

# Informal Insurance and Income Inequality\*

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## Abstract

This paper examines the effects of income inequality in a risk sharing model with limited commitment, that is, when insurance agreements have to be self-enforcing. In this context, numerical dynamic programming is used to examine three questions. First, I consider heterogeneity in mean income, and study the welfare effects when inequality together with aggregate income increases. Second, subsistence consumption is introduced to see how it affects consumption smoothing. Finally, income is endogenized by allowing households to choose between two production technologies, to look at the importance of consumption insurance for income smoothing.

**Keywords:** risk sharing, limited commitment, inequality, technology choice, developing countries

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# 1 Introduction

In low-income village economies we often observe incomplete markets. Financial instruments or formal insurance contracts are often lacking. However, growing empirical evidence suggests that households enter into informal risk sharing arrangements, and achieve some, though not perfect insurance. The questions are then, (i) how this partial insurance can be modeled, (ii) what are its implications for the consumption and welfare of households, and (iii) what policies are appropriate in this context. This paper considers a model where informal insurance is characterized by limited commitment, in other words, insurance arrangements have to be self-enforcing (Kocherlakota (1996), Ligon, Thomas, and Worrall (2002)). This setting allows us to explain the observed partial insurance and shed some light on the mechanisms involved.

Examining informal risk sharing in the context of developing countries is important for two main reasons. On the one hand, people living in low-income, rural areas often face a huge amount of risk. Revenue from agricultural production is usually low and volatile, further, outside job opportunities are often lacking. On the other hand, financial instruments, or formal, legally enforceable insurance contracts are often not available to smooth consumption inter-temporally or across states of nature. The question is then, how can people in these kinds of environments somehow mitigate the effects of risk they face. Growing empirical evidence suggests that households achieve something better than autarky, but not quite perfect risk sharing (see the seminal paper by Townsend (1994), among many others), by transfers, gifts, quasi-credit, and the like among relatives, neighbours, or friends (see, for example, anthropological work by Platteau and Abraham (1987) and Platteau (1997)). This means that consumption reacts to idiosyncratic changes in income, but the variance of consumption is less than that of income.

Informal insurance is modeled in this paper by supposing that contracts have to be self-enforcing, because often no authority exists to enforce insurance agreements in poor villages in developing countries, while informational problems are less important. This approach yields partial insurance, which is consistent with empirical evidence. The model has a wide range of interpretations. In addition to thinking about households in a village, we may consider members of a family (Mazzocco, 2007), an employee and an

employer (Thomas and Worrall, 1988), or countries (Kehoe and Perri, 2002).

In this paper an infinite-horizon model is considered with risk-averse households, whose income follows some exogenous, discrete stochastic process, that is common knowledge. I concentrate on insurance across states of nature, and ignore savings, or storage. I look for a constrained-efficient solution, maximizing a utilitarian social welfare function subject to resource constraints and enforcement constraints. That is, it is required that, for each household at every period and every state of the world, staying in the informal risk sharing contract be better than reverting to autarky. If income is independently and identically distributed (iid) or follows a Markov-process, we have the following important property characterizing the solution: the current ratios of marginal utilities between households, and therefore the consumption allocation, depends only on current income realizations and the ratios of marginal utilities in the previous period. In addition, unlike in the perfect risk sharing case, the allocation in the limited commitment solution depends not only on aggregate income, but also on its distribution. This is because individual income determines the utility a household may get were she in autarky, that is, her threatpoint. This threatpoint in turn determines the household's bargaining, or decision power.

This paper examines the interaction of income inequality and self-enforcing risk sharing contracts. To do this, three types of simulation exercises are performed in the context of the model of risk sharing with limited commitment. In all cases I assume that only two households populate the village economy, and that each household's income may take only two values, for clarity and computational ease.

First, I consider a "poor" household interacting with a "rich" one. The households have the same isoelastic, or constant-relative-risk-aversion (CRRA) utility function, and they differ in their mean income, while they face the same amount of risk in the sense that the standard deviation of their income process is the same. I perform a comparative statics exercise: while keeping the income process of the poor the same, the mean income of the rich is increased, thereby increasing inequality together with aggregate and per-capita income. Note that we do not expect this type of inequality to have any adverse effects, since what happens is just that in each state of the world we give more income to the rich, while leaving the income of the poor unchanged. However, for some reasonable parameter values, the poor is worse off when inequality

together with per-capita income increases. This is in contrast with Genicot (2006), who emphasizes the possible positive effects of inequality, keeping aggregate income constant. The intuition behind my result is that the poor household's relative decision power decreases vis-a-vis the rich, thus she can secure smaller net transfers in the limited commitment solution. Another way of putting it is that the rich demands less insurance, she behaves in a less risk-averse fashion, thus the rich does not value the contract much. The result warns of the possible adverse consequences of inequality for the poor even when per-capita income increases in the community, the reason being that the poor is more and more excluded from informal insurance arrangements.

Second, I take just one pair of income processes, but “subsistence consumption”, or, a “subsistence level” is added. In other words, I suppose decreasing relative risk aversion (Ogaki and Zhang, 2001). The effects of changes in the subsistence level is examined in this example. A higher subsistence level makes insurance more valuable for both agents, thus it may make perfect risk sharing self-enforcing. Here it is interesting to look at the properties of the consumption process, since income does not change. The consumption of the poor becomes less volatile as the subsistence level increases, but she has to sacrifice mean consumption to compensate the rich for the insurance she provides. Further, when perfect risk sharing is self-enforcing, aggregate risk can be shared more efficiently.

Finally, in the last example economy, income is endogenized. In particular, the possibility to choose between two production technologies is introduced, to examine the consequences of lack of insurance for income smoothing (Morduch, 1995). A technology is described by the income process it generates. As in the first example, households have standard CRRA utility functions. In two numerical examples, I consider two types of heterogeneity in turn, (i) the rich household has some exogenous wealth that yields a fixed revenue every period, and (ii) the two households differ in their risk preferences. Note that also in case (i), the rich behaves in a less risk-averse fashion. Further, in each period, households may choose between two technologies, an “old”, safer technology with lower expected values, and a “new”, riskier, but more profitable technology. I look at households' technology choice both with and without informal insurance. In both numerical examples, when an informal risk sharing contract becomes available, one household switches to the riskier technology with higher expected profits, in all states

of the world and time periods. This result illustrates the importance of consumption insurance for production choices, and the negative consequences high risk aversion may have on expected profits, for example when households living near the subsistence level are willing to bear very little risk.

The rest of the paper is structured as follows. Section 2 discusses some related literature. Section 3 outlines the model of risk sharing with limited commitment, and talks about some characteristics of the solution. An algorithm to numerically solve the model is described in the appendix. Section 4 presents simulation results to examine the interaction between informal risk sharing and income inequality. Section 5 concludes.

## 2 Related Literature

There is a growing literature on informal insurance in rural communities in developing countries. It has been recognized that even without formal contracts, households enter into risk sharing arrangements. In a world with complete information and perfect commitment, informal insurance would even achieve the first best, or full insurance, that is, the ratios of marginal utilities would stay the same in all states of nature and across time. This perfect risk sharing outcome can be imagined as the case where incomes are pooled in the village, and then redistributed according to some predetermined weights. A number of papers test the hypothesis of full insurance in low-income village economies (see Townsend (1994) for Indian villages in the semi-arid tropics, Grimard (1997) using data from Ivory Coast, Dubois (2000) on Pakistan, Dercon and Krishnan (2003a, 2003b) working with Ethiopian data, Laczó (2005) using Bangladeshi data, and Mazzocco and Saini (2007) for India, among others). Perfect insurance is rejected, but a remarkable amount of risk sharing is found. Thus a next step is to think about partial insurance, how and why households achieve something better than autarky, but not full insurance.

In modeling partial insurance, we may relax the assumption of complete information or perfect commitment. Ligon (1998) introduces private information in a dynamic setting. He derives Euler-equation type reduced form restrictions to test the private information model against the alternatives of full insurance and the permanent income hypothesis. Ligon (1998) finds that consumption in two of the three Indian villages examined is best explained by the private information model, while in the third village

different households seem to belong to different regimes, but most of them are classified as belonging to the permanent income regime. Wang (1995) establishes some theoretical results for the model of risk sharing with private information, and provides an algorithm to compute the solution.

The second approach is to relax the assumption of perfect commitment, and instead require contracts to be self-enforcing. One may argue that this way of modeling partial insurance in small, rural communities is more appropriate, since households are able to observe what their neighbors are doing and shocks they face (crop damage, or illness for example), but there is no commitment device, like an independent authority, to enforce contracts. In addition, arguably this model is also appropriate when one thinks about risk sharing within the family, since husband and wife are free to end the contract, that is, they may divorce. Introducing lack of commitment extends the standard collective model of the household (Browning and Chiappori, 1998) in an interesting way (see Mazzocco (2007)). Another interpretation is long-term labour contracts, where both employer and employee may choose to end the contract in favour of an outside option (Thomas and Worrall, 1988). A further application concerns the interaction between two countries, since a country may default on its sovereign debt, facing possible exclusion from future international trade and financial contracts (see Kehoe and Perri (2002)). Schechter (2007) uses the model to explain the interaction between a farmer and a thief.

