The Asset Pricing Model Of Exchange Rate And Its Test On Survey Data

Anna Naszodi[1]

This paper gives a solution to a particular type of asset pricing exchange rate model. According to the closed-form solution, the exchange rate is a non-linear function of the following stochastic factors: (i) fundamental, (ii) market expectation for the exchange rate, (iii) discount factor. Our three-factor model is found to have better out-of-sample performance than simpler models with less factors or with linear functional form.

JEL: F31, F36, G13.

Keywords: asset-pricing exchange rate model, factor model, time-varying parameter, survey data, disconnect puzzle, excess volatility puzzle.

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This research project has been started while the author was at the Sveriges Riksbank. The views expressed are those of the author and do not necessarily reflect the official view of the Sveriges Riksbank (National Bank of Sweden) and Magyar Nemzeti Bank (National Bank of Hungary) or that of any other institution the author has been or will be affiliated with.

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Dec 4 2008

1 Introduction

This paper introduces an exchange rate model that is the standard asset pricing model with time-varying discount factor. We derive that in this model the exchange rate is a non-linear closed form function of the following three factors: the fundamental, the market expectation for the exchange rate, and the discount factor.

The empirical part of the paper tests our asset pricing model against some alternative models by using survey data on exchange rate forecasts. The alternative models are simpler models with less factors. One is the random walk model, the other is the constant parameter model, and the third one is the linear model. Our three-factor model is found to have better out-of-sample performance than any of its alternatives. This finding supports the view that our non-linear time-varying parameter model is close to the one that forecasters have in their minds. Moreover, according to a standard measure, the forecasting performance of our model is found to be better than that of the random walk for almost all the analyized exchange rates and forecast horizons. Therefore, our non-linear time-varying parameter model can represent not only the model used by the forecasters, but it is relevant also for the realized exchange rate.

What supports the asset pricing model is not only its forecasting ability, but also the fact that it can account for some well known puzzles and anomalies. The disconnect puzzle, the excess volatility puzzle, the near random walk behaviour of the exchange rate, the inability of macro models to provide reliable exchange rate forecasts for the short and medium horizons, the seemingly frequent exchange rate regime switches and the fat tail distribution of the returns can be explained by our three-factor model.

This paper unifies standard, although not commonly used, building blocks of the exchange rate theory. These building blocks are the following: the asset pricing view of the exchange rate [1] (APV), time-varying parameter approach (TVP), filtering the fundamental, and using survey-based exchange rate expectations.

[1] The asset pricing model of the exchange rate has different names in the literature. It is called "asset market view model" by Frenkel and Mussa (1980), the "canonical model" by Krugman (1992) and by Gardeazabal et al (1997) and the "rational expectations present-value model" by Engel and West (2005).

According to the APV, the exchange rate is a function of the fundamental and a forward looking element. The fundamental captures the current state of the economy relevant for the exchange rate. The forward looking part is the present value of the expected future exchange rate.

Although the APV has become a widely used building block in the exchange rate literature, it has been supported only by a few empirical works and has been rejected by a number of others.[2] The rejections can be attributed to the following reasons. First, the misspecification of the structural macro models that define the fundamental and the discount factor in the asset pricing model. Second, the law of iterated expectations may not necessarily hold.[3] Therefore, the process of one of the underlying factors, *i.e.*, the expected future exchange rate, is not necessarily martingale. The functional relationship between the exchange rate and the factors depends highly on the processes of the underlying factors. Consequently, the functional relationship that is subject to the empirical tests is also sensitive to the type of expectations.

In view of these problems leading to the false rejection of the APV, we treat in this paper both the fundamental and the discount factor with special care while sticking to the the rational expectation hypothesis and assuming to have representative agent. Therefore, the law of iterated expectation is not violated.

Almost all papers in the literature assume the discount factor to be constant for the sake of simplicity or for the analytical tractability. The few counter examples are Wolff (1987), Alexander and Thomas (1987), Schinasi and Swamy (1989), Wu and Chen (2001), and this paper. These papers apply the TVP approach by allowing the discount factor to change over time. The TVP literature of exchange rate have the following remarkable results in chronological order. First, the introduction of time-varying parameters ala Wolff (1987) enhances the forecasting performance of the structural models. Second, the model by Schinasi and Swamy (1989) can even outperform the random walk model in terms of out-of-sample forecasting ability of the exchange rate. Third, the TVP model of Wu and Chen (2001) is not only able to beat the random walk, but its out-of-sample prediction performance is proved to be significantly better than that of the random walks.

In their seminal paper, Meese and Rogoff (1983) find that the simple, linear, macro models with constant parameters can not outperform the random walk in terms of exchange rate forecasting ability on the short and medium horizons. Meese and Rogoff note that the disappointing forecasting performance of these models is most likely to be attributable to simultaneous equation bias, sampling error, misspecification, or parameter instability. The above mentioned papers of the TVP literature of exchange rate contribute to the Meese and Rogoff literature,[4] by showing that it is mainly the parameter instability that is responsible for the poor forecasting ability of previously examined models.

[2]The empirical works by Gardeazabal et al (1997) and Naszodi (2008b) are exeptions as providing direct support for the APV. The papers by Engel and West (2005) and Engel et al (2007) also find empirical evidences that are consistent with the APV, but their findings can result also from some alternative mechanisms, other than the APV.

[3]Bacchetta and van Wincoop (2003) prove that the law of iterated expectations is violated in a heterogeneous agent and higher order beliefs framework.

[4]It is worth to mention that these papers do not revert the Meese and Rogoff (1983) finding. It is still unrejected.

The time-varying feature of the parameters may be rationalized on a number of grounds. First, parameters are likely to change in response to policy regime changes as an example of a Lucas critique (see: Lucas (1976)). Second, there can easily be implicit instability in the money demand equation. For instance, instability in empirical money demand functions have been documented by Hondroyiannis et al (2001). Third, the time-varying behavior of parameters can also be attributed to heterogeneous agents with highly variable market shares. As it is noted by Schinasi and Swamy (1989), even if each participant reacted to macroeconomic developments according to a stable constant coefficient reaction function, it would be difficult to argue that macroeconomic variables are related to exchange rates by a simple fixed coefficient relationship, without also assuming that individual reaction functions were identical.

It is a common practice in the exchange rate literature to start with a structural macro model and define the fundamental accordingly. In contrast, this paper uses the time-series of the exchange rate and survey data to filter out the fundamental. The filtering approach is applied also by some other empirical papers (see: Wu and Chen (2001), Naszodi (2008b), Gardeazabal et al (1997), Sarno and Valente (2008)).[5] The advantage of the filtering approach is that it allows us to bypass the problem of choosing a structural macro model. We have no reason to believe that it is not the macro variables and the expectations on their future evolutions that are the most important determinants of the exchange rate besides some short term effects coming from microstructure noise or short-living asymmetric information for instance.[6] But we do not think that the commonly used structural macro models with constant coefficients can sufficiently capture the rich dynamics of the fundamental. These ideas have gained empirical support by Sarno and Valente (2008). They claim that the exchange rate disconnect puzzle is unlikely to be caused by lack of information in the fundamental, and it is more likely due to frequent shifts in the set of fundamental driving exchange rates.[7]

Besides the frequent shifts in the fundamentals, there are some data limitations that can also explain the failor of previous empirical studies to find the relevant macro fundamentals. The commonly used data are not forward looking in the sence that they do not capture the market expectation on the future evolution of the macro fundamentals. In contrast to the majority of the empirical studies, we use survey-based exchange rate expectations in the filtering exercise. It is argued by Frankel and Froot (1987) that survey

[5] De Grauwe et al. (1999b), Burda and Gerlach (1993) also filters the fundamental from the exchange rate instead of constructing its time series from macro data using questionable macro models, but they do not report the filtered fundamental.

[6] Bacchetta and Wincoop (2006) demonstrate that asymmetric information with higher order beliefs can cause the relationship between the fundamental and the exchange rate to be weaker then in the common knowledge, full information framework. By that, they provide a potential theoretical explanation for the disconnect puzzle and excess volatility puzzle. However, the empirical work by Chaboud et. al. (2007) suggest that participants in the foreign exchange market learn quickly each other's beliefs. Consequently, higher order beliefs are not likely to be the substantial cause of the failures of many empirical papers to find the link between the fundamental and the exchange rate.

[7] Practitioners in the foreign exchange market regularly change the weight they attach to different economic variables as evidenced in a variety of survey studies (see, for instance, Cheung and Chinn (2001)).

measures of exchange rate expectations are very poor forecasters and the expectations, themselves, are frequently internally inconsistent. Therefore, we test the forecasting ability of the survey data and check their internal consistency.

The paper is structured as follows. Section 2 introduces the exchange rate model. Section 3 tests the general non-linear three-factor model against some restricted models. Section 4 tests the general non-linear three-factor model against the random walk model based on the forecasting performance. Section 5 presents the filtered factors. Section 6 provides explanation for the disconnect puzzle, the excess volatility puzzle and some other anomalies. Finally, Section 7 concludes.

2 Exchange Rate Model

The exchange rate model is the conventional asset-pricing exchange rate model generalized by having time-varying parameter. In the conventional asset-pricing model, the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks.

$$s_t = v_t + c_t \frac{E_t(ds_t)}{dt} \quad . \tag{1}$$

Here, s is the log exchange rate, and v is the fundamental, and $\frac{E_t(ds_t)}{dt}$ is the expected instantaneous change of the log exchange rate. The only parameter of this model is the discount factor c_t that determines the relative importance of the forward looking term $\frac{E_t(ds_t)}{dt}$ in the exchange rate s_t.

Macro models that rationalize the asset-pricing exchange rate model offer the following interpretation of parameter c_t. It links the nominal and the real variables. In the monetarist models, for instance, c_t is the semi-elasticity of money demand (see: Engel and West (2005) and Svensson (1991) about these models).[8] Unlike most of the papers in the literature, we assume that this parameter is time-varying.

The fundamental v_t could be defined as a function of some macro variables by using Equation (4f) of the money income model for instance. However, we opt for using neither the definition of c_t in Equation (1f), nor that of v_t in Equation (4f), nor the corresponding macro data, mainly because of the possibility of misspecification of the underlying macro model, but also because of the low frequency and the substantial measurement errors of these data. However, these definitions motivate the choice of the processes of the underlying factors and the interpretation of the results. It is worth to note that the misspecification problem is not specific to the money income model, but all the alternative structural macro models are potentially subject to this problem.

As we will see later, the expected instantaneous change of the exchange rate $\frac{E_t(ds_t)}{dt}$ depends on the fundamental v_t, the stochastic discount factor c_t, and a third factor as

[8]The simplest model among them is the four-equations money income model.

(1f) $m_t - p_t = \alpha y_t - c_t i_t \quad \alpha > 0 \quad c_t > 0$ money market equilibrium

(2f) $q_t = s_t + p_t^* - p_t$ real exchange rate

(3f) $\psi_t = i_t - i_t^* - \frac{E(ds_t)}{dt}$ instantenous risk premium

(4f) $v_t = -\alpha y_t + q_t + c_t \psi_t - p_t^* + m_t + c_t i_t^*$ fundamental.

well not mentioned yet. The third factor is the market expectation for the $T-t$ ahead log exchange rate denoted by $x_{T,t}$. We assume that expectations are formed rationally in the sense that the *subjective expectation of the market participants for the $T-t$ ahead log exchange rate* is the mathematical expected value conditional on all the information available at the time the expectation is formed

$$x_{T,t} = E_t(s_T) \quad . \tag{2}$$

As we will see, there is more than one equilibria in this model and expectations determine which of the equilibria is attained. Or in other words, expectations are self fulfilling, because no matter what are the expectations, the exchange rate converges to the expected exchange rate.

2.1 Dynamics

This Section specifies the processes of the factors $x_{T,t}$, v_t, and c_t. These processes will be used to derive the process of the exchange rate.

The factors are assumed to follow Brownian motions. This assumption can be decomposed into an assumption on the martingale property of the processes and into the Gaussian distribution of the innovations. The Gaussian distribution of the innovations is assumed only for technical reasons. The martingale property of $x_{T,t}$ and v_t, however, can be explained along the following lines.

First, if the *law of iterated expectations* holds then the process of $x_{T,t}$ is martingale. The law of iterated expectations can be captured by the following formula: $E_t(E_{t+1}(s_T)) = E_t(s_T)$. By substituting Equation (2), the definition of $x_{T,t}$, into the previous formula, we get that $E_t(x_{T,t+1}) = x_{T,t}$. That is the process of $x_{T,t}$ is martingale.

Second, it makes sense not to have a constant trend in the process of the fundamental v_t, because the fundamental is usually defined as the relative values of some macro variables in the foreign and domestic countries. If we strongly believe that the countries are not diverging in terms of these variables, then we can rule out to have constant trend in v_t. Stochastic trend, like that of an error correction model could be considered, however, we opt to work with the simplest model. The martingale property of v_t allows us to focus entirely on the dynamics caused by the expectations and the stochastic discount factor, as opposed to the effects of predictable future changes in the fundamental.

The process of the market expectation for any $T-t$ ahead log exchange rate $x_{T,t}$ is

$$dx_{T,t} = \begin{cases} \sigma_{x,T,t} dw_{x,T,t} \ , & \text{if } t < T \\ 0 \ , & \text{otherwise} \end{cases} \tag{3}$$

Where $dw_{x,T,t}$ is a Wiener process. The parameter $\sigma_{x,T,t}$ is the time-varying volatility.

The assumed process for v_t is

$$dv_t = \sigma_{v,t} dw_{v,t} \quad . \tag{4}$$

Where $\sigma_{v,t}$ is the volatility of the fundamental.

The discount factor c_t, the third factor of the exchange rate model, is also assumed to follow a stochastic process. In contrast, the restriction of having constant c_t is almost always routinely imposed in the exchange rate literature. Here, we relax this assumption.

Motivated by the possible interpretation of the discount factor as being the semi-elasticity of money demand, we treat it as being non-negative. Therefore, a geometric Brownian motion is assumed to generate the time series of $\frac{1}{c_t}$:

$$\frac{d\frac{1}{c_t}}{\frac{1}{c_t}} = \sigma_{c,t} dw_{c,t} \quad , \tag{5}$$

The Wiener process $dw_{v,t}$ is not necessarily independent of $dw_{x,T,t}$. Similarly, the Wiener process $dw_{c,t}$ may correlate with $dw_{x,T,t}$, and $dw_{v,t}$. For technical reasons, we impose the following restriction on the correlations

$$\rho(dw_{c,t}, dw_{x,T,t})\sigma_{x,T,t} - \rho(dw_{c,t}, dw_{v,t})\sigma_{v,t} = (T-t)\,(x_{T,t} - v_t)\frac{\sigma_{c,t}}{c_t} \quad . \tag{6}$$

2.2 Functional Relationship Between The Exchange Rate And The Underlying Factors

This section derives the functional relationship $s_t = f\left(t, v_t, x_{T,t}, \frac{1}{c_t}\right)$ between the exchange rate on the one hand and the fundamental v_t, the market expectations for the $T-t$ ahead exchange rate $x_{T,t}$ and the stochastic discount factor c_t. This function should satisfy not only Equation (1), but also the following terminal condition. Expectations formed at time T on the spot exchange rate $x_{T,T}$ should simply be equal to the spot exchange rate s_T.

$$f\left(T, v_T, x_{T,T}, \frac{1}{c_T}\right) = x_{T,T} \quad . \tag{7}$$

Section 2.2.1 presents the derivation under the assumption of having constant discount factor. Then this assumption is relaxed, and Section 2.2.2 derives the function in the general case with stochastic discount factor.

