B,P}$, or $\bar{y}_2^{B,P} > \bar{y}_2^{B,I}$ and $y_2 < \bar{y}_2^{B,I}$, then there is no bank financing.

□

Proof of Proposition 2: *We state first the relevant constraints beyond those already stated in the proof of Proposition 1. We then analyze which constraints need to be satisfied in the given financing regions.*

Private credit fund break-even condition. Private credit funds are competitive and break even in expectations, which yields equations (7) and (8).

Borrower incentive constraint when borrowing from the private credit fund. If $R_{PC} \leq y_1$, the borrower exerts effort (and thus $p_1 = \pi_1$) if and only if

$$\pi_1(y_1 - R_{PC} + y_2) \geq B_1 + (\pi_1 - \Delta_1)(y_1 - R_{PC} + y_2) \tag{16}$$

which yields (6) in the main text.

Feasibility of private credit. The private credit fund offers a feasible contract and constraint (5) holds.

Switching constraint. *The borrower prefers bank financing with low effort over private credit financing with high effort if (9) in the main text holds.*

Resulting financing regions:

i) *Risk-free bank loan. Same as in Proposition 1.*

ii) *Low-risk bank loan. Same as in Proposition 1.*

iii) *High-risk bank loan.* y_2 *must be high enough that a feasible contract exists even with low effort yet* y_2 *must be low enough that the borrower does not want to exert effort thus (2) is violated. Thus* $y_2 \in \left[\bar{y}_2^{B,P}, \bar{y}_2^{B,I}\right)$*. In addition, private credit financing is not preferred, which changes the lower bound on the above interval to what is in the proposition statement. If private credit is available, then* y_2 *is large enough that bank loan is preferred, which gives* $y_2 \in \left[\bar{y}_2^{B,P}, \bar{y}_2^{PC,I}\right)$*. In this range private credit cannot incentivize effort.*

iv) *Low risk private credit loan. Plugging in the break-even interest rates (8) of the private credit fund into the borrower's incentive constraint (6) yields*

$$y_2 \geq \bar{y}_2^{PC,I} = \frac{1 - \pi_1\left(y_1 - \frac{B_1}{\Delta_1}\right)}{\pi_1 + (1-\pi_1)\beta}. \tag{17}$$

v) *No financing.* y_2 *is too low and there is no feasible interest rate consistent with the incentive constraint of the borrower under either private credit (6) or bank financing (2) to exert effort.*

□

Proof of Lemma 4 : *For private credit to emerge in equilibrium, a cone-shaped area between* $\bar{y}_2^{PC,I}$ *as lower bound and* $\bar{y}_2^{B,I}$ *as upper bound must open up in the* (B_1, y_2) *plane, see Figure 3. First note that* $\bar{y}_2^{PC,I} = \frac{1-\pi_1\left(y_1 - \frac{B_1}{\Delta_1}\right)}{\pi_1 + (1-\pi_1)\beta}$ *and* $\bar{y}_2^{B,I} = \frac{1-\pi_1\left(y_1 - \frac{B_1}{\Delta_1}\right)}{\alpha}$ *intersect in the* (B_1, y_2) *plane at* $y_2 = 0$ *and* $B_1 = \Delta_1(y_1 - 1/\pi_1)$*. Since we only consider projects with* $y_2 > 0$*, the only projects for which we need to establish that* $\bar{y}_2^{B,I} > \bar{y}_2^{PC,I}$ *is possible are those for* $B_1 > \Delta_1(y_1 - 1/\pi_1)$*.* $\bar{y}_2^{B,I} > \bar{y}_2^{PC,I}$ *holds if and only if* $\pi_1 + (1-\pi_1)\beta > \alpha$*, which is the condition stated in the Lemma.*

Now we only need to establish that for these projects indeed private credit is preferred instead of risky bank financing. At the intersection of $\bar{y}_2^{PC,I}$ *and* $\bar{y}_2^{B,I}$ *(9) holds thus there are projects with* $B_1 > \Delta_1(y_1 - 1/\pi_1)$ *for which private credit financing is preferred to risky bank financing (if available) and there exists* $y_2 > 0$ *such that* $\bar{y}_2^{PC,I} < y_2 < \bar{y}_2^{B,I}$*. Note that* $B_1 = \Delta_1(y_1 - 1/\pi_1) < 0$ *is also possible and thus we have to make sure private credit financing happens when* $B_1 > 0$ *as well.*