One-sided limited commitment is relevant for principal-agent models, for example in the case of a contract between an insurance company and an insured, where the insurance company (the principal) is fully committed, while the insured (the agent) is not. For empirical evidence on one-sided limited commitment see the work of Hendel and Lizzeri (2003) on life insurance, and Crocker and Moran (2003) on health insurance. Two-sided limited commitment is introduced in a dynamic wage contract setting by Thomas and Worrall (1988). A very important result they derive is that contracts are history dependent, that is, past outcomes influence today's payoffs.

Kimball (1988) is the first to argue that informal risk sharing in a community may be achieved with voluntary participation of all members. He shows that for reasonable values of the discount factor and the coefficient of relative risk aversion, households could provide a substantial amount of insurance to one another. Early contributions

to modeling risk sharing with limited commitment include Coate and Ravallion (1993), who introduce two-sided limited commitment in a dynamic model, but they restrict contracts to be static. Their characterization of transfers is not optimal, once we allow for history-dependent contracts. On the other hand, Kocherlakota (1996) allows for dynamic contracts, and proves existence and some properties of the solution, but he does not give an explicit characterization. Early empirical evidence on dynamic limited commitment is provided by Foster and Rosenzweig (2001). They test the restriction that there is a negative relationship between the current transfer and aggregate past transfers, and they find some supporting evidence. Anthropological work by Platteau (1997) also points out the importance of limited commitment in informal risk sharing contracts. Charness and Genicot (2006) provide experimental evidence in support of the model.

Ligon, Thomas, and Worrall (2002) characterize and calculate the solution of a dynamic model of risk sharing with limited commitment. As a result, the authors are able to test in a structural manner the hypothesis of dynamic limited commitment against the alternatives of perfect risk sharing, autarky, and the static limited commitment model of Coate and Ravallion (1993). They find evidence in support of the dynamic limited commitment model, using data from Indian villages. In addition, Ligon et al. (2002) derive a number of theoretical properties of the solution. In particular, they look at the effect of changing the discount factor, relative income across different states of the world (or different riskiness of the environment), and the direct penalty faced by the household breaking the agreement. More risk raises the demand for insurance, while a higher discount factor and harsher penalties help to enforce more risk sharing.<sup>1</sup>

Attanasio and Ríos-Rull (2000) examine the effects of the introduction of an aggregate insurance scheme in a world with informal insurance and lack of commitment. They show, by an example, that aggregate insurance might reduce welfare. The reason is that aggregate insurance crowds out informal insurance, because it raises the value of autarky, and in some cases it even crowds out more insurance than it provides. The

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<sup>1</sup>Ligon et al. (2002) assume no savings. In another contribution (Ligon, Thomas, and Worrall, 2000) the authors look at the effects of savings, and show, by an example, that the possibility to save may decrease welfare. In general, it is difficult to allow for savings in a model with limited commitment, since savings enter the enforceability constraints, and I will assume away savings as well.

authors also present some suggestive empirical evidence on the crowding out of private transfers by public ones using data from Mexico, but their approach is reduced form, and they do not actually use the theoretical model to predict private transfers.

An important innovation of the above papers is the methodology used to calculate the solution of the problem. Ligon et al. (2002) use a Pareto-frontier approach to find the solution of the risk sharing with limited commitment model. Attanasio and Ríos-Rull (2000) and Kehoe and Perri (2002) apply a slightly different methodology, building on the work of Marcet and Marimon (1998). In this approach the social planner's problem is examined. The problem is a difficult one, since future decision variables enter into today's enforcement constraints, thus the problem is not recursive. However, the weights of households' utilities in the social planner's objective, equal to the ratio of marginal utilities in equilibrium, can be introduced as a co-state variable. With the new (co-)state variable the problem has a recursive structure. I use this later approach in this paper.

Some extensions of the model of risk sharing with limited commitment have been developed recently. Genicot and Ray (2003) consider possible deviations by a group of households in an informal risk sharing arrangement among  $n$  households. The main message of their paper is that the stability of a risk sharing group with respect to deviations by a smaller group is a complex issue, and there is not much we can say in general. One interesting result is that stable groups are limited in size. Wahhaj (2006) introduces public goods, and shows that in this case, private consumption of a member may increase when the community experiences an adverse aggregate shock. He argues that this result is consistent with empirical evidence provided by Duflo and Udry (2003) on intrahousehold allocation in Cote d'Ivoire. Dubois, Jullien, and Magnac (2007) consider both formal and informal contracts. Formal contracts are short-term, so households may complement these by self-enforcing, informal ones. The authors use semi-parametric techniques to test the model, and find that it explains well the consumption of Pakistani households. Hertel (2007) considers both limited commitment and private information. As a simplification, one household receives a fixed income each period, while the second household's income is stochastic, and its realization is her private information. The author shows that, with additional incomplete information, consumption adjusts slowly to income changes, while there still exists a unique

nondegenerate stationary distribution of utilities.

The literature examining the relation between insurance and inequality includes Morduch (1994), who draws attention to the fact that lack of insurance may exacerbate poverty. In a simple model, he shows that a lack of consumption credit may lead the poor to forego risky, but profitable investment opportunities. Fafchamps (2002) summarizes some results concerning different concepts of inequality (income, wealth, cash-in-hand, consumption, and welfare) in environments that differ in the type of assets available and in risk sharing opportunities. He briefly talks about the limited commitment case as well, and states that, the more efficient risk sharing is, the more persistent poverty is, and that limited commitment, as a departure from perfect risk sharing, allows for social mobility<sup>2</sup>. Furthermore, the author talks about the emergence of patronage in polarized societies, meaning that the rich provides insurance to the poor in exchange for net transfers from the poor on average. With positive returns to assets, patronage is transitory, because in the long run the poor also accumulates sufficient assets to self-insure. If returns are negative, patronage reinforces inequality in the short run, while in the long run all wealth is depleted.

Krueger and Perri (2006) explain the fact that, in the United States, cross-sectional consumption inequality has not increased as much as income inequality, using the model of risk sharing with limited commitment. However, the authors consider ex-ante identical households, only the income realizations differ in a given period, and are perfectly negatively correlated. In this setting, what they actually show is that there is partial insurance. Partial insurance implies, by definition, that consumption is less volatile than income across states of the world, which is equivalent to there being less consumption than income inequality in the cross-section.

Genicot (2006) examines similar issues as the present paper. The author considers the model of risk sharing with limited commitment as well. She argues that (i) in some cases wealth inequality may help risk sharing in the sense that perfect risk sharing is possible in a wider range of cases, and that (ii) total welfare may increase with inequality, keeping aggregate, or per-capita, wealth<sup>3</sup> constant. On the modeling side,

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<sup>2</sup>Note that Fafchamps (2002) defines welfare inequality as the ratio of marginal utilities, so there is no social mobility in terms of welfare in the perfect risk sharing case *by definition*.

<sup>3</sup>Note that in this context, “income” and “wealth” are essentially the same, meaning that we think

an important shortcoming of the paper is that it only considers static contracts, which have been proven not to be constrained-efficient in the dynamic case. The present paper allows for history-dependent contracts.

### 3 Modeling Informal Insurance

This section presents the basic model. First, we look at perfect risk sharing as a benchmark. Then, limited commitment is introduced, requiring contracts to be self-enforcing. The context is a stochastic, dynamic framework with common beliefs, and egoistic, risk-averse households consuming a private, perishable good.

For the sake of clarity, let us consider a village, or community, of two households. Extending the model to  $n$  households is straightforward<sup>4</sup>. The households live in an uncertain environment: their income realizations are unknown ex ante. Income realizations are common knowledge ex post. As a consequence, they might choose to insure through a formal or informal agreement against variation of incomes. Risk sharing can thus be defined as follows. “Any two [households] may be said to share risk if they employ state-contingent transfers to increase the expected utility of both by reducing the risk of at least one.” (Ligon, 2004)

In section 3.1, I describe the model of perfect risk sharing. Formally, households may sign an enforceable contract in period 0, in other words, we assume full commitment, and that income realizations are observable by both agents and verifiable by a third party. In section 3.2, households still observe the income realizations, but they cannot sign formal insurance contracts. Only informal risk sharing arrangements are possible instead, meaning that at each period and each state of the world, it is required that both households respect *voluntarily* the terms of the agreement.

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of both as an amount that is consumed in the current period. In other words, wealth means the fixed revenue from assets the household owns in each period, rather than the value of assets.

<sup>4</sup>The theoretical properties can easily be extended in both the perfect risk sharing and the limited commitment case. The algorithm to compute the solution also logically extends to  $n$  households, however, in the limited commitment case, computation time might be prohibitive with  $n$  large.

### 3.1 Perfect Risk Sharing

Let us consider a dynamic model of risk sharing. Assume that the economy is populated by two infinitely-lived households, indexed 1 and 2. Their preferences are identical, and separable over time and across states of nature. The utility function  $u()$  is defined over a private, perishable consumption good  $c$ , and it is assumed to be monotone increasing, strictly concave (so households are risk averse), and twice continuously differentiable. Households live in an uncertain environment, and income of each individual  $y_i$  follows some exogenous discrete stochastic process, that is common knowledge. In other words, beliefs about the distribution of the state of nature, or the “income state” (the vector (income of 1, income of 2)), are homogeneous. In mathematical terms, each agent  $i$  seeks to maximize the following von Neumann-Morgenstern expected utility:

$$E_0 \delta^t u(c_{it}), \quad (1)$$

where  $E_0$  is the expected value at time 0 calculated with respect to the probability measure describing the common beliefs,  $\delta \in (0, 1)$  is the discount factor, and  $c_{it}$  is consumption of household  $i$  at time  $t$ . I concentrate on insurance across state of nature, and assume no savings, or storage.