The derivation has the following two steps in both cases. First, the process of the log exchange rate s_t is derived from the processes of the factors by using Ito's stochastic change-of-variable formula. Second, we obtain that the function satisfying the derived process, Equation (1), and the terminal condition (7) is

$$s_t = f\left(t, v_t, x_{T,t}, \frac{1}{c_t}\right) = \left(1 - e^{-\frac{T-t}{c_t}}\right) v_t + e^{-\frac{T-t}{c_t}} x_{T,t} \quad . \tag{8}$$

It is important to notice that the exchange rate s_t does not depend on the arbitrarily choosen expectation horizon $T-t$. Therefore, the exchange rate is the same no matter the expectation horizon is 1-year (1Y), or 2-years (2Y), or it has any other non-negative value $T-t$.

$$s_t = f\left(t, v_t, x_{t+1Y,t}, \frac{1}{c_t}\right) = \left(1 - e^{-\frac{1Y}{c_t}}\right) v_t + e^{-\frac{1Y}{c_t}} x_{t+1Y,t} \quad . \tag{9}$$

$$s_t = f\left(t, v_t, x_{t+2Y,t}, \frac{1}{c_t}\right) = \left(1 - e^{-\frac{2Y}{c_t}}\right) v_t + e^{-\frac{2Y}{c_t}} x_{t+2Y,t} \quad . \tag{10}$$

Equation (8) shows that the log exchange rate is the weighted average of the fundamental and the expected $T-t$ ahead log exchange rate. The relaitve weights depend on two things: the expectation horizon and the discount factor. If the horizon is infinite, or in other words $T-t=\infty$, then the weight of the fundamental is one and the weight of the expected exchange rate is zero. As the time until T decreases, the weight of the expected exchange rate increases. Finally, as the time until T approaches zero, the weight of the expected exchange rate approaches one.

Similarly, the relative weight of the expected exchange rate is increasing in c_t. If $c_t = 0$, then the weight of the fundamental is one and the weight of the expected exchange rate is zero. While in the other extreme case, when $c_t = \infty$, the weight of the fundamental is zero and the weight of the expected exchange rate is one. As we will see in Section 6, it is highly important to analyze the relative weights, because the relative weights influence substantially the behavior of the exchange rate.

2.2.1 Constant Discount Factor

In this Section, the discount factor is assumed to be constant $c_t = c$. According to Ito's formula, the function $f\left(t, v_t, x_{T,t}, \frac{1}{c}\right)$ should satisfy (11).

$$df = \left[\frac{\partial f}{\partial t}+\frac{\partial f}{\partial v_t}\mu_{v,t}+\frac{\partial f}{\partial x_{T,t}}\mu_{x,T,t}+\frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2+\frac{1}{2}\frac{\partial^2 f}{\partial x_{T,t}^2}\sigma_{x,T,t}^2+\frac{1}{2}\frac{\partial^2 f}{\partial x_{T,t}\partial v_t}\rho\left(dw_{v,t}, dw_{x,T,t}\right)\sigma_{v,t}\sigma_{x,T,t}\right]dt+ \\ +\frac{\partial f}{\partial v_t}\sigma_{v,t}dw_{v,t}+\frac{\partial f}{\partial x_{T,t}}\sigma_{x,T,t}dw_{x,T,t} \quad . \tag{11}$$

The different μ's denote the drift terms, whose values are zero in Equations (3) and (4).

The solution for $f\left(t, v_t, x_{T,t}, \frac{1}{c}\right)$ that satisfies (1), (7) and (11) is given by (8). Appendix B presents the proof for the general case with stochastic discount factor.

By substituting (3), (4), and (8) into Equation (11), we obtain the dynamics of the exchange rate:

$$ds_t = \frac{1}{c}\frac{e^{-\frac{T-t}{c}}}{1-e^{-\frac{T-t}{c}}}\left(x_{T,t}-s_t\right)dt+\left(1-e^{-\frac{T-t}{c}}\right)\sigma_{v,t}dw_{v,t}+e^{-\frac{T-t}{c}}\sigma_{x,T,t}dw_{x,T,t} \quad . \tag{12}$$

Equation (12) shows that the dynamics of the exchange rate is such that it converges to the actual market expectation for the future exchange rate. Moreover, the shorter the expectation horizon, the faster the convergence is. The deviation from this trend is due to the stochastic innovations ($dw_{v,t}$, $dw_{x,T,t}$) of the factors; consequently, the instantaneous volatility of the exchange rate depends on the joint distribution of these innovations.

2.2.2 Stochastic Discount Factor

Here, it is assumed that the discount factor c_t is stochastic and its process is given by (5). The function $f\left(t, v_t, x_{T,t}, \frac{1}{c_t}\right)$ is derived under the assumption of stochastic discount factor similarly to the deterministic case. The solution is again given by (8), however, this finding depends on restriction (6).

The Ito calculus can be used again to find the function $f\left(t, v_t, x_{T,t}, \frac{1}{c_t}\right)$. By using Ito's stochastic change-of-variable formula, we obtain a similar expression for df as previously with constant discount factor, however some new terms appear in the formula.

$$df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_{T,t}}\mu_{x,T,t} + \frac{\partial f}{\partial \frac{1}{c_t}}\mu_{c,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_{T,t}^2}\sigma_{x,T,t}^2 + \right.$$
$$+ \frac{1}{2}\frac{\partial^2 f}{\partial \left(\frac{1}{c_t}\right)^2}\left(\frac{\sigma_{c,t}}{c_t}\right)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial \frac{1}{c_t}\partial x_{T,t}} Cov\left(d\frac{1}{c_t}, dx_{T,t}\right) +$$
$$\left. + \frac{1}{2}\frac{\partial^2 f}{\partial \frac{1}{c_t}\partial v_t} Cov\left(d\frac{1}{c_t}, dv_t\right) + \frac{1}{2}\frac{\partial^2 f}{\partial x_{T,t}\partial v_t} Cov\left(dv_t, dx_{T,t}\right)\right] dt +$$
$$+ \frac{\partial f}{\partial v_t}\sigma_{v,t} dw_{v,t} + \frac{\partial f}{\partial x_{T,t}}\sigma_{x,T,t} dw_{x,T,t} + \frac{\partial f}{\partial \frac{1}{c_t}}\frac{\sigma_{c,t}}{c_t} dw_{c,t}. \quad (13)$$

The parameters $\mu_{v,t}$, $\mu_{x,T,t}$, and $\mu_{c,t}$ are zero, because the processes of the factors are driftless.

We obtain again that the function satisfying Equation (1), the terminal condition (7) and the derived process (13) is given by (8). The proof can be found in Appendix B.

In order to examine the exchange rate dynamics of the model, we substitute (3), (4), (5), (6) and (8) into Equation (13). The dynamics of the exchange rate with stochastic discount factor is:

$$ds_t = \frac{1}{c_t}\frac{e^{-\frac{T-t}{c_t}}}{1-e^{-\frac{T-t}{c_t}}}\left(x_{T,t} - s_t\right) dt +$$
$$+ \left(1 - e^{-\frac{T-t}{c_t}}\right)\sigma_{v,t} dw_{v,t} + e^{-\frac{T-t}{c_t}}\sigma_{x,T,t} dw_{x,T,t} - e^{-\frac{T-t}{c_t}}\left(T-t\right)\left(x_{T,t} - v_t\right)\frac{\sigma_{c,t}}{c_t} dw_{c,t} \quad . \quad (14)$$

Similarly to the constant discount factor model, the dynamics of the exchange rate is such that it converges to the actual market expectation for the future exchange rate. Moreover, the closer is T, and the higher is the discount factor c_t, the faster the convergence is. The deviation from trend is not only due to the stochastic innovations $dw_{v,t}$ and $dw_{x,T,t}$, but also due to $dw_{c,t}$.

2.3 Expectated Exchange Rate As A Function Of The Horizon

Equation (8) provides us not only the functional relationship between the factors and the exchange rate, but also the expectations as a function of the expectation horizon. We refere to this function as the *expectation scheme*. The exchange rate expectations for any horizon can be expressed as a function of the spot exchange rate, the discount factor c_t and the fundamental v_t. By rearranging Equation (8) we obtain

$$x_{T,t} = e^{\frac{T-t}{c_t}}\left(s_t - v_t\right) + v_t \quad . \quad (15)$$

Figure 2 demonstrates the expectation scheme by using a simple numerical example. In this numerical example $s_t = 1$, $v_t = .9$, and $c_t = 2$. It is important to notice that the expected future exchange rate is highly non-linear in $T - t$. Whether this type of non-linearity is an essential property of the model or not, will be tested in the next Section.

Equation (15) is not very useful, since it provides a formula for the expected future exchange rate as a function of two latent factors, the fundamental v_t, and the discount factor c_t. It is better to have the expectation scheme as a function of some observable variables. Figure 2 suggests that by fitting a curve on some data points, the spot exchange rate and the expected exchange rates for some different horizons, we obtain the exchange rate expectation for any other horizon. Since we have survey data on exchange rate forecasts for the 3-months, 1-year and 2-years ahead exchange rates, therefore not only the spot exchange rate is observable, but also the expectations. In the following, we show analytically that the curve representing the expectation scheme is the exponential function, and therefore it is sufficient to have only three data points in order to obtain any other point of the curve. For instance, it is sufficient to have data on the spot exchange rate and the expected 1-year and 2-years ahead exchange rates. If these are strictly monotonous in the forecast horizon ($s_t = x_{t,t} < x_{t+1Y,t} < x_{t+2Y,t}$ or $s_t = x_{t,t} > x_{t+1Y,t} > x_{t+2Y,t}$), then they can be used to express the c_t and v_t parameters of the exponential function (15) by rearranging Equations (9) and (10).

$$c_t = \begin{cases} 0 \, , & \text{if } x_{t+1Y,t} = \frac{x_{t+2Y,t}+s_t}{2} \\ -\frac{1}{log\left(\frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}}\right)} \, , & \text{otherwise} \end{cases} \tag{16}$$

$$v_t = \begin{cases} s_t \, , & \text{if } x_{t+1Y,t} = \frac{x_{t+2Y,t}+s_t}{2} \\ \frac{-x^2_{t+1Y,t}+s_t x_{t+2Y,t}}{s_t+x_{t+2Y,t}-2x_{t+1Y,t}} \, , & \text{otherwise} \end{cases} \tag{17}$$

By substituting the expressions (16) and (17) for c_t and v_t into Equation (15) we obtain (18).

$$x_{T,t} = \begin{cases} (1-T+t)s_t + (T-t)x_{t+1Y,t} \, , & \text{if } x_{t+1Y,t} = \frac{x_{t+2Y,t}+s_t}{2} \\ \frac{(s_t-x_{t+1Y,t})^2}{s_t+x_{t+2Y,t}-2x_{t+1Y,t}} \left(\frac{x_{t+2Y,t}-x_{t+1Y,t}}{x_{t+1Y,t}-s_t}\right)^{T-t} + \frac{s_t x_{t+2Y,t}-x^2_{t+1Y,t}}{s_t+x_{t+2Y,t}-2x_{t+1Y,t}} \, , & \text{otherwise} \end{cases} \tag{18}$$

Equation (18) will be used in the remaining part of the paper for testing the asset pricing exchange rate model. Equation (18) is equivalent to Equation (8). Equation (8) was derived from Equations (1)-(6). Therefore, if the data support Equation (18), then they are also consistent with Equation (1), the main equation of the asset pricing view of the exchange rate. If the empirical test happen to reject Equation (18), then it is better to foreget the exchange rate theory of the last 20 years. The good news for the asset pricing theory is that our test favours model (18) again some commonly used alternatives.

3 Survey-Based Test Of The Exchange Rate Model

This Section demonstrates the superiority of the non-linear time-varying parameter exchange rate model of Equation (8) relative to the simple linear model and to the constant parameter model. We use monthly survey data of the Consensus Economics on the expected 1-year and 2-years ahead exchange rates and the spot exchange rates on the days of the surveys. (See Figure 3). [9] First, we fit the models on these data. Than, we calculate

[9]The reported forecasts are not the expected log exchange rates, but the expected exchange rates. We approximate the expected log exchange rates by the log of the reported expected exchange rates in all

some measures on the goodness of fit of each of the competing models on the expected 3-months ahead exchange rate. In this way we get measures on the out-of-sample fit of the models, since the survey data on the 3-months forecasts are not used in the first step of estimation. Finally, the models are compared based on their out-of-sample performance.

We introduce a new notation for the expected θ ahead log exchange rate: $z_{t+\theta,t}$. The definition of $z_{t+\theta,t}$ is that it is the expectation formed at time t on the log exchange rate of time $t+\theta$, *i.e.*, $z_{t+\theta,t} = E_t(s_{t+\theta})$. The survey data on the forecasted 3-months, 1-year, and 2-years ahead exchange rates will be denoted by $\tilde{z}_{t+.25Y,t}$, $\tilde{z}_{t+1Y,t}$, and $\tilde{z}_{t+2Y,t}$ respectively.

We need these new notations for the following reasons. First, the survey data $\tilde{z}_{t+\theta,t}$ may deviate from its true value $z_{t+\theta,t}$, because of measurement error in the surveys. Second, although we have already introduced the notation $x_{T,t}$ for the expectation, the time series of $x_{T,t}$ is different from that of $z_{t+\theta,t}$. One of the expectations is formed on the future exchange rate of time T, whereas the other is formed on the θ ahead exchange rate. Figure 1 demonstrates the difference between the two concepts. We give examples on both types of expectations in order to make clear the difference. The time series of the first type of expectations can be obtained from a regular survey on the exchange rate of the year 2525, for instance.[10] Our time series data on the expected 3-months ahead exchange rate, for instance, is of the second type as consisting of monthly forecasts of the market analysts on the 3-months ahead exchange rate. Obviously, $x_{T,t} = z_{t+\theta,t}$, when the constant T happens to be equal to $t+\theta$. In that sence, there is a unique bijection between $x_{T,t}$ and $z_{t+\theta,t}$.