For this to hold, $\bar{y}_2^{PC,I}$ needs to intersect the $B_1 = 0$ line below (9) $\frac{\Delta_1 y_1 - B_1}{(1-\pi_1)(1-\beta)}$ which is the case if

$$\frac{\Delta_1 y_1}{(1-\pi_1)(1-\beta)} > \frac{1-\pi_1 y_1}{\pi_1 + (1-\pi_1)\beta} \tag{18}$$

If $\Delta_1 > -\frac{\pi_1((1-\beta)(1-\pi_1))}{\beta+(1-\beta)\pi_1}$ (which follows from $-\frac{\pi_1((1-\beta)(1-\pi_1))}{\beta+(1-\beta)\pi_1} < 0$) this reduces to

$$y_1\pi_1 > \frac{(1-\beta)(1-\pi_1)}{\Delta_1\left(1+\frac{\beta}{\pi_1}-\beta\right)+(1-\beta)(1-\pi_1)} \tag{19}$$

since the right hand side is smaller than 1, this holds for sure if $y_1\pi_1 \geq 1$.

Thus we have established that there are some projects with specific B_1 and y_2 that are financed by private credit.

□

Proposition 5. Optimal financing contract with commitment. *In the optimal contract, the project is financed with:*

i) <u>Low risk loan with continuation</u> if $y_2 \geq \bar{y}_2^{B,I}$: the borrower exerts effort and the project is financed by a loan with $\gamma = 0$;

ii) <u>High-risk loan with continuation</u> if $y_2 \in \left[\bar{y}_2^{B,P}, \bar{y}_2^{PC,I}\right)$ or $y_2 \in \left[\max\left(\bar{y}_2^{B,P}, \bar{y}_2^{switch,o}\right), \bar{y}_2^{B,I}\right)$: the borrower shirks and the project is financed by a loan with $\gamma = 0$;

iii) <u>Low-risk loan with probabilistic liquidation</u> if $y_2 \geq \bar{y}_2^{PC,I}$ and $y_2 < \min\left(\bar{y}_2^{B,I}, \bar{y}_2^{switch,o}\right)$: the borrower exerts effort and the project is financed by a loan with $\gamma \in (0,1]$, furthermore γ is strictly increasing in y_2, $\gamma = 0$ when $y_2 = \bar{y}_2^{B,I}$, and $\gamma = 1$ when $y_2 = \bar{y}_2^{PC,I}$;

If $y_2 < \min\left(\bar{y}_2^{PC,I}, \bar{y}_2^{B,P}\right)$, the project is not financed. $\bar{y}_2^{switch,o}$ is given in the Appendix.

Proof of Proposition 5: *First note that setting $R_l = \alpha y_2$ is weakly optimal. If $R_l < \alpha y_2$ was chosen, it could be increased such that the break-even constraint still holds by decreasing R_h. Both increasing R_h and decreasing R_l is relaxes the incentive constraint.*

Also minimizing γ is optimal as it increases the payoff of the borrower, relaxes the incentive constraint and at the same time break-even constraint can be made to bind by lowering R_h since $R_l = \alpha y_2 > \beta y_2$.

We now prove the regions one by one:

i) <u>Risk-free loan without liquidation</u>

If $\alpha y_2 \geq 1$ then the loan can be repaid even in the outcome with low intermediate payoff. No liquidation is necessary since the borrower does not have to incentivized and the loan is risk-free.

ii) *Low-risk loan without liquidation*

Whenever the incentive constraint can be satisfied with $\gamma = 0$ then high effort with $p_1 = \pi_1$ will be chosen. $p_1 = \pi_1 - \Delta_1$ is not optimal in this case as switching to $p_1 = \pi_1$ as it increases the payoff and at the same time the break-even constraint can be fulfilled by simply choosing a lower R_h. Substituting $\gamma = 0$ into the incentive constraint and using the break-even constraint to to fix R_h one gets that high effort without liquidation is possible if $y_2 \geq \bar{y}_2^{B,I} = \frac{1-\pi_1\left(y_1 - \frac{B_1}{\Delta_1}\right)}{\alpha}$.