Let  $s_t$  (with a lower index  $t$ ) denote the income state at time  $t$ , and  $s^t = (s_1, s_2, \dots, s_{t-1}, s_t)$  (with an upper index  $t$ ) the history of income states up to  $t$ . Let us first consider autarky as a benchmark. In autarky, each household consumes her own income in every state and every period, since there is no possibility to save or borrow. In this case, household  $i$  receives the following expected lifetime utility:

$$\sum_{t=1}^{\infty} \delta^t \pi(s^t) u(y_i(s^t)), \quad (2)$$

where  $\pi(s^t)$  is the probability of history  $s^t$  occurring, and  $y_i(s^t)$  denotes the income of individual  $i$  at time  $t$  when history  $s^t$  has occurred.

Now, suppose that households may sign an enforceable risk sharing contract. A risk sharing contract specifies transfers that may depend, a priori, on the whole history of income states  $s^t$ . The timing is the following. At time 0, a risk sharing contract may be signed, then, at time 1 and each subsequent period, the income state is realized, then transfers are made according to the contract, and finally, consumption takes place.

First, the properties of the contract are described, given that it is signed. Then, we examine under what conditions agents are ready to actually sign the contract at time 0, in other words, we look at the ex-ante participation constraints.

In the presence of complete information, that is, in each period each household perfectly observes the other household's income realization, and under full commitment, the ex-ante Pareto-optimal allocations can be found by considering the social planner's problem. The social planner's objective is to maximize a weighted sum of households' lifetime utilities,

$$\max_{\{c_i(s^t)\}} \sum_i \lambda_i \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi(s^t) u(c_i(s^t)), \quad (3)$$

where  $\lambda_i$  is the weight the social planner assigns to household  $i$ , and  $c_i(s^t)$  denotes the consumption of individual  $i$  at time  $t$  when history  $s^t$  has occurred; subject to the resource constraint

$$\sum_i c_i(s^t) \leq \sum_i y_i(s^t), \quad (4)$$

for all histories  $s^t$ .

The Lagrangian is

$$\sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi(s^t) \left[ \sum_i \lambda_i u(c_i(s^t)) + \gamma(s^t) \left( \sum_i y_i(s^t) - c_i(s^t) \right) \right], \quad (5)$$

where  $\delta^t \pi(s^t) \gamma(s^t)$  is the multiplier on the resource constraint at history  $s^t$ . Note that we can reverse the order of the summation signs because of two properties, (i) the linearity of the expected utility function, and because (ii) the social planner's objective is additive in households' lifetime utilities (utilitarian social welfare function).

The first order condition for household  $i$ , if history  $s^t$  has occurred, is

$$\lambda_i u'(c_i(s^t)) = \gamma(s^t) \quad (6)$$

Combining the first order conditions for the two households at history  $s^t$ , we have

$$\frac{u'(c_1(s^t))}{u'(c_2(s^t))} = \frac{\lambda_2}{\lambda_1} \equiv x_0 = cste, \quad (7)$$

where  $x_0$  is the (initial) relative weight assigned to household 2.

Equation (7) indicates that the ratio of marginal utilities is constant across states and over time in the case of perfect risk sharing (Wilson, 1968). (7) is also called the Borch rule. Dividing the first order conditions across periods yields

$$\frac{u'(c_1(s^t))}{u'(c_1(s^{t-1}))} = \frac{u'(c_2(s^t))}{u'(c_2(s^{t-1}))}, \forall s^t \supset s^{t-1}, \quad (8)$$

which means that the growth path of marginal utilities of all households is the same. Note that the expectations operator does not appear in this condition, which is the hallmark of full insurance.

Equations (7) and (8) give us the three major implications of efficient risk sharing in this framework. First, the (relative) Pareto weight  $x_0$  is constant across time. Second, the consumption allocation at time  $t$  depends only on  $s_t$ , the income realizations at time  $t$ , and is independent of the history of income states  $s^{t-1}$ . Third and moreover, the consumption allocation, depends only on aggregate income, and is independent of the distribution of income. Income pooling together with the constant relative weight determine the consumption of each agent, and assure *ex-ante Pareto efficiency*.

To summarize, the consumption allocation at time  $t$ , given the current income state  $s_t$ , only depends on aggregate income  $y_1(s_t) + y_2(s_t)$ , and the relative weight the social planner assigns to household 2,  $x_0$ , which pins down a point on the Pareto-frontier. Denote  $c_i(s_t, x_0)$ ,  $i = 1, 2$ , the solution to (7) and (4), noting once again that the solution  $c_i()$  only depends on  $s_t$  and is independent of  $s^{t-1}$ .  $c_i(s_t, x_0)$  is called the *sharing rule*.

Clearly, an *ex-ante participation constraint* should also be satisfied, that is, at time 0 it must be that the expected lifetime utility for each household signing the contract is at least as high as in autarky. Technically, this implies that some points of the Pareto-frontier, or some  $x_0$ 's, cannot be attained under the risk sharing contract.

To introduce the participation constraints, we have to calculate each agent's expected lifetime utility at the moment of contracting, and make sure that it is greater than the expected lifetime utility under autarky. Assuming that the income state follows a Markov-process allows us to express agents' lifetime utility recursively. This is because with the Markov assumption, the current state  $s_t$  tells us everything we need

to know about the income process in the past. In mathematical terms, the conditional distribution of the income state at time  $t + 1$  only depends on the realization of the income state at  $t$ , and not on the whole history.

The Bellman-equation can be written, when the state of the world is  $s_t$ , as

$$U_i^{aut}(s_t) = u(y_i(s_t)) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | s_t) U_i^{aut}(s_{t+1}), \quad (9)$$

where  $U_i^{aut}(s_t)$  is the lifetime utility, or welfare, of household  $i$  in autarky, given today's state  $s_t$ , or, in other words,  $U_i^{aut}()$  is the autarkic value function; and  $\pi(s_{t+1} | s_t)$  is the conditional probability of state  $s_{t+1}$  occurring tomorrow if state  $s_t$  occurs today, which is common knowledge.  $U_i^{aut}(s_t)$  can easily be found by successive iteration using the contraction mapping property of the Bellman-equation.

Suppose that the unconditional distribution of the income state at time 1 is known. Now, we may also compute the expected lifetime utility for agent  $i$  at time 0, when the risk sharing contract may be signed. Ex ante, at time 0, the expected value of autarky for agent  $i$ , denoted  $EU_i^{aut}$  is

$$EU_i^{aut} = E_0 U_i^{aut}(s_1)$$

Let us now turn to calculating the lifetime utility of household  $i$  in the case of perfect risk sharing, like we did for autarky. Assuming once again that the income process is Markovian, we have a recursive problem. The value function of agent  $i$  at state  $s_t$  and with weight  $x_0$ , in the case of perfect risk sharing, can be written recursively as

$$U_i^{prs}(s_t, x_0) = u(c_i(s_t, x_0)) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | s_t) U_i^{prs}(s_{t+1}, x_0), \quad (10)$$

where  $U_i^{prs}(s_t, x_0)$  is the value of the infinite consumption stream in case of full insurance, given today's state  $s_t$  and relative weight  $x_0$ . As the autarkic utility, the value of perfect risk sharing can easily be found by successive iteration.

Given  $x_0$  and the unconditional distribution of the income state at time 1, the expected value of the full insurance solution for agent  $i$  is denoted  $EU_i^{prs}(x_0)$ , and is given by

$$EU_i^{prs}(x_0) = E_0 U_i^{prs}(s_1, x_0).$$

At last we may return to the ex-ante participation constraints. So we require that

$$EU_i^{prs}(x_0) \geq EU_i^{aut}, \forall i, \quad (11)$$

that is, the value of the perfect risk sharing allocation must be as great as the value of autarky. (11) rules out for example that one agent makes a transfer to the other whichever income state occurs. For all  $x_0$  such that (11) is satisfied, a contract ensuring perfect risk sharing is signed at time 0, and is implemented in all subsequent periods. For other  $x_0$ 's one agent prefers to stay in autarky, thus no insurance contract is signed.

### 3.2 Risk Sharing with Limited Commitment

In this section we consider the case when agents are unable to commit, and there is no authority to enforce risk sharing contracts either, building on Attanasio and Ríos-Rull (2000), Kocherlakota (1996), Ligon, Thomas, and Worrall (2002), and others. The objective (3) is maximized, subject to the resource constraints (4), and additional *enforcement constraints*. At each time  $t$ , after each history  $s^{t5}$ , and for  $i = 1, 2$ , the following inequality must be satisfied:

$$\sum_{r=t}^{\infty} \sum_{s^r} \delta^{r-t} \pi(s^r | s^t) u(c_i(s^r)) \geq U_i^{aut}(s^t), \quad (12)$$

where  $\pi(s^r | s^t)$  is the probability of history  $s^r$  occurring given that history  $s^t$  occurred up to period  $t$  ( $r \geq t$ ).