In order to distinguish between the two expectations, we will refere to $z_{t+\theta,t}$ as the Zexpectation© and to $x_{T,t}$ as the eXpectation©.[11]

One might ask the question why do we have eXpectation© in the model, if our data are on Zexpectation© and there is a unique bijection between the two. The answer to this question is that it is easier to build a model with factors that have martingale processes. The process of eXpectation© is martingale if the low of iterated expectations holds. Surprisingly, Zexpectation© is not martingale unless the market participants expect the same exchange rate for every horizons.[12] Appendix A proves that if the expectation scheme is not flat, *i.e.*, the market does not believe in the random walk behaviour of the exchange rate, then the process of Zexpectation© is not martingale. Moreover, by assuming that

calculations and estimations. An even more precise approximation would be based on adjusting by half of the variance. If the percentage change of the exchange rate has Gaussian distribution, the expected log exchange rate is usually approximated by the log of the expected exchange rate decreased by half of the variance. In our case, the distribution is different from the Gaussian distribution, as the exchange rate being the weighted average of two lognormally distributed variables. Still, both approximations work well according to a simulation-based test. The difference between the approximations are negligible, therefore, we apply the simple one. All results obtained with the other approximation are available from the author upon request.

[10] "In the year 2525, if the man is still alive..."

[11] It is highly important to notice that the interpretation of s_t is not effected by the introduction of Zexpectation© and eXpectation©. It remains the spot exchange rate and we do not change it to Sexpectation©.

[12] In the literature of exchange rate, the martingale property of Zexpectation© is often assumed mistakenly.

the process of Zexpectation© is martingale, one also assumes implicitely that the process of the exchange rate is martingale.

We compare four models based on their fit on the survey data. The first model is the non-linear time-varying parameter model that nests all the other models. Therefore, we refere to it as the *general model*. The second model is a *linear model*. The third model is the *random walk model (RW)*. Finally, the fourth model is a *constant parameter model*.

The estimates on the expected 3-months ahead log exchange rate $\widehat{z}_{t+.25Y,t}$ can be obtained for the *general model* by substituting $x_{t+1Y,t} = \tilde{z}_{t+1Y,t}$, $x_{t+2Y,t} = \tilde{z}_{t+2Y,t}$, and $T = t + .25Y$ into Equation (18).

$$\widehat{z}^{general}_{t+.25Y,t} = \begin{cases} .75s_t + .25\tilde{z}_{t+1Y,t}\ , & \text{if } \tilde{z}_{t+1Y,t} = \frac{\tilde{z}_{t+2Y,t}+s_t}{2} \\ \frac{(s_t-\tilde{z}_{t+1Y,t})^2}{s_t+\tilde{z}_{t+2Y,t}-2\tilde{z}_{t+1Y,t}}\left(\frac{\tilde{z}_{t+2Y,t}-\tilde{z}_{t+1Y,t}}{\tilde{z}_{t+1Y,t}-s_t}\right)^{.25Y} + \frac{s_t\tilde{z}_{t+2Y,t}-\tilde{z}^2_{t+1Y,t}}{s_t+\tilde{z}_{t+2Y,t}-2\tilde{z}_{t+1Y,t}}\ , & \text{otherwise} \end{cases} \tag{19}$$

The estimates on $\widehat{z}_{t+.25Y,t}$ under the other three model specifications are given by Equation (19) and the parameter restictions of each models.

In the *second model*, *i.e.*, the *linear model* with restriction $e^{-\frac{1Y}{c_t}} = 0$, the estimated expected 3-months ahead log exchange rate is the linear interpolation of the expectations with the two closest horizons. [13] Therefore, we interpolate the reported expected 1-year ahead log exchange rate $\tilde{z}_{t+1Y,t}$ and the expected 0-year ahead log exchange rate, *i.e.*, the log spot exchange rate s_t.

$$\widehat{z}^{linear}_{t+.25Y,t} = .75s_t + .25\tilde{z}_{t+1Y,t} \quad . \tag{20}$$

In the *third model* with restriction $e^{-\frac{1Y}{c_t}} = 1$, the expected exchange rates of any future T are equal to the spot exchange rate. (See Equation (15).) This feature of the model motivates us to call it the *random walk model*.

$$\widehat{z}^{RW}_{t+.25Y,t} = s_t \quad . \tag{21}$$

Finally, the parameter restriction of the *fourth model* is that $e^{-\frac{1Y}{c_t}} = e^{-\frac{1Y}{c}}$ and $e^{-\frac{1Y}{c_t}} \neq 0$ and $e^{-\frac{1Y}{c_t}} \neq 1$. In this *constant parameter model*, the estimates for $z_{t+.25Y,t}$ is given by Equations (22) and (23).

$$\widehat{z}^{const}_{t+.25Y,t} = e^{\frac{.25Y}{\widehat{c}}}(s_t - \widehat{v}_t) + \widehat{v}_t \quad . \tag{22}$$

$$min_{c,v_{\underline{\tau}},\ldots,v_{\overline{\tau}}} \sum_{t=\underline{\tau}}^{\overline{\tau}} (\tilde{z}_{t+1Y,t} - \widehat{z}^{const}_{t+1Y,t})^2 + (\tilde{z}_{t+2Y,t} - \widehat{z}^{const}_{t+2Y,t})^2 \quad , \tag{23}$$

where $\widehat{z}^{const}_{t+1Y,t} = e^{\frac{1Y}{\widehat{c}}}(s_t - \widehat{v}_t) + \widehat{v}_t$, and $\widehat{z}^{const}_{t+2Y,t} = e^{\frac{2Y}{\widehat{c}}}(s_t - \widehat{v}_t) + \widehat{v}_t$. And the sample period is between date $\underline{\tau}$ and $\overline{\tau}$.

Equation (22) can be derived by substituting $T - t = .25Y$ and estimates for the constant parameter $\widehat{c}$ and for the time-varying fundamental $\widehat{v}_t$ into Equation (15). The

[13] As an alternative to the linear interpolation between two data points, we could fit a linear model on all three data points s_t, $\tilde{z}_{t+1Y,t}$ and $\tilde{z}_{t+2Y,t}$. However, the relative out-of-sample performance of the model estimated on this alternative way is worse than that of the linearly interpolated one.

estimates $\widehat{c}$ and $\widehat{v}_t$ are obtained by the least square method (LS). These satisfy the optimization problem of Equation (23).

The minimization problem is multi-dimensional. In general, multi-dimensional optimizations rais numerical problems. Luckily, our problem can be reduced into a single-dimensional optimization problem by utilizing the following analytical solution. Equation (24) solves the minimization problem of Equation (23) for any given constant $c \neq \pm\infty$, and time $t \in [\underline{\tau}, \overline{\tau}]$.

$$\widehat{v}_t = \frac{2e^{\frac{1Y}{c}} s_t[\tilde{z}_{t+1Y,t} - 1 + (\tilde{z}_{t+2Y,t})(e^{\frac{1Y}{c}} + 1)e^{\frac{1Y}{c}}]}{2 - e^{\frac{2Y}{c}} - e^{\frac{3Y}{c}}}, \text{if } e^{\frac{1Y}{c}} \neq 1 \quad . \tag{24}$$

With the analytical solution of Equation (24) in hand, what remains to be done numerically, is only the optimization of the objective function with respect to the constant c. We have found the optimum for almost all currency pairs except for CAD/USD, USD/EUR, CHF/EUR.

Once we have the estimates of each models on the 3-months ahead exchange rate forecast, we compare them by using some standard measures on the goodness of fit. The goodness of fit on the survey forecast is measured by the mean absolute error (MAE) and the root mean squared error (RMSE).

$$\text{MAE} = \text{mean}(\text{abs}(\tilde{z}_{t+.25Y,t} - \widehat{z}_{t+.25Y,t})) \quad , \tag{25}$$

$$\text{RMSE} = (\text{mean}(\tilde{z}_{t+.25Y,t} - \widehat{z}_{t+.25Y,t})^2)^{\frac{1}{2}} \quad , \tag{26}$$

where $\tilde{z}_{t+.25Y,t}$ and $\widehat{z}_{t+.25Y,t}$ denote the survey based expectation and the estimated expectation on the 3-months ahead log exchange rate respectively.

Table 1 shows that the non-linear time-varying parameter model performs better for almost all currency pairs according to both measures (MAE, RMSE). The only exception is the JPY/USD, where the random walk model fits almost perfectly the forecasted 3-months ahead exchange rate. This is not surprising, because the Japanese economy is often argued to be characterized by liquidity trap [14] in the investigated period. Theoretically, we have liquidity trap exactly at those times when the parameter restriction of the random walk model is fulfilled, $i.e.$, $e^{-\frac{1Y}{c}} = 1$.

Whether the out-of sample fit of the general model is significantly better than that of the nested models should be tested statistically. [15] However, we find our estimates convincing enough to say that the differences in the goodness of fit under different model specifications are not only due to sample variations. Our reasons for that are the following. First, the estimates are carried out on twelve currency pairs among which eleven do support our hypothesis of the dominance of the general model. Second, the following

[14]Liquidity trap occurs when the monetary authority is unable to stimulate the economy with traditional monetary policy tools. For instance, if the interest rate semi-elasticity of money demand (parameter c in the money income model) is infinitly large. When the nominal interest rate is close or equal to zero, like in Japan in the last few decades, then there is a high chance to be in this unfavorable situation.

[15]Granger and Newbold (1977), Ashley et al. (1980), Hansen (1982), Diebold and Mariano (1995), West (1996) and Clark and West (2006) propose different test statistics for that purpose. Unfortunately, non of them can be applied here, because of the nested nature of the competing models and the non-linearity of the general model.

theoretical consideration makes it needless to test the significance of the difference in order to reject the null. In case of having nested models, the out-of sample performance of the broader model is never better than that of the restricted model under the null that the data are generated by the restricted model. This finding is proved analytically by Clark and West (2006) and has been demonstrated by simulations by McCracken (2004). The intuitive explanation for the finding is that the broader model is flexible enough to learn sample specific regularities that are disadvantageous in the out-of sample prediction. [16]

It is worth to discuss the sample sizes reported by Table 1. Our sample period is spanned by January 11, 1999 and June 11, 2007. The surveys are on a monthly frequency; therefore, the size of the time dimension of the entire sample is 101. However, the number of observations differ from 101 for almost all the currency pairs in Table 1, because we have worked only on subsamples. The exchange rate specific subsamples are defined by the monotony condition. We either have $s_t = z_{t,t} \leq \tilde{z}_{t+1Y,t} \leq \tilde{z}_{t+2Y,t}$, or $s_t = z_{t,t} \geq \tilde{z}_{t+1Y,t} \geq \tilde{z}_{t+2Y,t}$ in the subsample. Unfortunately, the reported forecasts in the entire sample do not always fulfill the monotony condition. [17] The failure to fulfill the monotony condition contradicts not only to the non-linear time-varying parameter model, but also to all the other three alternative models. Whenever the monotony condition is not fulfilled by a given cross-sectional data, *i.e.*, the reported averaged forecasts of different horizons in a monthly survey, we exclude these observations.

The exclusion of these observations do not influence the following findings. First, the discount factor is time-varying. Second, the results summarized by Table 1 clearly show that the non-linear time-varying parameter exchange rate model has better out-of-sample performance than the simple linear model and the constant parameter model. This result has another interpretation that focuses on the number of factors: our three-factor model dominates the models with one or two factors. We have demonstrated these findings on the largest sample of internally consistent survey data that these models can be estimated on.

4 Testing The Exchange Rate Model Based On Its Forecasting Performance

Our three-factor model has gained empirical support in the previous Section by having been tested whether the way market analysts generate their forecasts is closer to the one implied by the three-factor model or to the one implied by simpler models with less factors. It turned out that the representative professional exchange rate forecaster has

[16] The reason why the finding can be somewhat surprising is the fact that exactly the opposite holds for the in-sample fit, *i.e.*, the broader model can not perform worse than the restricted one.

[17] The most likely reason why the monotony condition is violated by the reported averaged forecast is that it reflects the aggregated views of heterogeneous agents. Even if the expectation scheme of each forecaster is monotone in the forecast horizon, the averaged forecasts is not necessarily monotone. The simplest example that can demonstrate this idea is the one with only two market analysts. One with increasing expectations scheme, the other with decreasing one.

an exchange rate model in her mind that can be represented by our three-factor model the best among the four models. Moreover, the survey-based estimates suggests that the exchange rate is largely determined by the expectations. Therefore, it was not extremely surprising if our model would fit the realized exchange rate just as well as it fits the survey data.

This Section tests the forcasing ability of the model and the survey data. We calculate some measures of the forecast accuracy (MAE, RMSE) and a measure of profitability of a simple trading strategy. Than, these measures are used to compare the forecasting performance of the general model and the random walk model. For correct comparison the measure of the forecasting performance of the random walk alternative is calculated only for those months when the general model provided us a forecast, *i.e.*, when the monotony condition was fulfilled.

The profitability measure is the same as the one used for instance by MacDonald and Marsh (1996), Boothe (1983), Boothe and Glassman (1987). If the domestic currency is forecasted to be stronger than that indicated by the forward rate ($z_{t+\theta,t} < s_t + (i_{t,\theta} - i^*_{t,\theta})\theta$), then the currency is bought. If the domestic currency is forecasted to be weaker ($z_{t+\theta,t} > s_t + (i_{t,\theta} - i^*_{t,\theta})\theta$) then it is sold. Along these lines, it is the sign of $z_{t+\theta,t} - s_t - (i_{t,\theta} - i^*_{t,\theta})\theta$ that determines whether to buy or sell the currency. The percentage profit earned on each trade is the difference between the log realized exchange rate at time $t + \theta$ and the log forward rate $s_t + (i_{t,\theta} - i^*_{t,\theta})\theta$. The profits and losses of these trades are cumulated and than divided by the number of forecasts. Finally, the profit is annulaized by multiplying it by $\frac{1Y}{\theta}$. In that way we obtain a measure on the percentage annualized profit per trade π.

$$\pi = \text{mean}\left[\left(s_{t+\theta} - s_t - (i_{t,\theta} - i^*_{t,\theta})\theta\right) \text{sign}(\widehat{z}_{t+\theta,t} - s_t - (i_{t,\theta} - i^*_{t,\theta})\theta)\right] \frac{1Y}{\theta} \quad . \tag{27}$$

This measure of profit is calculated both for the random walk model and for the general model. [18] The forecast of the random walk model is simply the spot exchange rate $\widehat{z}^{RW}_{t+\theta,t} = s_t$. Whereas, for the general model, it is given by the model of Equation (18) fitted on three data points s_t, $\tilde{z}_{t+1Y,t}$ and $\tilde{z}_{t+2Y,t}$.