iii) *High-risk loan without liquidation*

Whenever the incentive constraint cannot be satisfied even with $\gamma = 1$ then low effort with $p_1 = \pi_1 - \Delta_1$ has to be chosen. The lowest y_2 for which $\gamma = 1$ still yields incentives to exert effort is set by combining the break-even constraint with the incentive constraint yielding $y_2 \geq \bar{y}_2^{PC,I} = \frac{1-\pi_1\left(y_1 - \frac{B_1}{\Delta_1}\right)}{\pi_1 + (1-\pi_1)\beta}$. Thus whenever $y_2 < \bar{y}_2^{PC,I}$ even $\gamma = 1$ cannot incentivize the borrower. Thus in this case the break-even condition yields $R_h = \frac{1-(1-\pi_1+\Delta_1)\alpha y_2}{\pi_1 - \Delta_1}$. Note that the feasibility constraint still has to bind $R_h \leq y_1 + \alpha y_2$ yielding $y_2 \geq \bar{y}_2^{B,P} = \frac{1-(\pi_1-\Delta_1)y_1}{\alpha}$. Note that the borrower may still choose not be incentivized which is the case if $y_2 \geq \bar{y}_2^{switch,o}$ which we derive below.

iv) *Low-risk loan with probabilistic liquidation*

Since the borrower wants to minimize the probability of termination γ in case of low intermediate outcome, the incentive constraint has to be binding in this case. The incentive constraint in this case yields:

$$y_1 + \gamma y_2 - R_h + (1-\gamma)\alpha y_2 = \frac{B_1}{\Delta_1}$$

and

$$\pi_1 R_h + (1-\pi_1) y_2 (\gamma\beta + (1-\gamma)\alpha) = 1.$$

Combining the two yields:

$$R_h = \frac{B_1(\pi_1 - 1)(\alpha - \beta) + \Delta_1(\beta\pi_1 y_1 + \alpha(-\pi_1 y_1 + (\beta - 1)(\pi_1 - 1)y_2 + y_1 + 1) - \beta y_1 - 1)}{\Delta_1(\alpha - \beta + (\beta - 1)\pi_1)} \tag{20}$$

$$\gamma = \frac{\Delta_1(\pi_1 y_1 + \alpha y_2 - 1) - B_1\pi_1}{\Delta_1 y_2(\alpha - \beta + (\beta - 1)\pi_1)} \tag{21}$$

Now if both probabilistic termination with effort and continuation without effort is available, then the borrower chooses probabilistic termination if and only if yields a higher expected payoff

$$\pi_1(y_1+y_2)+(1-\pi_1)(\alpha(1-\gamma)y_2+\beta\gamma y_2)-1 > B_1+(\pi_1-\Delta_1)(y_1+y_2)+y_2(\Delta_1-\pi_1+1)-1 \quad (22)$$

which yields

$$y_2 < \bar{y}_2^{switch,o} = \frac{B_1\Delta_1(-\alpha+\beta-\beta\pi_1+\pi_1) - B_1(\pi_1-1)\pi_1(\alpha-\beta) + \Delta_1^2 y_1(\alpha-\beta+(\beta-1)\pi_1) + \Delta_1(\pi_1-1)(\alpha-\beta)(\pi_1 y_1 - 1)}{\Delta_1(\pi_1-1)(-\alpha+\beta+(\alpha-1)(\beta-1)\pi_1)}. \quad (23)$$

The derivative of γ with respect to y_2 in this region is:

$$\frac{\partial\gamma}{\partial y_2} = \frac{B_1\pi_1 + D_1(-\pi_1)y_1 + D_1}{D_1 y_2^2(\alpha-\beta+(\beta-1)\pi_1)} \quad (24)$$

$\frac{\partial\gamma}{\partial y_2} < 0$ *follows from the following observations.*

The denominator is positive because $1 - \pi_1\left(y_1 - \frac{B_1}{D_1}\right) > 0$ *since in the relevant region* $\bar{y}_2^{B,I} > \bar{y}_2^{PC,I} > 0$. *Also the denominator is negative since by Lemma 4 the region between* $\bar{y}_2^{PC,I}$ *and* $\bar{y}_2^{B,I}$ *only exists if* $\alpha < \beta(1-\pi_1) + \pi_1$.

$\gamma = 0$ *when* $y_2 = \bar{y}_2^{B,I}$, *and* $\gamma = 1$ *when* $y_2 = \bar{y}_2^{PC,I}$ *follow by substitution.*

□