In words, (12) means that each household's expected utility from staying in the informal risk sharing contract must be greater than her expected utility if she deviates and consumes her own income thereafter. This condition is based on the assumption that if one household deviates, the other household does not enter into any risk sharing with her any more. Note that reversion to autarky is the most severe subgame perfect punishment in this environment (Abreu, 1988). We might call reversion to autarky a trigger strategy, or the breakdown of trust. We may also call (12) an *ex-post participation constraint*, meaning that it requires each agent to voluntarily "sign" the contract after any realization of the history of states. Obviously, this is a stronger requirement

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<sup>5</sup>I speak about histories again, to write the basic model in a general form.

than the ex-ante participation constraints that have to be satisfied in the case of perfect risk sharing.

Notice that adding the constraints (12) substantially complicates the analysis, because future decision variables enter into today's enforcement constraints. Thus the problem at hand no longer has a recursive structure, even with a Markov-process assumption on incomes, and the whole history of states might matter. Following Marcat and Marimon (1998), Attanasio and Ríos-Rull (2000), and Kehoe and Perri (2002), I reformulate the problem. By adding a co-state variable, in particular the relative weight in the social planner's problem, or, in other words, the ratio of marginal utilities, the problem can be written in a recursive form.

Denoting the multiplier on the enforcement constraint of household  $i$  by  $\delta^t \pi(s^t) \mu_i(s^t)$ , and the multiplier on the resource constraint by  $\delta^t \pi(s^t) \gamma(s^t)$  when history  $s^t$  has occurred, the Lagrangian is

$$\begin{aligned} & \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi(s^t) [\sum_i \lambda_i u(c_i(s^t)) + \\ & + \mu_i(s^t) (\sum_{r=t}^{\infty} \sum_{s^r} \delta^{r-t} \pi(s^r | s) u(c_i(s^r)) - U_i^{aut}(s^t)) + \\ & + \gamma(s^t) (\sum_i y_i(s^t) - c_i(s^t))] \end{aligned} \quad (13)$$

The Lagrangian can also be written in the following form:

$$\begin{aligned} & \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi(s^t) [\sum_i M_i(s^{t-1}) u(c_i(s^t)) + \\ & + \mu_i(s^t) (u(c_i(s^t)) - U_i^{aut}(s^t)) + \gamma(s^t) (\sum_i y_i(s^t) - c_i(s^t))] \end{aligned} \quad (14)$$

where  $M_i(s^t) = M_i(s^{t-1}) + \mu_i(s^t)$  with  $M_i(s^0) = \lambda_i$ . In words,  $M_i(s^t)$  is the initial weight on agent  $i$  plus the sum of the Lagrange multipliers on her enforcement constraints along the history  $s^t$ .

The first order condition with respect to  $c_i(s^t)$  is

$$\delta^t \pi(s^t) M_i(s^t) u'(c_i(s^t)) - \gamma(s^t) = 0. \quad (15)$$

We also have standard first order conditions relating to the resource and enforcement constraints, with complementarity slackness conditions. Combining the first order conditions (15) for the two households for history  $s^t$  at time  $t$ , we have

$$\frac{u'(c_1(s^t))}{u'(c_2(s^t))} = \frac{M_2(s^t)}{M_1(s^t)} = \frac{\lambda_2 + \mu_2(s^1) + \mu_2(s^2) + \dots + \mu_2(s^t)}{\lambda_1 + \mu_1(s^1) + \mu_1(s^2) + \dots + \mu_1(s^t)} \equiv x(s^t), \quad (16)$$

where  $x(s^t)$  can be thought of as the relative weight assigned to household 2 when history  $s^t$  has occurred. Notice that, unlike in the perfect risk sharing case, where  $\mu_i(s^r) = 0, \forall i, \forall s^r$ , in the case of limited commitment, the relative weight  $x(s^t)$  will vary over time and across states. We would like to keep  $x$  constant (as in first best), but when an enforcement constraint binds, we cannot do that. However, intuitively we will try to keep  $x(s^t)$ , for all  $s^t \supset s^{t-1}$ , as close as possible to  $x(s^{t-1})$ .

The relative weight  $x(s^t)$ , defined in (16) is used as an additional co-state variable in order to rewrite the problem in a recursive form. This idea is due to Marcat and Marimon (1998). To do this, suppose once again that the state of the world with respect to income follows a Markov process, so that we may write  $\pi(s^t | s^{t-1}) = \pi(s_t | s_{t-1})$ . Still, the current income state  $s_t$  does not tell us everything we need to know about the past, only  $(s_t, x_{t-1})$  does, where  $x_{t-1}$  is the relative weight inherited from the previous period. Denote  $x_t$  the new relative weight we have to find at time  $t$ . We are looking for policy functions for the consumption allocation and the new relative weight, with support over the extended state space  $(s_t, x_{t-1})$ , that is, we want to know  $c_i(s_t, x_{t-1})$ ,  $\forall i$ , and  $x_t(s_t, x_{t-1})$ . At last, the value functions can be defined recursively as

$$V_i(s_t, x_{t-1}) = u(c_i(s_t, x_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | s_t) V_i(s_{t+1}, x_t(s_t, x_{t-1})). \quad (17)$$

We may also call  $c_i(s_t, x_{t-1})$  the *sharing rule*. Note that since policies and values depend on  $x_{t-1}$ , the contract is *history dependent*.

Numerical dynamic programming allows us to solve for the consumption allocation and lifetime utilities, given the income processes, utility functions and discount rates for the two households, and the initial relative weight in the social planner's objective. The appendix explains how in details. The next section uses the algorithm to generate comparative static results to examine issues related to the interaction of inequality and informal risk sharing contracts.

What are the properties of the solution? First of all, it is easy to see that, if the discount factor  $\delta$  is sufficiently large, then the perfect risk sharing solution is self-enforcing for some  $x_0$ 's (folk theorem), while if  $\delta$  is sufficiently small, there does not

exist any non-autarkic allocation that is sustainable with voluntary participation. Now, suppose that there exists a non-autarkic solution, but the first best is not self-enforcing for any  $x_0$ .

The limited commitment solution can be fully characterized by a set of state-dependent intervals on the relative weight of household 2, or ratio of marginal utilities,  $x$ , that give the possible relative weights in a given income state. Note that there is a one-to-one relationship between the relative weight and the consumption allocation, given the income state (see (16)). These are *optimal* intervals, meaning that they correspond to optimally chosen future promised utilities as well. Once we have found the intervals we know everything there is to know about the solution. Denote the interval for state  $s$  by  $[\underline{x}^s, \bar{x}^s]$ .

Suppose we have inherited some  $x_{t-1}$  from last period, and today the income state is  $s$ .  $x_t$  is determined by the following updating rule:

$$x_t = \begin{cases} \bar{x}^s & \text{if } x_{t-1} > \bar{x}^s \\ x_{t-1} & \text{if } x_{t-1} \in [\underline{x}^s, \bar{x}^s] \\ \underline{x}^s & \text{if } x_{t-1} < \underline{x}^s \end{cases} \quad (18)$$

To see how this works, suppose that the two households are identical ex ante,  $u(\cdot) = \log(\cdot)$ , and their income may only take two values,  $y^h$  (high) or  $y^l$  (low), with  $y^h > y^l > 0$ . There are four income states,  $hh$ ,  $hl$ ,  $lh$ , and  $ll$ , where the first argument refers to household 1's income, and the second to household 2's income. Suppose that the intervals overlap, except for states  $hl$  and  $lh$ , so  $\bar{x}^{hh}, \bar{x}^{ll} > \underline{x}^{lh} > 1 > \bar{x}^{hl} > \underline{x}^{hh}, \underline{x}^{ll}$ .<sup>6</sup> Take  $x_0 = 1$ , so the two agents have equal weights in the social planner's objective. Now, suppose that at time 1 the state is  $hh$ . In this case,  $x$  can be kept constant, because  $1 \in [\underline{x}^{hh}, \bar{x}^{hh}]$ , so  $x_1 = x_0$ , and no transfer is made. Suppose that at time 2 the state is  $lh$ . We cannot keep  $x$  constant any more, because household 2 is not willing to share aggregate income equally (she would prefer to revert to autarky instead), her enforcement constraint is binding. We set  $x_2 = \underline{x}^{lh} > x_1$ , agent 2 is making a transfer, but not as large as she would in the perfect risk sharing solution. Suppose that at

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<sup>6</sup>Take the numerical example from Ligon, Thomas, and Worrall (2002), that is  $y^l = 1$  and  $y^h = 2$ , and suppose that the discount factor  $\delta = 0.95$ . Then the optimal intervals are  $[\underline{x}^{hh}, \bar{x}^{hh}] = [\underline{x}^{ll}, \bar{x}^{ll}] = [0.934, 1.070]$ ,  $[\underline{x}^{hl}, \bar{x}^{hl}] = [0.5, 0.961]$ , and  $[\underline{x}^{lh}, \bar{x}^{lh}] = [1.041, 2]$ , so the  $hl$  and  $lh$  intervals do not overlap, but both overlap with the interval for the symmetric states.

time 3 the income state is  $hh$  once again. But, unlike at time 1, now we would like to set  $x_3 = x_2 > 1$ . We can do so, since we have supposed that the  $hh$  and  $lh$  intervals overlap. Notice that we are in a symmetric state, the incomes of the two households are equal, but household 1 is making a transfer to household 2, because of the *history dependence* of the contract. In this way household 1 partly reciprocates the transfer she got in period 2, so risk sharing with limited commitment has a *quasi-credit* element (Fafchamps, 1999). Now, suppose that at time 4 the state is  $hl$ . The best we can do is to set  $x_4 = \bar{x}^{hl}$ . Now household 1 is helping out household 2, who has a bad income realization. If at time 5 we are at a symmetric state again, agent 2 pays back some part of the “credit” she got the previous period. The credit of period 2 is *forgotten* forever, what matters is only who was constrained last, thus we may say that the economy is displaying *amnesia*. Further, after a sufficient number of periods  $x$  only takes two different values,  $\underline{x}^{lh}$  and  $\bar{x}^{hl}$ , thus *the consumption allocation converges weakly to the same distribution, independently of the initial relative Pareto weight  $x_0$* .<sup>7</sup>

## 4 Consequences and Sources of Income Inequality

This section examines the interaction of income inequality and self-enforcing risk sharing contracts in the context of the model presented in section 3. To do this, three types of simulation exercises are performed. In all cases I assume that only two households populate the village economy, and that each household’s income may take only two values. Households are allowed to be heterogeneous in either (i) the characteristics of their income process, (ii) some predetermined wealth, the returns of which are fixed each period, or (iii) their risk preferences.