The lower the MAE and RMSE, the better is the forecasting performance. Whereas in case of the third measure of the forcasting performance, the profit, this relationship is just the opposit. The higher the profit is, the better is our forecast. In order to avoid problems comming from the different signs, we report in Tables 3-6. the negative profit, *i.e.*, the loss of the model based trading strategies.

According to the measures of MAE and RMSE, the random walk model is better than the general model for all the exchange rates for the 3-months forecast horizon. For the longer horizons (one year and two years) the forecasting performance of the general model is somewhat better. It can beat the random walk for some exchange rates.

[18] **I have used a simpler version of Equation (27), becuse of not having collected the interest rate data yet. This simpler model is the following. Buy if the currency is expected to strenthen. Sell if the currency is expected to weaken. And do not take into account the profit coming from the interest rate differential.**

$$\pi = \text{mean}\left[(s_{t+\theta} - s_t)\,\text{sign}(\widehat{z}_{t+\theta,t} - s_t)\right] \frac{1Y}{\theta} \quad . \tag{28}$$

.

In contrast to MAE and RMSE the profitability measure of the forecasting performance clearly shows that the general model performes better than the random walk. This highly remakable performance of the general model comes partly from the survey data, or in other words from the ability of the forecasters to predict the direction of the changes in the exchange rate. And it is partly due to the model that provides a theoretical link between the forcasts of different forcast horizons. The contribution of the model to the forecasting performance is evident from the comparison of the results in Table 3 and 4. Table 3 reports the profit that can be earned by using the survey data on the 3-months forecasts. Whereas Table 4 reports the profit that can be earned by applying the model based forecast of the same horizon. The profit is higher for 8 currencies out of 11 if we use the model and not only the survey data.

An interesting finding that emerged from our evaluation of the potential profitability of following the advice given by the forecasters is that even when the performance of forecasters is poor, as measured by standard statistical criteria (RMSE, MAE), they may still produce forecast advice that would have been profitable to follow. In a finite sample, however, the mere existence of positive returns is not sufficient evidence of forecast ability since the profit also needs to compensate for the risk associated with the trading positions.

MacDonald and Marsh (1996) use panel data of exchange rate forecasts of individual forecasters. They data is also from the Consensus Economics. The sample is spanned between October 1989 September 1992. They calculate the same measures on profitability and report the number of forecasters having excess profit relative to the random walk forecast. Out of 6 combinations of currencies and horizons (currencies: $\frac{DEM}{USD}$, $\frac{JPY}{USD}$,$\frac{USD}{GBP}$, horizons: 3-months, 12-months) there is only one, where the forecasters with positive excess return are in majority. In the other 5 cases less than half of the forecasters could out-perform the random walk. They do not report any statistics on the forecasing performance of the mean forecast, therefore it is not straightforward how to compare their results with ours. However, if the aggregated forecast is just as disappointing as the forecasting performeance of that of the individuals, then we could say that either the forecasters have improoved over time or the exchange rates have become more forecastabale.

Recently, Darvas and Schepp (2007) were able to systematically beat the random walk at forecasting the exchange rate with their statistical model using interest rate data. We think that the highly remarkable forecasting ability of their model can be explained by the following. First, Hondroyiannis et al (2001) show that the interest rate semi-elasticity of money demand, that is equivalent to the inverse of the discount rate in some macro models, is not only time varying, but it is a decreasing function of the interest rate differential. Therefore, the interest rate data used by Darvas and Schepp (2007) can proxy the discount factor. Second, as it is documented by this paper, the discount factor has a highly important role at determining the exchange rate.

5 Survey-Based Estimated Time Series Of The Factors

The survey data and the general model can be used not only to estimate the expected 3-months ahead log exchange rate, but also to estimate the factors c_t, v_t and $x_{T,t}$ for any

$T > t$. This Section presents the estimates for the time series of c_t and v_t.

Just like in the previous Section, we will use only three data points s_t, $\tilde{z}_{t+1Y,t}$, $\tilde{z}_{t+2Y,t}$ for estimation, but not $\tilde{z}_{t+.25Y,t}$. The advantage of this approach is that we can simply invert out c_t, v_t, $x_{T,t}$ from the three observations by substituting the survey data into Equations (18), (16) and (17). In contrast, if we would use the data on $\tilde{z}_{t+.25Y,t}$ as well, then we had over-identification. Estimation in case of over-identification requires to make assumptions on the errors the survey data are contaminated with. In this Section we will not take into account these errors. However, we are aware of that the estimated factors $\widehat{c}_t$, $\widehat{v}_t$, $\widehat{x}_{T,t}$ can deviate from their true values c_t, v_t, $x_{T,t}$.

$$\widehat{c}_t = c_t + \epsilon_{c,t} \quad , \tag{29}$$

$$\widehat{v}_t = v_t + \epsilon_{v,t} \quad , \tag{30}$$

$$\widehat{x}_{T,t} = x_{T,t} + \epsilon_{x,T,t} \quad . \tag{31}$$

5.1 Filtered Discount Factor

Figure 4 and 5 show the time series of $e^{-\frac{1Y}{\widehat{c}_t}}$ estimated under different model specifications. It is restricted to zero in the linear specification and to unity in the random walk specification. Therefore, the interesting results are obtained with the other two models, the general model and the constant parameter model.

What the market analysts think about the importance of the fundamental at determining the exchange rate relative to that of the one-year ahead forecast is implied by the estimated relative weight $e^{-\frac{1Y}{c_t}}$. By analyzing Equation (8), we obtain that the relative absolute weight of the expectation $x_{T,t}$ is higher than that of the fundamental v_t, *i.e.*, $\text{abs}\left(e^{-\frac{T-t}{c_t}}\right) > \text{abs}\left(1 - e^{-\frac{T-t}{c_t}}\right)$, if and only if $e^{-\frac{T-t}{c_t}} > .5$. One can see from Figure 4 and 5 that $e^{-\frac{1Y}{\widehat{c}_t}}$ estimated by the general model exceeds the one-half tresh hold level most of the times for all currency pairs. Moreover, all estimates in the constant parameter specification are above one-half. These empirical results makes us think that the relative importance of the expectation is higher than that of the fundamental.

It is important to keep in mind that the previous finding is conditional on the expectation horizon. Even if the relative importance of the fundamental is smaller than that of the expected one-year ahead log exchange rate, it can be higher than that of the expected two-years ahead log exchange rate. Obviously, we prefere to say something about the relationship among the exchange rate, the fundamental, and the expectations that is not conditional on the arbitrarily choosen expectation horizon. Here we repeat and interpret such a previous result of the paper. We could reject the hypothesis that the fundamental is the only driving force of the exchange rate as rejecting the linear model against the general model in Section 3.

It is important to notice, that the sign of the estimated $\widehat{c}_t$ parameter is mostly negative. In these cases the transformed $e^{-\frac{1Y}{\widehat{c}_t}}$ is above the red line of unity. This finding disables us to interpret c_t as the interest rate semi-elasticity of money demand, because the latter should be positive. However, we can still interpret c_t as the discount factor that is determined by the interest rate differential and the risk premia. The interest rate differential is the difference between the domestic and foreign interest rates earned on risk free assets, like government notes or bonds with almost zero chance to default and with predetermined pay-off at maturity. And the risk premia is for compensating the risk associated with the stochastic nature of the future exchange rate, *i.e.*, the non-predetermined future pay-off.

Theoretically, the sign of the estimated c_t parameter depends on whether the survey forecasts are convex or concave and increasing or decreasing in the expectation horizon. Parameter c_t is positive if and only if the forecasts are monotone increasing and convex in $T-t$ or monotone decreasing and concave. [19] Whereas in case of increasing and concave expectations or decreasing and convex expectations scheme, c_t is negative. In case of linearity, c_t is zero. (See Appendix C).

We have missing data in Figure 4 and 5 for the general model for those months and countries, where the monotony condition is violated by the reported forecasts. As we see, $\widehat{c}_t$ is highly time-varying for all the analyzed exchange rates under the general specification. However, this can be taken only as a weak evidence for the time-varying nature of the discount factor, because it can not only be due to the time-varying nature of the true c_t, but also to the time-varying nature of the error $\epsilon_{c,t}$ that $\widehat{c}_t$ is contaminated with. The strong evidence that favors the time-varying specification is based on the out-of-sample performance discussed in Section 3.

To rewrite : In the following we compare our estimates on $e^{-\frac{1Y}{c}}$ with those used by Engel and West (2005). Appendix D shows that $e^{-\frac{1Y}{c}}$ is equivalent to the dicount factor b of the model by Engel and West (2005). Engel and West (2005) argue that b is close to one. Our empirical results contradict to their finding as our estimes on $e^{-\frac{1Y}{c_t}}$ are far from unity both in the time-varying parameter specification and in the constant parameter specification. This contradiction may come from the fact that we estime $e^{-\frac{1Y}{c_t}}$ directly from the data, whereas Engel and West (2005) use estimates on the semi-elasticity of money demand c in order to calculate their discount factor b. They simply plug in some estimates of Bilson (1978), Frankel (1979), Stock and Watson (1993) and Obstfeld and Rogoff (2003) for c into Equation (52) that provides a theoretical relationship between parameter c and b. The poential problems with this method are the following. First, it fails to take into account that the estimates on c are subject to errors and therefore the estimates on b will be biased as b is a non-linear function of the error. Second, the misspecification of the money demand equation can also result in biased estimates.

5.2 Filtered Fundamental

Figure 6 and 7 show the survey-based filtered fundamental v_t under different model specifications. The fundamental is not identified by the survey data under the random walk specification. Moreover, we could not estimate the constant parameter model for the exchange rates CAD/USD, USD/EUR and CHF/EUR. Therefore, Figure 6 and 7 can not show estimates for all specifications and for all currency pairs.

[19] One potential reason for having negative estimates for $\widehat{c}_t$ can be the aggregation. Even if the expectation scheme of each forecaster is in line with the theory, it is not necessarily true for the averaged forecasts. The simplest example that can demonstrate this idea is again the one with only two market analysts. One with increasing and convex expectations scheme, the other with decreasing and concave one.

Theoretically, the filtered fundamental could be used to find those macro variables that are the main determinants of the exchange rate. This is a highly challenging objective, because many papers, including this one, find that the fundamental is not necessarily always the most important determinant of the exchange rate. Moreover, model uncerataint may play an importantant role, that is reflected by the differences between the filtered fundamentals v_t under different model specifications. Finally, the link between the fundamental and the exchange rate is not stable over time and the time-varying relative weight of the fundamental is subject to estimation error. For all these reasons, the exchange rate can only weakly identify the fundamental.

Still, the survey-based method introduced by this paper may be useful at selecting the relevant macro fundamentals for the following reasons. First, it provids us some hint about the relative importance of the fundamental in the exchange rate. Once, we know which are those periods when the fundamental is likely to matter more, we can restrict our analysis to this subsample.

Second, the filtering method can be improved by taking into account simultaneously the errors in the survey data and the innovations in the transition equations (3), (4) and (5). In this paper, estimates on the fundamental have been carried out either by using only the transition equations or by using only the observation equations (29), (30), and (31). In the constant parameter specification, we had the implicit assumption that the variance of the innovations in the transition equation for c_t is zero. Whereas in the estimation of the general model the variance of the errors in the observation equation was set to zero. Once we have reliable estimates on the system covariance matrix and the covariance matrix of the errors in the survey data, we can set up the filtering problem as consisting of both the transition equations and the observation equations. [20]

6 Explaining Some Characteristics Of The Exchange Rate

This Section provides explanation for the disconnect puzzle, the excess volatility puzzle, the near random walk behaviour of the exchange rate, and some findings on the exchange rate forecasting ability of macro models. Moreover, it demonstrates that our model mimics some well known empirical anomalies of the exchange rate literature such as the frequent regime switches detected by statistical models and the fat tail distribution of the percentage changes in the exchange rate.

The theoretical model of this paper is not the first one, that can explain these puzzles and anomalies of the empirical exchange rate literature. Other multi-factor models with non-linear dynamics, like the heterogeneous agent model built by De Grauwe and Grimmaldi (2005) is also able to do that. In their paper the exchange rate is determined by the forecasts of different types of agents, the chartists and the fundamentalists. The advantage of our model relative to such heterogeneous agent model lies in its empirical applicability. Our model is so parsimonious that its single parameter that determines the

[20]The covariances can be estimated from the dispersion of the expectations of individual forecasters and option prices with different maturities. Naszodi (2008) utilizes option prices in a similar filtering problem.

relative importance of the fundamental in the exchange rate can be estimated even from survey data. Whereas it is more difficult to estimate the relative share of fundamentalist in the heterogeneous agent model.

6.1 Disconnect Puzzle

The disconnect puzzle refers to the failure to find empirical support for the link between the macro fundamentals and the exchange rate. Or in other words, the exchange rate is found to substantially deviate from its fundamentally justified value even for relatively long periods.

Our model provides the following theoretical explanation for the puzzle. The key feature of the model is that it is not only the fundamental that drives the exchange rate, but also two other factors. These factors can detour the exchange rate from the fundamental. Therefore, in light of our theoretical three-factor model it is not natural at all to require the exchange rate to be close to its fundamental value. Moreover, it would be surprising if the fundamental was the dominant determinant of the exchange rate despite of the fact that market analysts seem to think just the opposit according to our test.

The empirical works by Goodhart (1989), Goodhart and Figlioli (1991), and Faust et al. (2003) also support the multi-factor approach by showing that most of the changes in the exchange rates occur when there is no observable news in the macro fundamentals.

Nevertheless, requiring the fundamental to be the main driving force of the exchange rate is just as unintuitive as to require the current annual dividend of a public company to be the main determinant of the stock price. [21] We know that expectations on the future dividends are usually even more important than the current one. Analogously, the expectations should be more important for determining the exchange rate than the fundamental capturing only the current state of the economy.

As part of the empirical investigation of the link between the exchange rate and the fundamental, we analyze the wedge between the two under different model specifications. The *linear model* and the *random walk model* are the two extreme specifications in the sense, that no deviation is possible under the *linear model*. Whereas, no link can be expected between the fundamental and the exchange rate in the *random walk model*.

In the other two models, we calculate the deviation of the filtered fundamental from the log exchange rate. Its average magnitude is measured by the mean absolute deviation (MAD) and the root mean squared deviation (RMSD).

$$\text{MAD} = \text{mean}(\text{abs}(\widehat{v}_t - s_t)) \quad , \tag{32}$$

$$\text{RMSD} = (\text{mean}(\widehat{v}_t - s_t)^2)^{\frac{1}{2}} \quad , \tag{33}$$

Table 2 shows that the deviation of the exchange rate from its fundamentum can be very high. For the general model, the RMSD is above 20% for 6 exchange rates out of 11. And for the constant model, the 20% trash hold is exceeded by the RMSD

[21] The analogy between the stock price and exchange rate and between the fundamental and the dividend is straightforward from the asset pricing view that can be applied both to stocks and currencies. The asset pricing view is applied to stock prices inter alia by Campbell and Shiller (1987) (1988), West (1988).

of 4 exchange rates out of 8. The deviation is higher for the *general model* than for the *constant parameter model* acording to both measures for some exchange rates. (ILS/USD, JPY/USD, NGN/USD, ZAR/USD). In these cases, the three-factor model can account for larger deviation than the commonly used two-factor model.

6.2 Excess Volatility Puzzle

Many empirical works find that the volatility of the exchange rate is higher than the one implied by the fundamental. This is called the excess volatility puzzle.

The excess volatility puzzle can be explained along the same lines as the disconnect puzzle. Since it is not only the fundamental that determines the exchange rate and its volatility, the latter should not be equal to the volatility of the fundamental. Among the models we have investigated, it is only the linear model that implies $\sigma_{s,t} = \sigma_{v,t}$ by restricting $s_t = v_t$. Since the linear model has been rejected against the general three-factor model, it is not surprising that the volatility of the exchange rate depends not only on the volatility of one single factor, the fundamental, but also on the varinces and covariances of all three factors.

The instantaneous volatility of the exchange rate can be derived from Equation (13).

$$\begin{aligned}\sigma_{s,t}^2 = &\left(1-e^{-\frac{T-t}{c_t}}\right)^2\sigma_{v,t}^2 + e^{-2\frac{T-t}{c_t}}\sigma_{x,T,t}^2 + e^{-2\frac{T-t}{c_t}}(T-t)^2\left(x_{T,t}-v_t\right)^2\frac{\sigma_{c,t}^2}{c_t^2}+\\ &-2e^{-\frac{T-t}{c_t}}\sigma_{x,T,t}e^{-\frac{T-t}{c_t}}(T-t)\left(x_{T,t}-v_t\right)\frac{\sigma_{c,t}}{c_t}\rho(dw_{x,T,t},dw_{c,t})+\\ &+2\left(1-e^{-\frac{T-t}{c_t}}\right)\sigma_{v,t}e^{-\frac{T-t}{c_t}}\sigma_{x,T,t}\rho(dw_{x,T,t},dw_{v,t})+\\ &-2\left(1-e^{-\frac{T-t}{c_t}}\right)\sigma_{v,t}e^{-\frac{T-t}{c_t}}(T-t)\left(x_{T,t}-v_t\right)\frac{\sigma_{c,t}}{c_t}\rho(dw_{c,t},dw_{v,t}) \quad . \qquad (34)\end{aligned}$$

If we have other sources of uncertainties than the future evolution of the fundamental, then the volatility of the exchange rate can easily be higher than that of the fundamental. It can be demonstrated by using Equation (34) and reasonable parameter values for $x_{T,t}$, v_t, c_t, $\sigma_{x,T,t}$, $\sigma_{v,t}$, $\sigma_{c,t}$, $\rho(dw_{x,T,t},dw_{c,t})$, $\rho(dw_{x,T,t},dw_{v,t})$ and $\rho(dw_{c,t},dw_{v,t})$.

6.3 Near Random Walk Behavior Of The Exchange Rate

In the following, we demonstrate that our exchange rate model with its special structure of the relative weights is able to explain the following finding of the empirical literature. The martingale or random walk behavior of the exchange rate can not be rejected in sample sizes that are typically available.

The explanation is the following. We have already argued in this paper that the process of the eXpectation© $x_{T,t}$ is martingale. If v_t is also martingale and the relative weight $e^{-\frac{T-t}{c_t}}$ is constant, then the exchange rate is also martingale. The more interesting case is, when the process of the fundamental is not martingale. Than, its relative weight in the exchange rate determines whether the process of the exchange rate is martingale or not. If $\left(1-e^{-\frac{T-t}{c_t}}\right)$ is zero, then the process of the exchange rate is martingale no matter what is the process of the fundamental. If the relative weight of the fundamental

$\left(1 - e^{-\frac{T-t}{c_t}}\right)$ is not zero, but sufficiently close to zero, then the process of the exchange rate will not be martingale, but it will be hard to distinguish it from a martingale process.

The importance of the relative weights and especially that of the discount factor at determining the process of the exchange rate has already been recognized by Engel and West (2005), although in a different model. In their model, the forward looking term of the exchange rate is not the eXpected© exchange rate, but the one period risk premia. The eXpectation© follows martingale process for sure, but not the risk premia. Therefore, it is not guaranteed automatically in their model, that any of the factors follow unit root process. Consequently, the condition they impose in order to get a near random walk process for the exchange rate is not only to have the discount factor close to unitiy, but also that at least one factor follows random walk.

6.4 Exchange Rate Regime Switches

See Figure 8 ...

To write:

6.5 Fat Tail Distribution

The distribution of the percentage changes of the exchange rate is often assumed to have Gaussian distribution despite of the rejection of this hypothesis by a number of empirical studies. In our model, the distribution is different from the Gaussian. We can get fat tail distribution for the returns simulated with reasonable parameter values. See Figure 8 ...

To write:

6.6 Exchange Rate Forecast Ability

As it is shown by Meese and Rogoff (1983) the exchange rate can not be forecasted in the short and medium horizons by using macro fundamentals. But the forecasting performance improves as we turn to the longer horizons.

This finding can also be explained by our exchange rate model. The explanation is again based on the structure of the relative weights, just like the explanation for the near random walk property of the exchange rate.

The forecasts are the weighted averages of the spot exchange rate and the fundamental. See Equation (15). The absolute relative weight of the fundamental is increasing in the forecast horizon. Therefore, the fundamental is a more important determinant in the forecast of the long horizons, than in the short or medium ones. This idea is examplified by the following. If $\left(1 - e^{-\frac{T-t}{c_t}}\right)$ equals .1 for the one-year forecast horizon $T - t = 1Y$, then the expected one-year ahead exchange rate is $x_{t+1Y,t} = \frac{1}{1-.1}s_t + (1 - \frac{1}{1-.1})v_t = \frac{10}{9}s_t - \frac{1}{9}v_t$. If we increase the forecast horizon to 5 years, then the expected five-years ahead exchange rate is $x_{t+5Y,t} = \frac{1}{(1-.1)^5}s_t + (1 - \frac{1}{(1-.1)^5})v_t = \left(\frac{10}{9}\right)^5 s_t + \left(1 - \left(\frac{10}{9}\right)^5\right) v_t$. By comparing the relative weights of the fundamental in these two examples, it is clear, that it is much higher for the 5-years forecast horizon, than for the 1-year horizon.

These theoretical considerations clearly show that the empirical models that use macro fundamentals to forecast the exchange rate while controlling for the spot exchange rate should do better on the long-run, then on the shorter-run.

7 Conclusion

This paper has introduced a theoretical model for the exchange rate. The model is the generalized asset-pricing exchange rate model with stochastic discount factor and subjective expectations. In this model the exchange rate is derived to be a closed-form non-linear function of three factors: the fundamental, the market expectation for the exchange rate, and the discount factor.

Our three-factor model has gained empirical support by having been tested whether the way market analysts generate their forecasts is closer to the one implied by the three-factor model or to the one implied by simpler models with less factors. The fundamental is the only factor that drives the exchange rate in one of the simpler models, the linear model. The fundamental together with the subjective expectations are the factors in another restricted model, the constant parameter model. Both the linear model and the constant parameter model have been rejected in favor of the three-factor model based on the comparison of their out-of-sample performance. Therefore, we have a good reason to assume that according to the forecasters thinking it is not only the fundamental that determines the exchange rate, but other factors as well. Moreover, the forecasters do not think the discount factor to be constant. All in all, the representative professional exchange rate forecaster has an exchange rate model in her mind that can be well represented by our three-factor model.

Usually the length of the time series limits us to test theoretical models on exchange rate data. Still, one can test the link between the theoretical model and the realized exchange rate along the following lines. First, this paper shows that survey-based expectations are in accordance with our theoretical model. Second, if the survey data can forecast the exchange rate sufficiently well, then the theoretical model is likely to be relevant not only for the expectation formation, but also for relaized exchange rate. Whether this model fits the realized exchange rate just as well as it fits the survey data is tested by using three measures of the forecasting performance. The survey forecasts could not systematically out-perform the random walk model based on the measures of MAE and RMSE. But they performed surprisingly well when the forcasting ability was measured by the profit of a simple forcast based trading strategy. The survey forecasts could beaten the random walk for 9 exchange rates out of 11 not only on the one-year horizon, but also on the 3-months horizon. Moreover, the model based forecasts turned out to be able to contribute the forcasting performance of the survey data. When we used the model based forcasts on the 3-months horizon instead of the pure survey data, then the profitability of the trading strategy increased for 8 exchange rates out of 11.

An alternative test of the model is based on its ability to mimic some characteristics of the exchange rate. Our model could account for some well known puzzles and anomalies, like the disconnect puzzle, the excess volatility puzzle, the near random walk behaviour of the exchange rate, the inability of macro models to provide reliable exchange rate forecasts for the short and medium horizons, the seemingly frequent exchange rate regime switches and the fat tail distribution of the returns.

Future research will strive at finding those macro variables that match the filtered fundamental and therefore are likely to be the main determinants of the exchange rate.

8 Acknowledgements

The author gratefully acknowledges comments and suggestions from András Fülöp, Júlia Király, Gergely Kiss, Tamás Kollányi, Tamás Papp, Lars Svensson, and from the participants of presentations at the Sveriges Riksbank.

Appendix A

In the paper we have argued that the process of $x_{T,t}$ is martingale. We show that in contrast to $x_{T,t}$, the process of $z_{t+\Theta,t}$ is not martingale if the market participants do not expect the same exchange rate for every forecast horizons.

This Appendix provides an indirect proof. First, we assume that the forecasts do vary across forecast horizons, *i.e.*, the expectation scheme of $x_{T,t}$ is not flat. Moreover, we assume that the process of z is martingale, *i.e.*, for all t and Θ

$$z_{t+\Theta,t} = E_t(z_{t+1+\Theta,t+1}) \quad . \tag{35}$$

Second, by following the logic of indirect proof, we show that the latter assumption contradicts to the assumption of having non-flat expectation scheme of $x_{T,t}$. The expectation scheme is not flat, if the expected exchange rate for at least two different forecast horizons $t+\Theta$ and $t+1+\Theta$ are different, *i.e.*,

$$x_{t+\Theta,t} \neq x_{t+1+\Theta,t} \quad . \tag{36}$$

By using the identity of $x_{T,t}$ and $z_{t+T-t,t}$ and the martingale property of x and z, we obtain

$$x_{t+\Theta,t} \quad = \quad z_{t+\Theta,t} \quad = \quad E_t(z_{t+1+\Theta,t+1}) \quad = \quad E_t(x_{t+1+\Theta,t+1}) \quad = \quad x_{t+1+\Theta,t} \qquad . \tag{37}$$

The first and third equalities in (37) are due to the identity of $x_{T,t}$ and $z_{t+T-t,t}$. The second and forth equalities are due the martingale property of z and x respectively.

We obtain from (37) that $x_{t+\Theta,t} = x_{t+1+\Theta,t}$. This equality contradicts to (36), therefore, the process of $z_{t+\Theta,t}$ is not martingale.

Appendix B

This Appendix proves that the derived function $s_t = f(t, v_t, x_{T,t}, c_t)$ of (8) satisfies the implicit relationship (1) between the exchange rate and fundamental.

By calculating the partial derivatives of (8) and by substituting these derivatives and $\mu_{v,t} = \mu_{x,T,t} = \mu_{c,t} = 0$ into (13), we obtain

$$ds_t = \left[\frac{1}{c_t}e^{-\frac{T-t}{c_t}}(x_{T,t} - v_t) + \frac{1}{2}e^{-\frac{T-t}{c_t}}(T-t)^2(x_{T,t} - v_t)\left(\frac{\sigma_{c,t}}{c_t}\right)^2 + \right.$$
$$\left. + \frac{1}{2}e^{-\frac{T-t}{c_t}}(T-t)\left(Cov\left(d\frac{1}{c_t}, dv_t\right) - Cov\left(d\frac{1}{c_t}, dx_{T,t}\right)\right)\right]dt +$$
$$+ \left(1 - e^{-\frac{T-t}{c_t}}\right)\sigma_{v,t}dw_{v,t} + e^{-\frac{T-t}{c_t}}\sigma_{x,T,t}dw_{x,T,t} - e^{-\frac{T-t}{c_t}}(T-t)(x_{T,t} - v_t)\frac{\sigma_{c,t}}{c_t}dw_{c,t}. \quad (38)$$

By using (38), the expected instantaneous change of the exchange rate can be expressed as

$$\frac{E_t(ds_t)}{dt} = \frac{1}{c_t}e^{-\frac{T-t}{c_t}}(x_{T,t} - v_t) + \frac{1}{2}e^{-\frac{T-t}{c_t}}(T-t)^2(x_{T,t} - v_t)\left(\frac{\sigma_{c,t}}{c_t}\right)^2 +$$
$$+ \frac{1}{2}e^{-\frac{T-t}{c_t}}(T-t)\left[Cov\left(d\frac{1}{c_t}, dv_t\right) - Cov\left(d\frac{1}{c_t}, dx_{T,t}\right)\right] \quad . \quad (39)$$

The implicit function (1) can be rewritten as

$$\frac{E_t(ds_t)}{dt} = \frac{1}{c}(s_t - v_t) \quad . \quad (40)$$

Consequently, if the right-hand-side (RHS) of Equation (39) is equal to the RHS of Equation (40), then the implicit function (1) is satisfied by (8). In order to prove the equality, it is sufficient to show that the first term of the RHS of (39) is equal to the RHS of (40), whereas the other terms of (39) sum up to zero.

By rearranging (8), we obtain that the first term of the RHS of (39) is equal to the RHS of (40).

$$\frac{1}{c}(s_t - v_t) = \frac{1}{c}e^{-\frac{T_t-t}{c}}(x_{T,t} - v_t) \quad . \quad (41)$$

What remains to prove is that the other terms of (39) sum up to zero. It follows trivially from Equation (6).

Appendix C

In the paper parameter c_t is estimated from survey data by using Equation (16). This Appendix proves that the sign of the estimated c_t parameter depends on whether the expectations are convex or concave and increasing or decreasing in the forecast horizon. More precisely, parameter c_t is positive if and only if the expectations are monotone increasing and convex in $T-t$ or monotone decreasing and concave in $T-t$. Here, we implicitly assume, that c_t can be estimated from the data s_t, $x_{t+1Y,t}$, $x_{t+2Y,t}$, *i.e.*, not all three are equal.