The first example illustrates the possible adverse consequences of inequality on the welfare of the poor, even if per-capita income increases in the economy. The second example looks at the effects of changes in the subsistence level on consumption smoothing, and shows, for example, that as the subsistence level increases, both the mean and

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<sup>7</sup>Note that, if perfect risk sharing is self-enforcing for some set of  $x$ , denote this interval  $[\underline{x}, \bar{x}]$ , then it does matter which  $x_0$  is chosen by the social planner. In particular, after a sufficient number of periods, with probability 1, the ratio of marginal utilities will be one of the following, in all periods and states:  $x_0$  if  $x_0 \in [\underline{x}, \bar{x}]$ ,  $\underline{x}$  if  $x_0 < \underline{x}$ , and  $\bar{x}$  if  $x_0 > \bar{x}$  (see Kocherlakota (1996)).

the volatility of the poor household's consumption process decrease. The third example is an attempt to look at the effects of informal insurance on income smoothing. In particular, I show that (i) the availability of informal insurance may improve efficiency, in the sense that expected income increases, (ii) lack of wealth and/or higher risk aversion may prevent the poor from adopting a riskier, higher yielding technology, and (iii) informal insurance may actually create income inequality.

All computations have been done using the software R ([www.r-project.org](http://www.r-project.org)).

## 4.1 Income inequality and welfare

This section examines the consequences of inequality on the welfare of the poor, given that only the income of the rich changes. This means that we do not look at changes in inequality in the usual sense, but rather, inequality increases together with aggregate, and per-capita, income. This exercise is interesting because we put ourselves in a disadvantageous environment to find any adverse consequences for welfare. In particular, I fix the income of the poor and give some additional income to the rich in each state of the world. Therefore, if the poor is worse off in terms of welfare as a result, it must somehow be due to the informal risk sharing arrangement.<sup>8</sup>

Suppose that there are two households, a poor and a rich one. Both households have standard constant-relative-risk-aversion (CRRA) preferences,

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma},$$

with identical coefficient of relative risk aversion ( $\sigma_1 = \sigma_2 \equiv \sigma$ ). Both households discount the future with discount factor  $\delta$ . Note that a higher  $\sigma$  increases the demand for insurance, while a higher  $\delta$  helps enforcement, thus allows more risk sharing (see Ligon, Thomas, and Worrall (2002)).

The two households differ in their income process. The poor household receives  $\underline{y} = 1.5$  or  $\bar{y} = 2.5$ , with equal probabilities, in each period. I perform a comparative statics exercise, changing the income process of the rich: starting from a situation close

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<sup>8</sup>Note that in autarky, the welfare of the poor does not change, while in the perfect risk sharing case, given  $x_0$ , the welfare of the poor increases as the income of the rich increases (provided that with the chosen  $x_0$ , the ex-ante participation constraints are still satisfied).

to equality, the rich getting  $\underline{y} = 2.5$  or  $\bar{y} = 3.5$ , with equal probabilities as well, to a situation where she is “a lot” richer, earning  $\underline{y} = 14$  or  $\bar{y} = 15$ , still with equal probabilities, and in each period. I take steps of 0.25, and all along I keep the riskiness of the income process constant, in the sense that its standard deviation stays the same.<sup>9</sup> My measure of inequality then is the ratio mean income of the rich over mean income of the poor. Note once again that in this way, inequality increases together with per-capita income. To specify preferences, let the discount factor  $\delta = 0.8$ , and the coefficient of relative risk aversion  $\sigma = 1.5$ .

I use the algorithm outlined in the appendix to find the solution of the model given the set of parameter values above<sup>10</sup>. The solution, that is, the constrained-efficient, informal contract, is given by a set of state-dependent intervals that tell us what ratios of marginal utilities are possible in each of the four states of the world. Once these optimal intervals have been computed, I allow the economy to run for 200 periods, that is, I generate a realization for the income state in each period, and let the contract tell us the consumption of the households. To calculate the lifetime utility of the poor, I take the last 100 periods.<sup>11</sup> Finally, to compute the expected welfare of the poor, I redo the above simulation 5000 times. Each time I take  $x_0 = 1$ , that is, the social planner would prefer an equal division of consumption and utilities in each period. The aim of this simulation is to pin down one point on the Pareto-frontier, the point that will be reached with probability 1 after a sufficient number of periods.

Figure 1 shows the expected lifetime utility of the poor as a function of inequality. Note that *welfare* is measured on an ordinal scale here, so only the slope is informative, the shape of the curve is not.

Figure 1 shows the main result of this subsection: it may happen that the welfare of the poor is decreasing with increasing inequality and per-capita income, even if her

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<sup>9</sup>We may also talk about the two households facing the same exogenous income process,  $\underline{y} = 1.5$  or  $\bar{y} = 2.5$ , with equal probabilities, in each period, but the rich household having some additional fixed revenue, that varies from 1 to 12.5.

<sup>10</sup>One also needs to choose the number of gridpoints, as the continuous variable  $x$  is discretized. Here I take a grid of 1200 intervals, considering the trade-off between precision and computation time.

<sup>11</sup>Note that 100 periods is sufficient for the economy to reach the stable distribution of consumption, regardless of the initial relative weight, with probability very close to one. Then, it is enough to take 100 periods to calculate lifetime utility, because  $\delta^{100} = 0.8^{100} = 2.037e^{-10}$ .

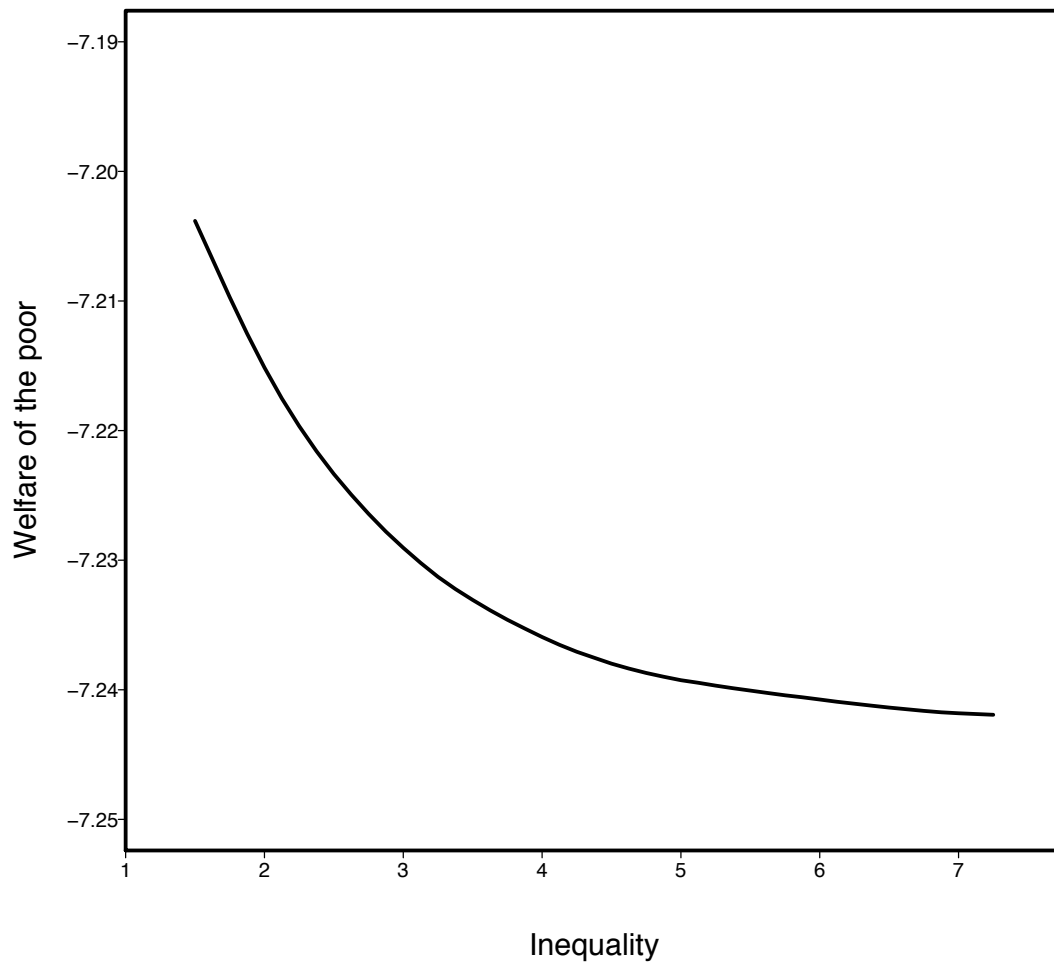


Figure 1: The welfare of the poor as a function of inequality. *Welfare* is expected lifetime utility at the limited commitment solution, supposing that the economy has run for a sufficient number of periods to reach the stationary distribution of consumption. *Inequality* is the ratio mean income of the poor over mean income of the rich, and it increases together with per-capita income. The income process of the poor is kept constant, and the standard deviation of the income process of the rich is kept constant. The curve has been smoothed to get rid off numerical error.

income does not change. The intuition behind this result is the following. As the rich gets richer, her outside option becomes more attractive, thus her decision power increases vis-a-vis that of the poor. A second point is that the rich behaves in a less risk-averse fashion, so the insurance the poor can provide becomes less valuable for the rich. These effects may outweigh the positive effects of higher per-capita income and the rich being able to bear more risk, thus the poor may be worse off.

To see what is happening to the self-enforcing level of insurance, it is useful to look at how the optimal intervals on  $x$ , the ratio of marginal utilities, change. Remember that, to achieve perfect risk, the intervals of all four states should overlap, while in autarky, each interval collapses to one point. Thus, roughly speaking, a wider interval means more insurance. Figure 2 shows the (natural logarithm of the) intervals in the four income states as a function of inequality. We see how the intervals shrink as inequality increases, meaning that there is less and less risk sharing between the two households.

One may further examine what is behind the welfare loss of the poor in terms of her consumption process. To do this, I compute the mean and the standard deviation of the consumption process, to see how they change with inequality. Panel (a) of Figure 3 shows the mean, while panel (b) shows the standard deviation of the poor household's consumption process, both as a function of inequality. We see that what causes the loss of welfare is higher volatility, which outweighs the positive effect of the increasing mean. Note also that both the mean and the standard deviation converge towards their autarkic values, 2 and 0.5, respectively.

Let us finally look at the constrained-efficient Pareto frontier for two levels of inequality. Figure 4 shows the Pareto frontiers for *inequality* = 1.5 (the rich earning  $\underline{y} = 2.5$  or  $\bar{y} = 3.5$ ) and *inequality* = 1.625 (the rich earning  $\underline{y} = 2.75$  or  $\bar{y} = 3.75$ ), together with the point on the Pareto frontier where the households end up with probability 1 after a sufficient number of periods. We see that the Pareto frontier moves outward when the rich gets richer, so a Pareto improvement is possible. However, the poor household's decision power decreases so much that the point selected on the Pareto frontier for higher inequality lies to the left of the point for lower inequality, meaning that the poor is worse off. The reason being that she can obtain less insurance.

To summarize, in the case of risk sharing with limited commitment, the poor may be more and more excluded from the informal insurance arrangement as the rich gets

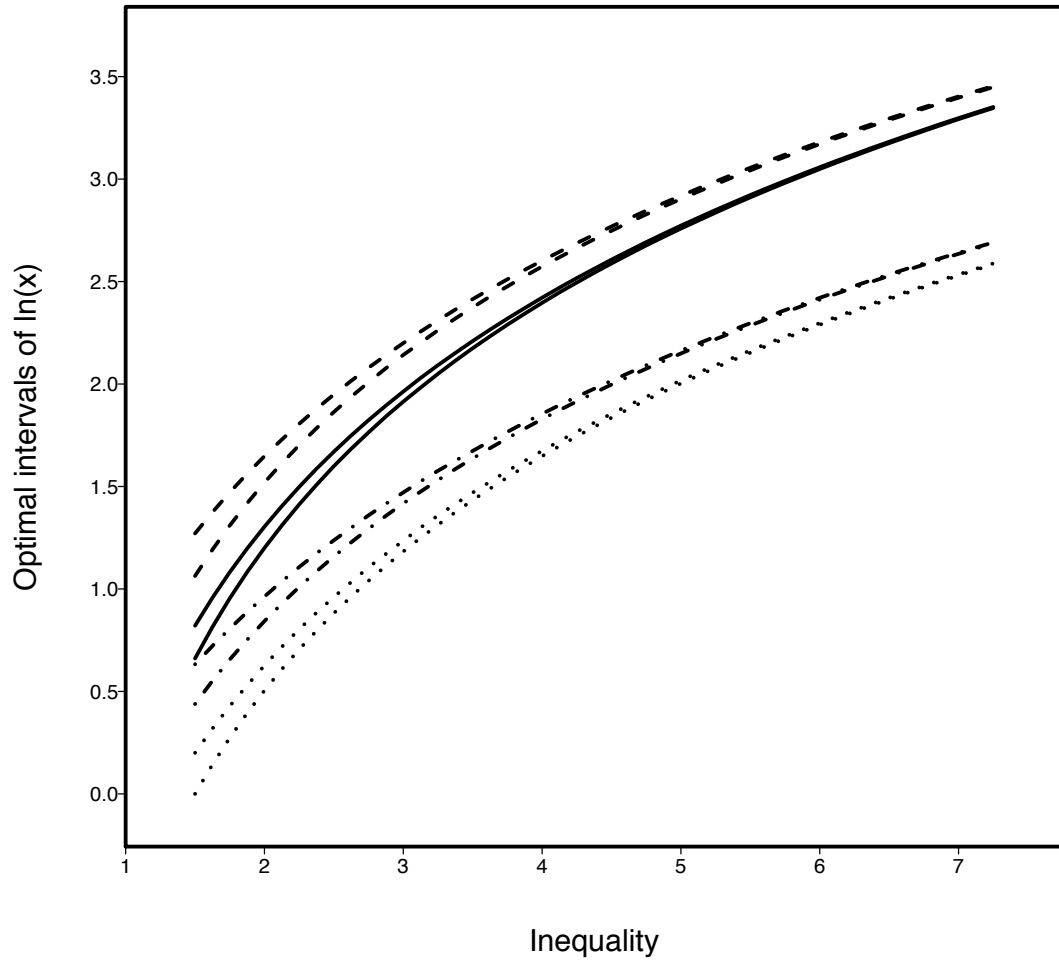


Figure 2: The optimal intervals of  $\ln(x)$ , the (natural logarithm of the) ratio of marginal utilities, as a function of inequality. The four income states are represented by the different line-types.

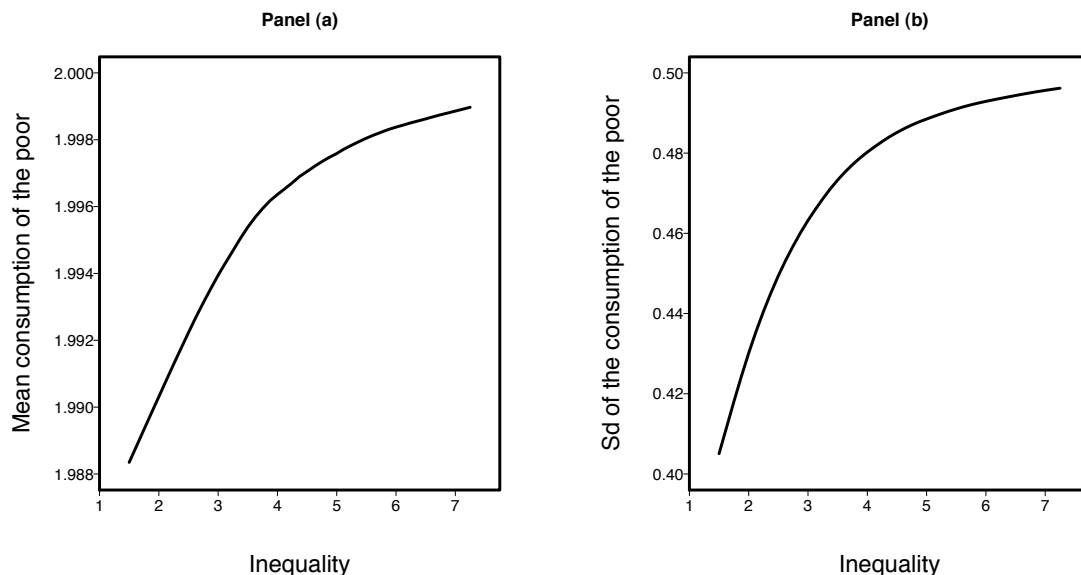


Figure 3: The mean and standard deviation of the consumption process of the poor as a function of inequality. The curves have been smoothed to get rid off numerical error.

richer. This loss of insurance may cause a decrease in welfare for the poor. This result warns of the possible adverse consequence of growth in per-capita income for the welfare of the poor, when the poor do not receive any of the additional income.

Empirical evidence on the exclusion of poor households of risk sharing networks includes Townsend (1994), who finds that landless households are less well insured in one of the three Indian villages in the study. Jalan and Ravallion (1999) reject perfect risk sharing most strongly for the poorest households in their sample from rural China, and estimate that 40% of income shocks the poor face are passed onto current consumption (while for the richest households, only 10%). Santos and Barrett (2006) find direct evidence that the poorest households are excluded from social networks in Ethiopia, in particular, they do not receive transfers in case of a negative income shock.