First, Equation (16) can be used to show that $\infty > c > 0$ is equivalent to $\frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}} < 1$.

$$\infty > c > 0 \Leftrightarrow log\left(\frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}}\right) < 0 \Leftrightarrow \frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}} < 1 \quad . \tag{42}$$

Second, we show that the restriction $\frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}} < 1$ is equivalent to the following. Either the expectations are monotone increasing and convex in $T-t$ or monotone decreasing and concave in $T-t$. If expectations are monotone increasing and convex, then $0 < x_{t+1Y,t} - s_t < x_{t+2Y,t} - x_{t+1Y,t}$. If expectations are monotone decreasing and concave, then $0 > x_{t+1Y,t} - s_t > x_{t+2Y,t} - x_{t+1Y,t}$. In both cases the ratio $\frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}}$ is less then 1. Whereas if expectations are monotone increasing and concave $0 < x_{t+2Y,t} - x_{t+1Y,t} < x_{t+1Y,t} - s_t$ or if expectations are monotone decreasing and convex $0 > x_{t+2Y,t} - x_{t+1Y,t} > x_{t+1Y,t} - s_t$ the ratio $\frac{x_{t+1Y,t}-s_t}{x_{t+2Y,t}-x_{t+1Y,t}}$ is greater then 1. In these cases parameter c_t is negative.

Appendix D

This Appendix derives the link between two asset pricing equations both used in the literature. One is a continous time model that is used by Froot and Obstfeld (1991) among others. The other is a discrete time model that is equally popular in the exchange rate literature. And it has been used recently by Engel and West (2005) for instance. We demonstrate that the difference between the two models is only the fact that one is in continous time and the other is in discrete time.

The first model is given by Equation (1) with constant c parameter that we repeat here for convinience.

$$s_t = v_t + c\frac{E_t(ds_t)}{dt} \quad 0 < c. \tag{43}$$

Here, s is the log exchange rate, and v is the fundamental, and $\frac{E_t(ds_t)}{dt}$ is the expected instantaneous change of the log exchange rate. Parameter c has the interpretation of beeing the semi-elasticity of money demand according to the monetarist model.

The second model is the following discrete time model (See Equation (7) in Engel and West (2005), where we have translated their notation to be consistent with ours.):

$$s_t = (1-b)v_t + bE_t(s_{t+\Delta t}) \quad 0 < b < 1. \tag{44}$$

The interpretation of s and v is the same as before. Parameter b is the discount factor. Although the discount factor b has no index, it corresponds to the Δt period. In order to make it explicit, we substitute $b = e^{-\rho \Delta t}$ into Equation (44), where the discount rate is restricted to be $0 < \rho < -\infty$.

$$s_t = (1 - e^{-\rho\Delta t})v_t + e^{-\rho\Delta t}E_t(s_{t+\Delta t}) \quad . \tag{45}$$

By substracting $e^{-\rho\Delta t}s_t$ from both sides of Equation (45) we obtain

$$(1 - e^{-\rho\Delta t})s_t = (1 - e^{-\rho\Delta t})v_t + e^{-\rho\Delta t}E_t(s_{t+\Delta t} - s_t) \quad . \tag{46}$$

After dividing by $1 - e^{-\rho\Delta t}$:

$$s_t = v_t + \frac{e^{-\rho\Delta t}}{1 - e^{-\rho\Delta t}}E_t(s_{t+\Delta t} - s_t) \quad . \tag{47}$$

In order to make the second model in discrete time comparable with the first model in continous time, we take the limit.

$$s_t = v_t + lim_{\Delta t \to 0}\left(\frac{e^{-\rho\Delta t}}{1 - e^{-\rho\Delta t}}E_t(s_{t+\Delta t} - s_t)\right) \quad . \tag{48}$$

The second term on the RHS of Equation (48) can be rearranged along the following lines

$$lim_{\Delta t \to 0}\left(\frac{e^{-\rho\Delta t}}{1 - e^{-\rho\Delta t}}E_t(s_{t+\Delta t} - s_t)\right) = lim_{\Delta t \to 0}\left(\frac{e^{-\rho\Delta t}\Delta t}{1 - e^{-\rho\Delta t}}\frac{E_t(s_{t+\Delta t} - s_t)}{\Delta t}\right) =$$
$$= lim_{\Delta t \to 0}\left(\frac{\Delta t}{e^{\rho\Delta t} - 1}\frac{E_t(s_{t+\Delta t} - s_t)}{\Delta t}\right) = \frac{1}{\rho}\frac{E_t(ds_t)}{dt} \quad . \tag{49}$$

By substiting $\frac{1}{\rho}\frac{E_t(ds_t)}{dt}$ into Equation (48) we obtain the continous version of the second model that can be directly compared with the first model of Equation (43).

$$s_t = v_t + \frac{1}{\rho}\frac{E_t(ds_t)}{dt} \quad . \tag{50}$$

It is straitforward from the comparison that the two models are identical under the condition $c = \frac{1}{\rho}$. By substituting the definition $b = e^{-\rho\Delta t}$ of parameter ρ into this condition we obtain the model identity condition for the original parameters c and b

$$b = e^{-\frac{1}{c}\Delta t} \quad . \tag{51}$$

Another form of the model identity condition can be obtained, if we express the relationship between the discount rate ρ and the discount factor b in discrete time, *i.e.*, $b = \left(\frac{1}{1+\rho}\right)^{\Delta t}$. If we repeat the derivation from Equation (45) by using this definition of the discount factor ρ, and by applying the following approximation $\frac{1}{log(1+\rho)} = \frac{1}{\rho}$, then the model identity condition is of the form of

$$b = \left(\frac{c}{1+c}\right)^{\Delta t} \quad . \tag{52}$$

The latter model identity condition is used by Engel and West (2005, page 497) when they relate the b parameter with the interest semi-elasticity of money demand.

The original interpretation of the c and b parameters were the semi-elasticity of money demand and the discount factor respectively. Using the identity $c = \frac{1}{\rho}$, we get another interpretation for parameter c. It is the inverse of the discount rate according to the second model.

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Tables and Figures

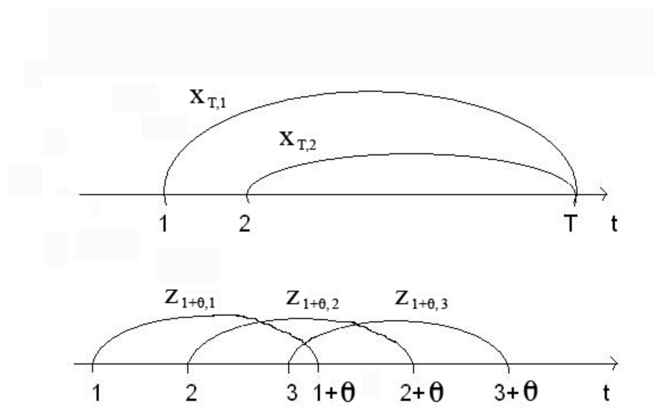


Figure 1: The difference between the two types of expectations of $x_{T,t}$ and $z_{t+\theta,t}$.

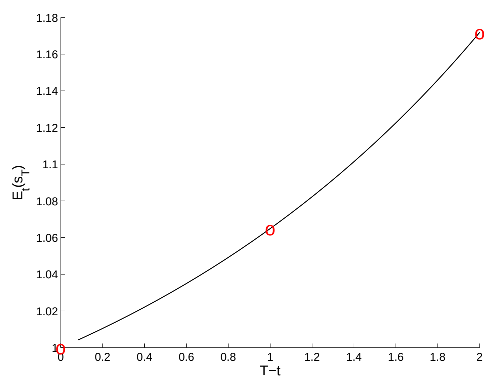
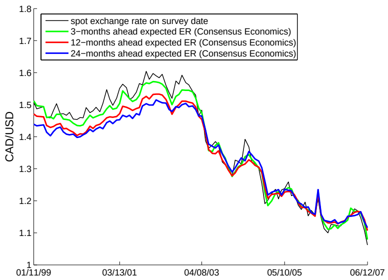


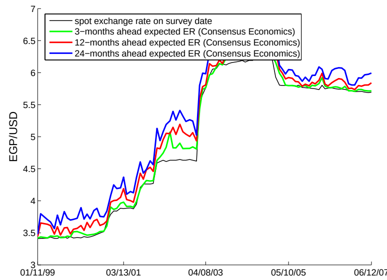
Figure 2: Stylized expectation scheme. In this numerical example $s_t = 1$, $v_t = .9$, and $c_t = 2$. The expected $T-t$ ahead log exchange rate is given by $x_{T,t} = e^{\frac{T-t}{c_t}}(s_t - v_t) + v_t$.

		mean absolute error (MAE)				root mean squared error (RMSE)			
Exchange rate	Num. obs.	unrestr. model	restricted models, $e^{-\frac{1Y}{c_t}} =$			unrestr. model	restricted models, $e^{-\frac{1Y}{c_t}} =$		
			$= 0$	$= 1$	$= e^{-\frac{1Y}{c}}$		$= 0$	$= 1$	$= e^{-\frac{1Y}{c}}$
		general	*linear*	*RW*	*const*	*general*	*linear*	*RW*	*const*
CAD/USD	71	**0.0059**	0.0076	0.0128	NaN	**0.0077**	0.0096	0.0154	NaN
EGP/USD	100	**0.0069**	0.0075	0.0129	0.0565	**0.0105**	0.0118	0.0194	0.0629
USD/EUR	71	**0.008**	0.0107	0.0227	NaN	**0.01**	0.0138	0.0288	NaN
ILS/USD	87	**0.0085**	0.0095	0.0147	0.0154	**0.011**	0.0123	0.0187	0.02
JPY/USD	52	0.0178	0.0123	**0**	0.034	0.0224	0.0151	**0**	0.0374
NGN/USD	101	**0.0101**	0.0105	0.0327	0.0545	**0.0141**	0.0148	0.039	0.0619
NOK/EUR	51	**0.0065**	0.007	0.0101	0.0115	**0.0098**	0.01	0.0125	0.0144
ZAR/USD	73	**0.0152**	0.0185	0.0317	0.0598	**0.0195**	0.0238	0.0411	0.0717
SEK/EUR	92	**0.0046**	0.0077	0.014	0.0115	**0.0063**	0.0093	0.0161	0.0142
CHF/EUR	69	**0.0039**	0.005	0.0089	NaN	**0.005**	0.0061	0.0102	NaN
USD/GBP	67	**0.0067**	0.0072	0.0106	0.04	**0.0084**	0.0095	0.0138	0.0444

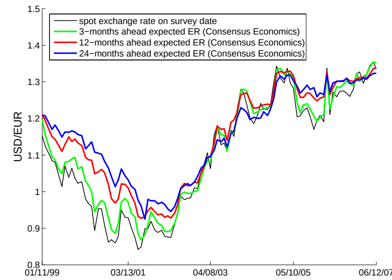
Table 1: Out-of-sample fit of the general model and that of some restricted models



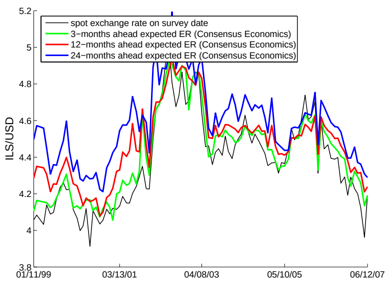
(a) Canadian Dollar



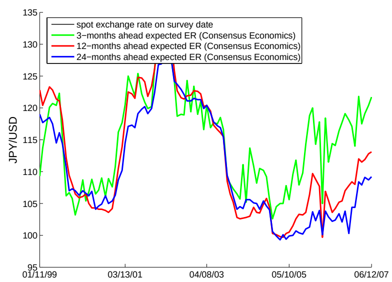
(b) Egyptian Pound



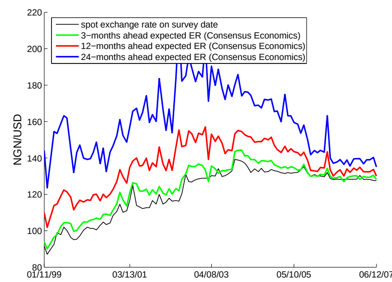
(c) Euro



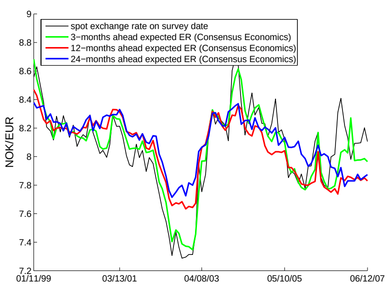
(d) Israeli Shekel



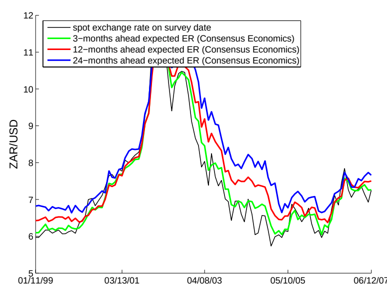
(e) Japanese Yen



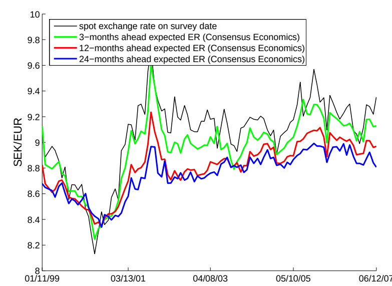
(f) Nigerian Naira



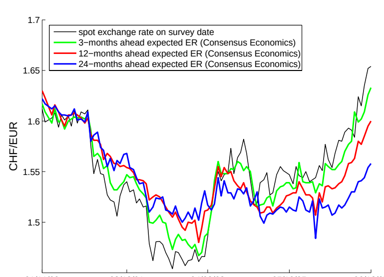
(g) Norwegian Krone



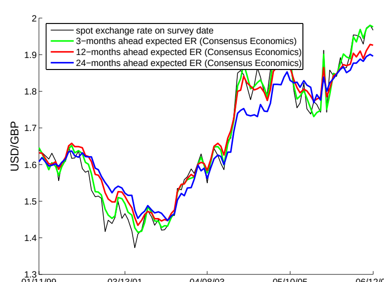
(h) South African Rand



(i) Swedish Krona

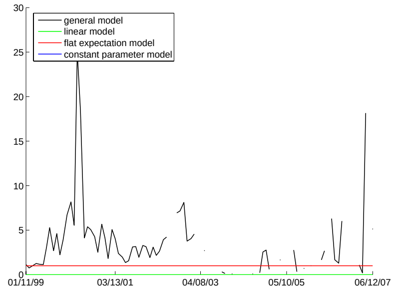


(j) Swiss Franc

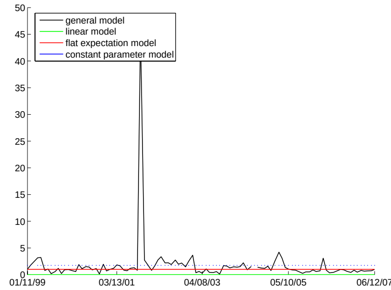


(k) United Kingdom Pound

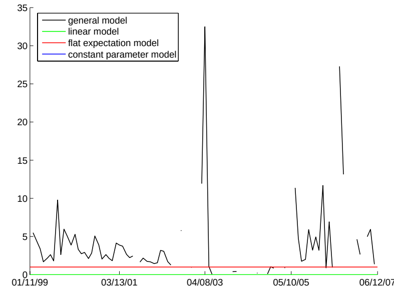
Figure 3: The spot exchange rate and the survey data.