## 4.2 The subsistence level and consumption smoothing

This section performs another type of comparative statics exercise. In particular, keeping the income process of the two households fixed, I change the subsistence level,

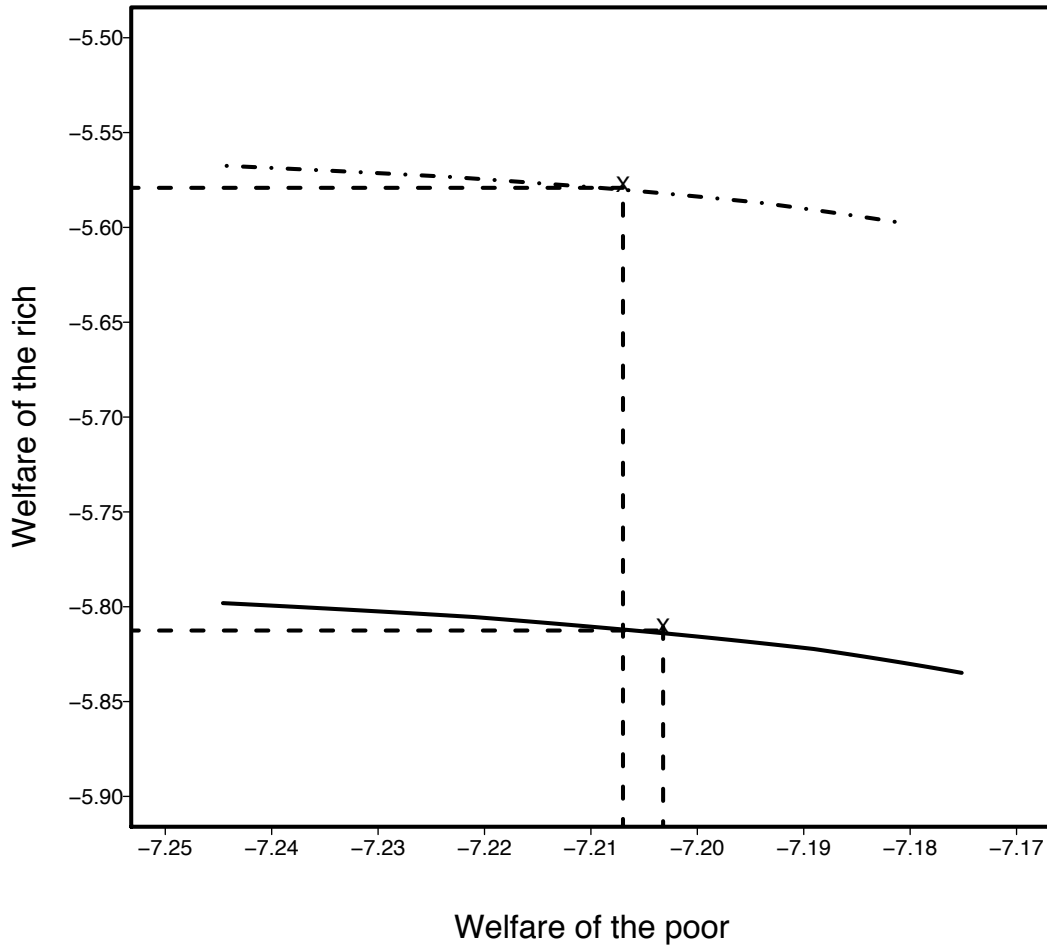


Figure 4: Pareto frontiers for two levels of inequality. The solid line is the Pareto frontier for *inequality* = 1.5 (the rich earning  $\underline{y} = 2.5$  or  $\bar{y} = 3.5$ ), the dot-dashed line is the Pareto frontier for *inequality* = 1.625 (the rich earning  $\underline{y} = 2.75$  or  $\bar{y} = 3.75$ ). The poor is getting  $\underline{y} = 1.5$  or  $\bar{y} = 2.5$  in both cases. The point on the Pareto frontier where the households end up with probability 1 after a sufficient number of periods is represented by X.

denoted  $subs$ . In this case, the utility function can be written as

$$u(c_{it}) = \frac{(c_{it} - subs)^{1-\sigma}}{1-\sigma}. \quad (19)$$

With  $subs > 0$  preferences are characterized by decreasing relative risk aversion (DRRA). Note that the utility function (19) implies that the coefficient of relative risk aversion is  $\sigma \left( \frac{c_{it}}{c_{it} - subs} \right)$ , which is decreasing in  $c_{it}$  for  $subs > 0$ . Empirical evidence on the relevance of a subsistence level, or decreasing relative risk aversion, in the case of perfect risk sharing is provided by Ogaki and Zhang (2001).

In the case of risk sharing with limited commitment, a first, natural result is that, when the subsistence level increases sufficiently, perfect risk sharing becomes self-enforcing. This is because insurance becomes more valuable for both households. The result follows from the fact that an increase in the subsistence level is equivalent to a decrease in some fixed revenue, or wealth. Thus, with a higher subsistence consumption, households behave in a more risk-averse fashion. At the limit, if in the worst state a household's income falls below the subsistence level, she becomes infinitely risk averse.

To take a closer look at the effect of changes in the subsistence level on consumption smoothing, consider two households once again, a poor and a rich one. The poor gets  $\underline{y} = 1.5$  or  $\bar{y} = 2.5$ , with equal probabilities, in each period, and the rich household earns  $\underline{y} = 6$  or  $\bar{y} = 10$ , with equal probabilities as well, in each period. The subsistence level changes between 0 and 1.2, and I take steps of 0.05. To specify preferences, take  $\sigma = 1.5$  and  $\delta = 0.85$ . Note that in this case, perfect risk sharing is self-enforcing for  $subs \geq 0.75$ .

Let us briefly consider welfare first. Note that preferences are changing as the subsistence level changes, further, the welfare of both households should decrease as subsistence consumption increases. This is what we see indeed. Figure 5 shows the expected lifetime utility of the poor (panel (a)), and the rich (panel (b)). The simulations are conducted as in section 4.1, except that now I keep the income processes constant, but increase subsistence consumption from 0 to 1.2.

It is more meaningful here to compare the properties of households' consumption processes, since incomes do not change. Figures 6 and 7 present the mean (panel (a)) and standard deviation (panel (b)) of the consumption process of the poor and the

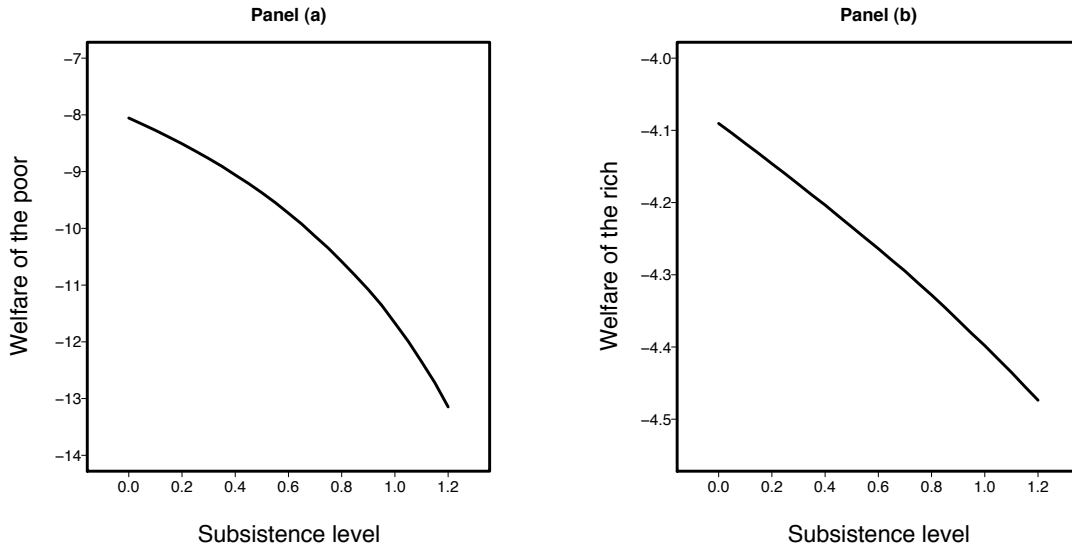


Figure 5: The welfare of the poor and the rich as a function of the subsistence level.

rich, respectively. We see reverse trends for the two households. Mean consumption and variance of consumption are both decreasing for the poor, while they both increase for the rich. The poor “buys” more insurance from the rich as she gets closer to the subsistence level, sacrificing mean consumption. As a result, the difference in expected consumption between the rich and the poor increases with the subsistence level.

When perfect risk sharing becomes self-enforcing, we see a kink in the line representing the standard deviation of consumption (see panel (b) of Figures 6 and 7). The poor household is getting relatively more insurance. As the insurance technology is better once perfect risk sharing is self-enforcing, aggregate risk can be shared efficiently, that is, the rich, less risk-averse household may bear more of the aggregate risk.