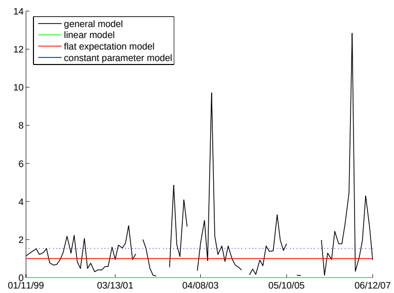
(a) Canadian Dollar



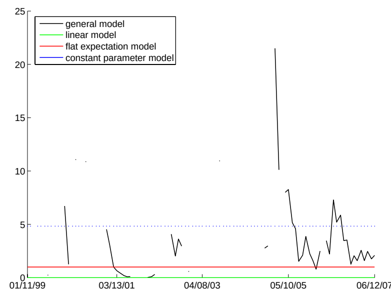
(b) Egyptian Pound



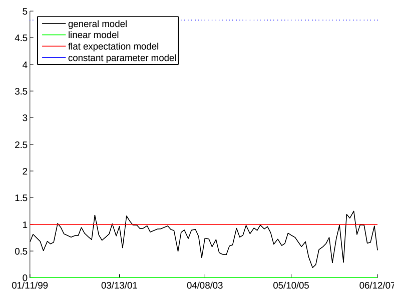
(c) Euro



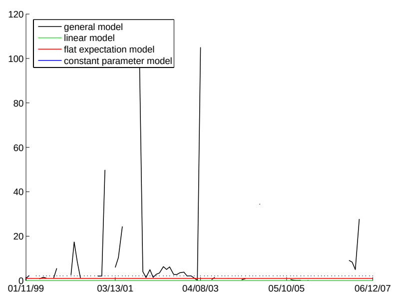
(d) Israeli Shekel



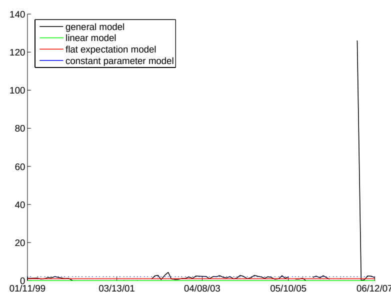
(e) Japanese Yen



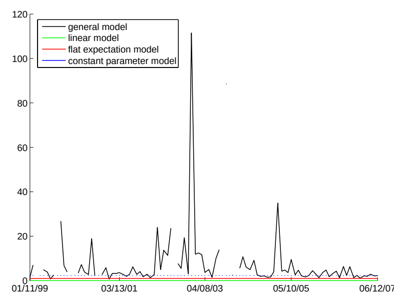
(f) Nigerian Naira



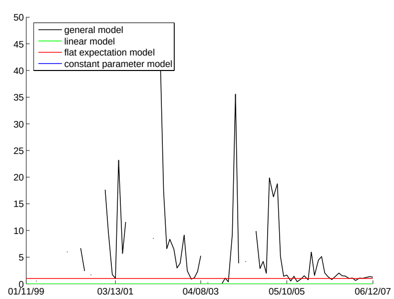
(g) Norwegian Krone



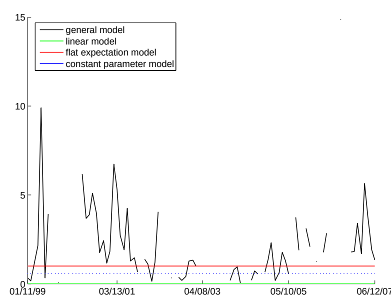
(h) South African Rand



(i) Swedish Krona

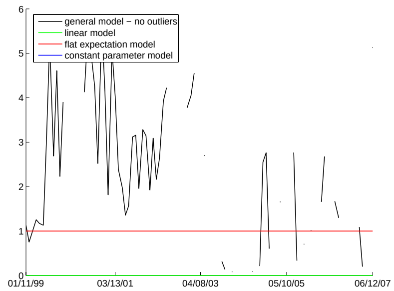


(j) Swiss Franc

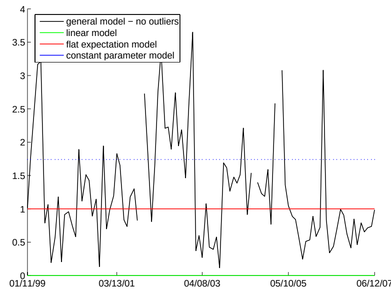


(k) United Kingdom Pound

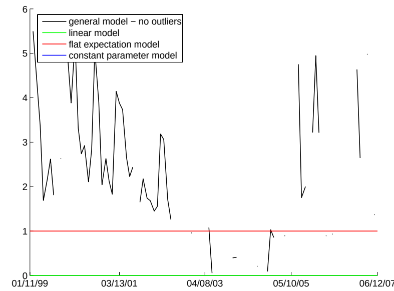
Figure 4: The survey-based estimates on the transformed discount factor $e^{-\frac{1Y}{\hat{c}_t}}$ under different model specifications.



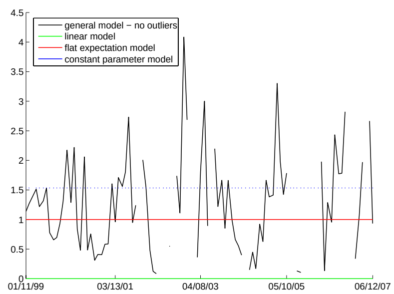
(a) Canadian Dollar



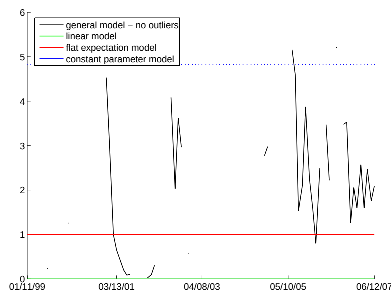
(b) Egyptian Pound



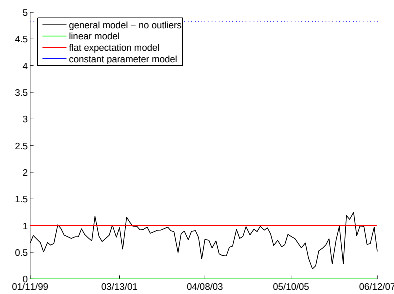
(c) Euro



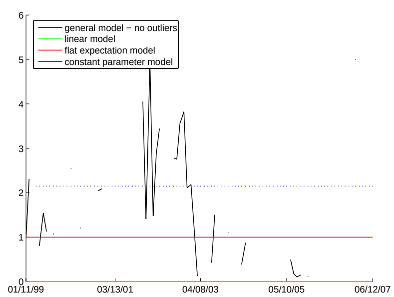
(d) Israeli Shekel



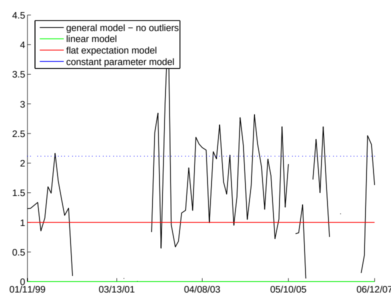
(e) Japanese Yen



(f) Nigerian Naira

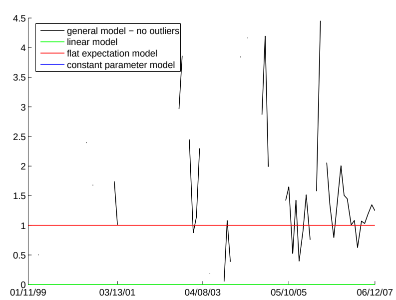


(g) Norwegian Krone

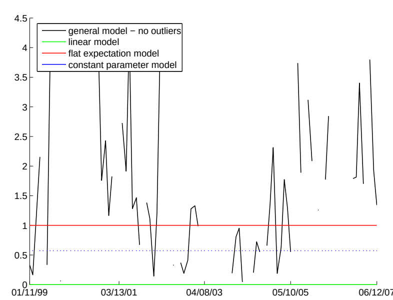


(h) South African Rand

(i) Swedish Krona

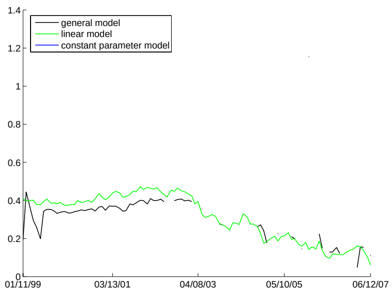


(j) Swiss Franc



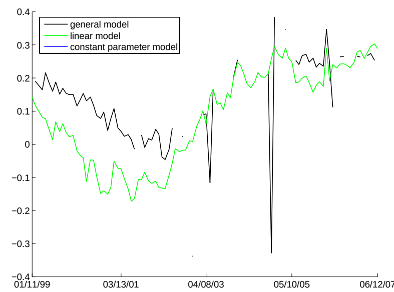
(k) United Kingdom Pound

Figure 5: The survey-based estimates on the transformed discount factor $e^{-\frac{1Y}{\hat{c}_t}}$ under different model specifications – without outliers. $e^{-\frac{1Y}{\hat{c}_t}}$ is considered to be an outlier if it exceeds the median by 3.

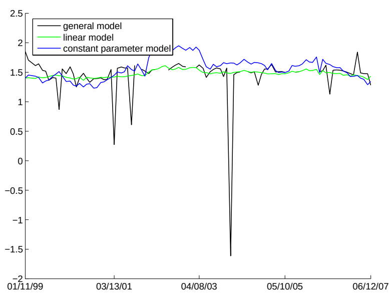


(a) Canadian Dollar

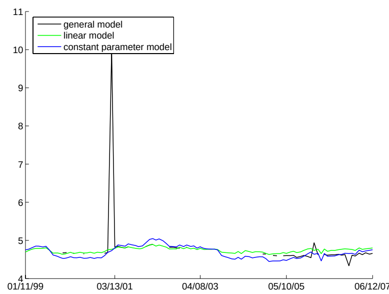
(b) Egyptian Pound



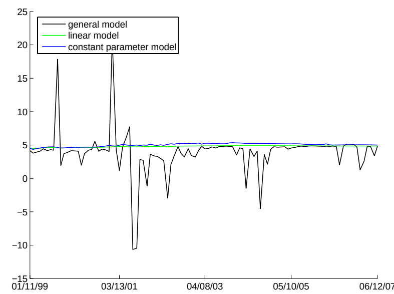
(c) Euro



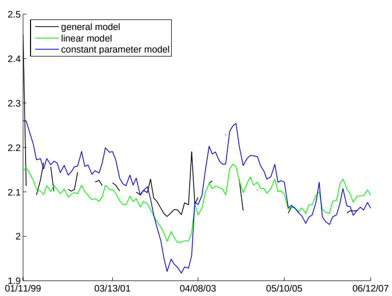
(d) Israeli Shekel



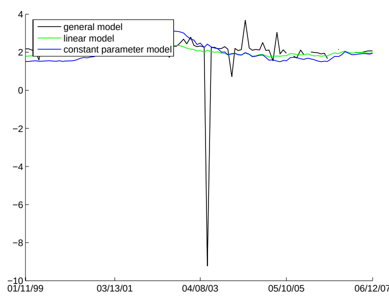
(e) Japanese Yen



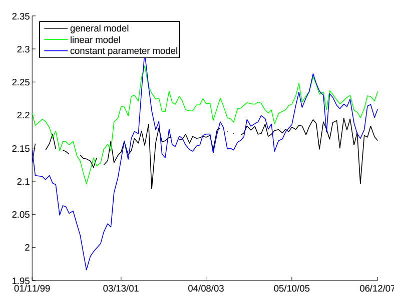
(f) Nigerian Naira



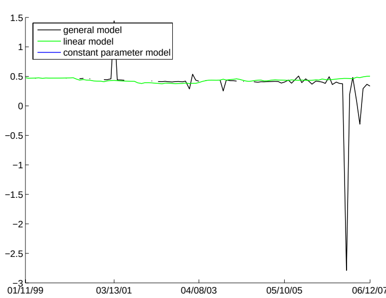
(g) Norwegian Krone



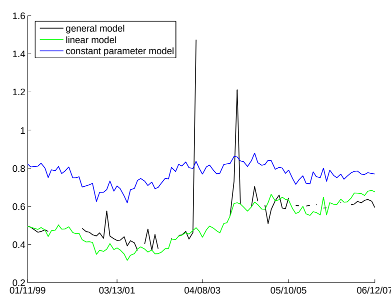
(h) South African Rand



(i) Swedish Krona

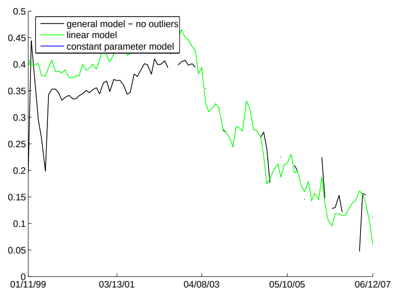


(j) Swiss Franc

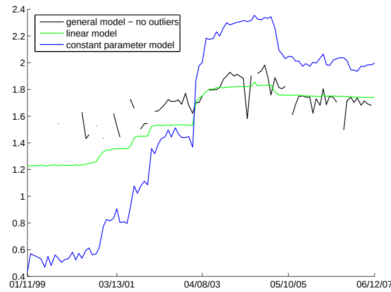


(k) United Kingdom Pound

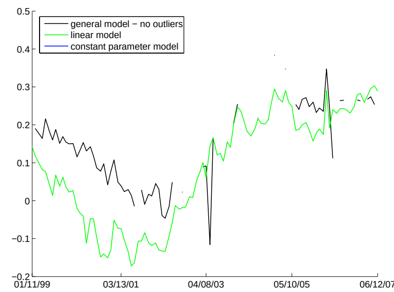
Figure 6: The survey-based estimates on the fundamental v_t under different model specifications.



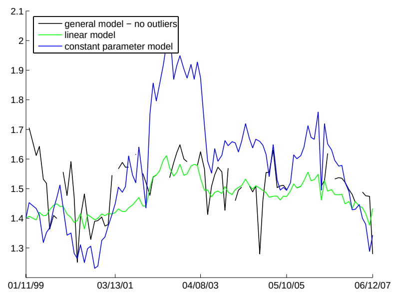
(a) Canadian Dollar



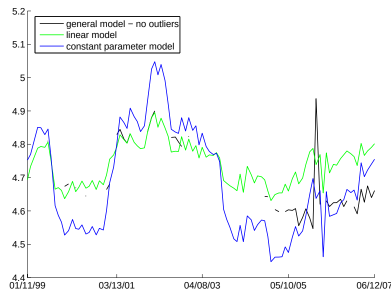
(b) Egyptian Pound



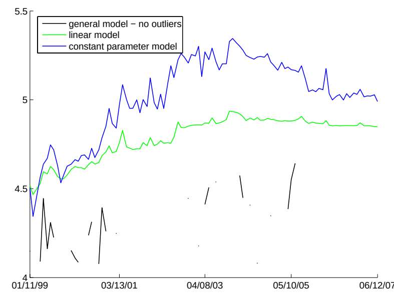
(c) Euro



(d) Israeli Shekel

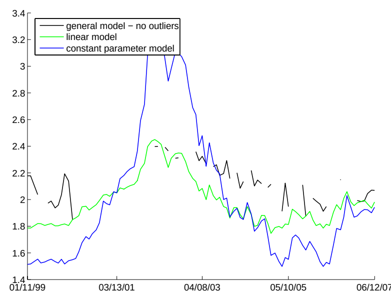


(e) Japanese Yen

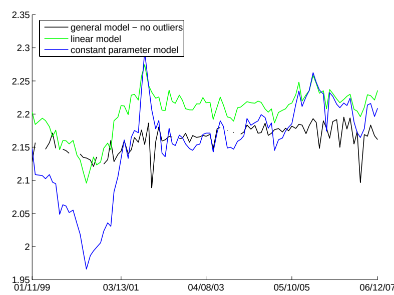


(f) Nigerian Naira

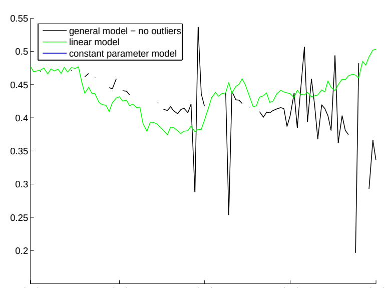
(g) Norwegian Krone



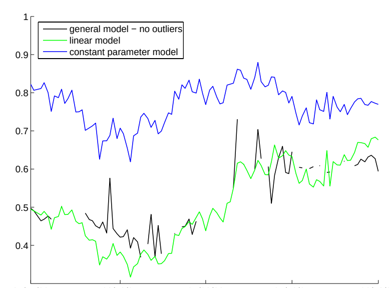
(h) South African Rand



(i) Swedish Krona



(j) Swiss Franc



(k) United Kingdom Pound

Figure 7: The survey-based estimates on the fundamental v_t under different model specifications – without outliers. v_t is considered to be an outlier if it exceeds the median by .3.

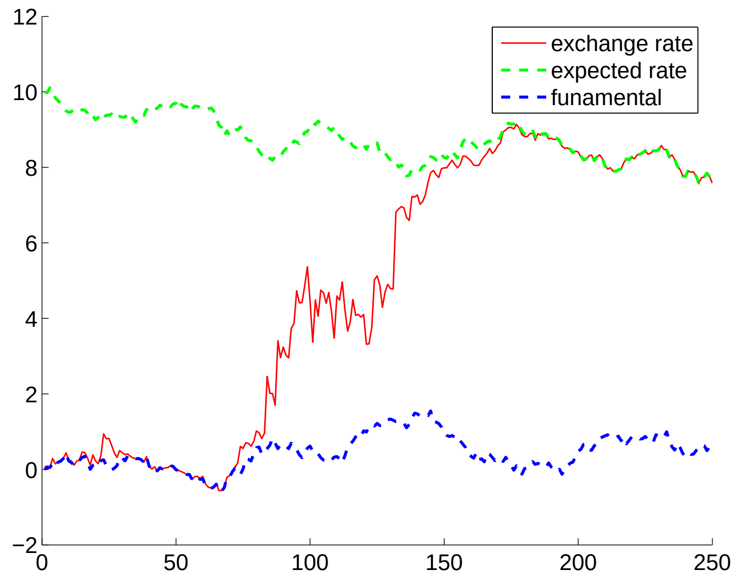


Figure 8: The simulated process of the exchange rate. Change it!!!

		mean absolute deviation (MAD)		root mean squared deviation (RMSD)	
Exchange rate	Num. obs.	unrestr. model *general*	restr. model, $e^{-\frac{1Y}{c_t}} = e^{-\frac{1Y}{c}}$ *const*	unrestr. model *general*	restr. model, $e^{-\frac{1Y}{c_t}} = e^{-\frac{1Y}{c}}$ *const*
CAD/USD	71	0.0627	NaN	0.1347	NaN
EGP/USD	100	**0.21**	0.3988	0.5122	**0.4463**
USD/EUR	71	0.113	NaN	0.145	NaN
ILS/USD	87	0.1445	**0.1209**	0.388	**0.156**
JPY/USD	52	0.1856	**0.1046**	0.7349	**0.115**
NGN/USD	101	1.6053	**0.2294**	3.5269	**0.2577**
NOK/EUR	51	**0.0423**	0.0496	0.0658	**0.0562**
ZAR/USD	73	0.4123	**0.2831**	1.3816	**0.3711**
SEK/EUR	92	**0.0456**	0.0483	**0.0512**	0.0602
CHF/EUR	69	0.123	NaN	0.4294	NaN
USD/GBP	67	**0.0658**	0.2552	**0.1515**	0.2701

Table 2: Deviation of the filtered fundamental from the exchange rate under the general model and the constant parameter model

		mean absolute error (MAE)		root mean squared error (RMSE)		mean loss ($-\pi$)	
Exchange rate	Num. obs.	Model: general	RW	Model: general	RW	Model: general	RW
CAD/USD	69	0.0277	**0.0251**	0.036	**0.0334**	**-0.005**	0
(t-test)		(-1.2483)		(-1.387)		(0.2192)	
EGP/USD	97	0.0271	**0.0224**	**0.044**	0.0465	**-0.0537**	0
(t-test)		(-1.5204)		(0.4976)		(1.9235)	
USD/EUR	69	0.0528	**0.0433**	0.0646	**0.0541**	0.0164	**0**
(t-test)		(-2.0285)		(-1.7554)		(-0.4256)	
ILS/USD	84	0.0263	**0.0237**	0.0338	**0.0321**	**-0.0066**	0
(t-test)		(-1.2934)		(-0.6429)		(0.3782)	
JPY/USD	49	0.0351	0.0351	0.0476	0.0476	0	0
(t-test)		(NaN)		(NaN)		(NaN)	
NGN/USD	98	0.0352	**0.0258**	0.0436	**0.0388**	**-0.0419**	0
(t-test)		(-2.0388)		(-0.8572)		(1.8645)	
NOK/EUR	51	0.025	**0.0225**	0.0314	**0.0298**	**-0.0018**	0
(t-test)		(-1.3128)		(-0.7674)		(0.0866)	
ZAR/USD	70	0.0746	**0.0635**	0.0964	**0.0852**	**-0.0444**	0
(t-test)		(-1.9856)		(-1.537)		(0.9022)	
SEK/EUR	89	0.0204	**0.017**	0.0254	**0.0212**	**-0.0034**	0
(t-test)		(-1.8131)		(-1.8466)		(0.2687)	
CHF/EUR	66	0.0145	**0.011**	0.0171	**0.0152**	**-0.0018**	0
(t-test)		(-3.0559)		(-1.2684)		(0.1602)	
USD/GBP	64	0.0266	**0.0255**	0.0338	**0.0319**	0.0007	**0**
(t-test)		(-0.5942)		(-0.8795)		(-0.0377)	

Table 3: Forecasting performance of the survey data and the random walk model on the 3-months horizons

		mean absolute error (MAE)		root mean squared error (RMSE)		mean loss ($-\pi$)	
Exchange rate	Num. obs.	Model: general	RW	Model: general	RW	Model: general	RW
CAD/USD	69	0.0271	**0.0251**	0.0353	**0.0334**	**-0.0119**	0
(t-test)		(-0.9144)		(-0.9339)		(0.5081)	
EGP/USD	97	0.0251	**0.0224**	**0.043**	0.0465	**-0.0643**	0
(t-test)		(-0.903)		(0.7789)		(1.9399)	
USD/EUR	69	0.051	**0.0433**	0.0634	**0.0541**	**-0.012**	0
(t-test)		(-1.5258)		(-1.4432)		(0.2795)	
ILS/USD	84	0.0249	**0.0237**	0.0324	**0.0321**	**-0.0165**	0
(t-test)		(-0.7175)		(-0.1413)		(0.6954)	
JPY/USD	49	0.0397	**0.0351**	0.0508	**0.0476**	**-0.0078**	0
(t-test)		(-1.6948)		(-1.1368)		(0.198)	
NGN/USD	98	0.0316	**0.0258**	0.0396	**0.0388**	**-0.0424**	0
(t-test)		(-1.4475)		(-0.1569)		(1.8766)	
NOK/EUR	51	0.0231	**0.0225**	**0.0289**	0.0298	**-0.0283**	0
(t-test)		(-0.2415)		(0.3021)		(0.8629)	
ZAR/USD	70	0.0719	**0.0635**	0.0933	**0.0852**	**-0.0065**	0
(t-test)		(-1.6791)		(-1.3847)		(0.0933)	
SEK/EUR	89	0.0205	**0.017**	0.0253	**0.0212**	0.0019	**0**
(t-test)		(-1.8307)		(-1.9076)		(-0.137)	
CHF/EUR	66	0.0136	**0.011**	0.0164	**0.0152**	0.0046	**0**
(t-test)		(-2.2986)		(-0.7552)		(-0.3671)	
USD/GBP	64	0.0262	**0.0255**	0.0328	**0.0319**	**-0.0115**	0
(t-test)		(-0.4139)		(-0.4232)		(0.4741)	

Table 4: Forecasting performance of the general model and the random walk model on the 3-months horizons

		mean absolute error (MAE)		root mean squared error (RMSE)		mean loss ($-\pi$)	
Exchange rate	Num. obs.	Model: general	RW	Model: general	RW	Model: general	RW
CAD/USD	64	0.0626	**0.05**	0.0726	**0.0692**	**-0.0026**	0
(t-test)		(-0.9915)		(-0.2314)		(0.0893)	
EGP/USD	88	**0.0751**	0.0918	**0.093**	0.1257	**-0.0711**	0
(t-test)		(1.0104)		(1.4851)		(1.6629)	
USD/EUR	64	0.1132	**0.0952**	0.138	**0.114**	**-0.0172**	0
(t-test)		(-0.5451)		(-0.6908)		(0.3258)	
ILS/USD	75	**0.0483**	0.0498	**0.0587**	0.0649	**-0.0151**	0
(t-test)		(0.1205)		(0.4201)		(0.6583)	
JPY/USD	40	0.0958	**0.0683**	0.1047	**0.0812**	0.0147	**0**
(t-test)		(-1.4684)		(-1.5898)		(-0.4931)	
NGN/USD	89	0.0948	**0.055**	0.1052	**0.0719**	**-0.0389**	0
(t-test)		(-1.946)		(-1.9809)		(1.6966)	
NOK/EUR	47	**0.046**	0.0524	**0.0586**	0.0684	**-0.0235**	0
(t-test)		(0.686)		(0.8438)		(0.8611)	
ZAR/USD	66	0.1852	**0.1597**	0.2216	**0.1894**	**-0.0063**	0
(t-test)		(-0.7255)		(-0.963)		(0.0856)	
SEK/EUR	80	0.0415	**0.0297**	0.0496	**0.0429**	**-0.0068**	0
(t-test)		(-2.3493)		(-1.7102)		(0.9561)	
CHF/EUR	57	0.0339	**0.0294**	0.0385	**0.0364**	**-0.0007**	0
(t-test)		(-0.5212)		(-0.2223)		(0.0385)	
USD/GBP	58	0.06	**0.0597**	0.0706	**0.0705**	**-0.0188**	0
(t-test)		(-0.0496)		(-0.0165)		(0.9938)	

Table 5: Forecasting performance of the survey data and the random walk model on the 12-months horizons

		mean absolute error (MAE)		root mean squared error (RMSE)		mean loss ($-\pi$)	
Exchange rate	Num. obs.	Model: general	RW	Model: general	RW	Model: general	RW
CAD/USD	57	0.1065	**0.1024**	**0.1162**	0.1238	**-0.0128**	0
(t-test)		(-0.2249)		(0.4365)		(0.5579)	
EGP/USD	76	**0.1527**	0.1952	**0.1749**	0.2332	**-0.0792**	0
(t-test)		(1.2616)		(1.851)		(1.6717)	
USD/EUR	53	**0.1604**	0.1615	**0.1888**	0.1902	**-0.0328**	0
(t-test)		(0.0493)		(0.0407)		(0.8871)	
ILS/USD	69	0.0676	**0.0652**	**0.0787**	0.0862	**-0.0157**	0
(t-test)		(-0.7)		(0.5651)		(0.9889)	
JPY/USD	29	0.1224	**0.1095**	0.1381	**0.1265**	**-0.0081**	0
(t-test)		(-0.6834)		(-0.4919)		(0.3285)	
NGN/USD	77	0.2532	**0.0989**	0.2661	**0.1217**	**-0.0393**	0
(t-test)		(-2.3327)		(-2.6854)		(1.4546)	
NOK/EUR	43	**0.0504**	0.0648	**0.0589**	0.0752	**-0.0134**	0
(t-test)		(1.7287)		(1.5387)		(1.0938)	
ZAR/USD	56	0.2975	**0.2753**	0.3482	**0.3236**	**-0.0215**	0
(t-test)		(-0.599)		(-0.5652)		(0.347)	
SEK/EUR	68	0.0532	**0.0315**	0.0582	**0.0447**	0.0017	**0**
(t-test)		(-4.4793)		(-5.0545)		(-0.4454)	
CHF/EUR	45	0.0399	**0.0394**	0.0477	**0.0444**	0.0003	**0**
(t-test)		(-0.0558)		(-0.3633)		(-0.0407)	
USD/GBP	51	**0.0963**	0.0981	**0.1147**	0.119	**-0.0165**	0
(t-test)		(0.1229)		(0.6069)		(1.9533)	

Table 6: Forecasting performance of the survey data and the random walk model on the 24-months horizons