Finally, let us look at the possible consumption values for the poor in the limited commitment solution as a function of the subsistence level (see figure 8). First, the spread decreases as subsistence consumption increases. Second, when perfect risk sharing becomes self-enforcing, the possible consumption values are reduced to four, the number of income states, and there are changes in the trends. In the asymmetric states, the trends are reversed, and consumption values in all states start getting closer to the mean. Finally, note that even when perfect risk sharing is possible, consumption is not

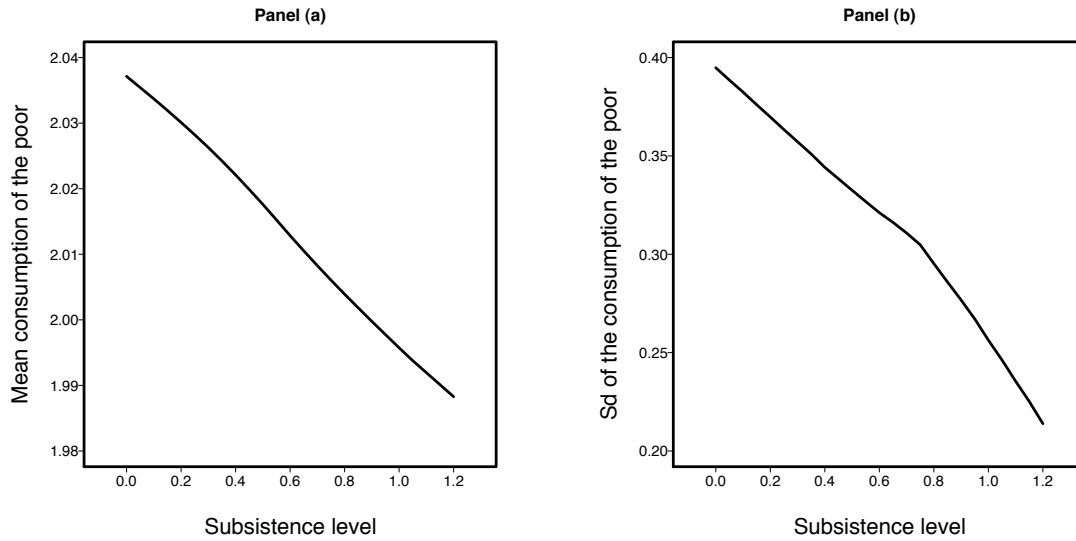


Figure 6: Mean and standard deviation of the consumption process of the poor as a function of the subsistence level. The curve representing the mean has been smoothed to get rid off numerical error.

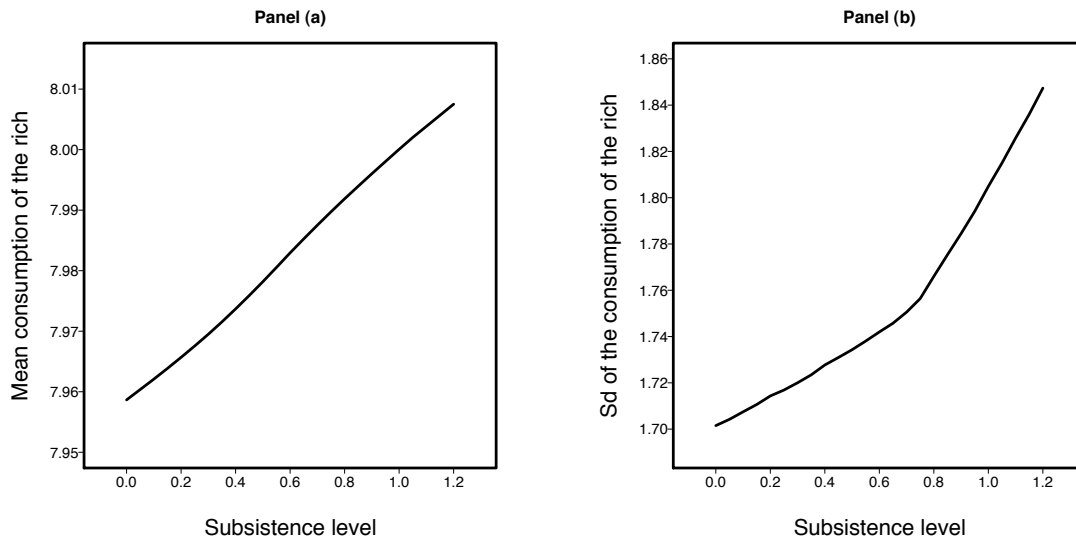


Figure 7: Mean and standard deviation of the consumption process of the rich as a function of the subsistence level. The curve representing the mean has been smoothed to get rid off numerical error.

constant, only the ratio of marginal utilities. This is because only idiosyncratic risk is insured perfectly, the households would need a third party to insure against aggregate risk.

### 4.3 Consumption insurance and income smoothing

By way of a third type of examples, this section examines how (i) the possibility to share risk, (ii) the availability of wealth that yields a fixed revenue each period, and (iii) heterogeneous risk preferences, may influence the choice of production technology. A production technology is described by the income process it generates. We will see that lack of insurance, and/or lack of wealth, or higher risk aversion may lead to more income smoothing, and thereby a loss in efficiency.

The importance of consumption smoothing possibilities in income decisions in low-income economies has been recognized by Morduch (1994, 1995). He convincingly argues that lack of credit and insurance not only affects the ability of households to smooth consumption given income, but also has important consequences for production decisions. Households have to choose safer income generating technologies in order to avoid big income fluctuations, with which they would be unable to deal. This might cause considerable efficiency losses. However, Morduch (1995) does not formalize these ideas, while Morduch (1994) considers lack of consumption credit.

Rosenzweig and Binswanger (1993) provide some empirical evidence that people with lack of consumption smoothing instruments have to sacrifice expected profits for less volatile income. They look at the effect of weather variation on the mean and variance of farm profits using data from Indian villages, and find that mean profits decrease with weather volatility for poorer households, but not for the rich. Kurosaki and Fafchamps (2002) find evidence that crop choices of households in Pakistan depend on price and yield risk. Even though efficient risk sharing among households of the same village cannot be rejected, aggregate shocks are not insured, and risk attitudes do affect production choices.

Modeling the case where households make production decisions taking into account that only informal insurance is available to smooth consumption, is thus an important problem. Here I aim to have some insights concerning the issue of income smoothing,

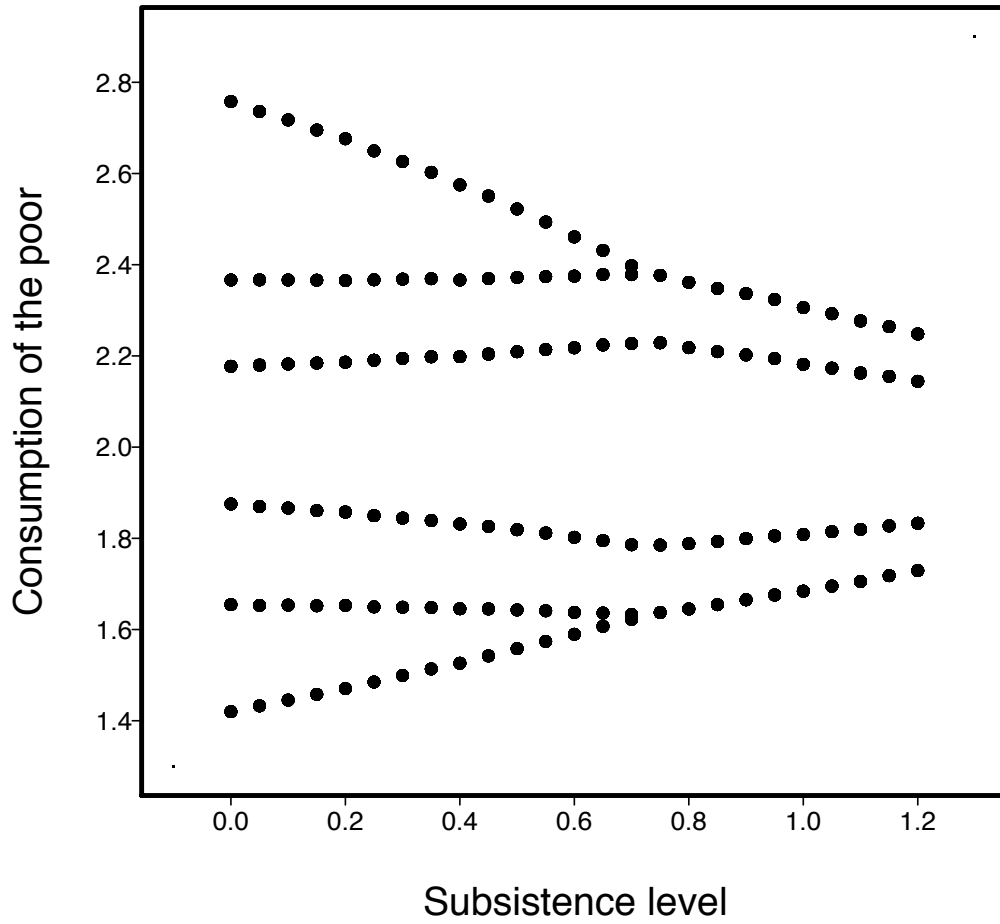


Figure 8: Possible consumption levels for the poor as a function of the subsistence level.

setting up a general model, but solving only a special case.

In general, adding technology choice complicates substantially the problem at hand, because households may switch between the technologies in any state of the world and any time period, whether they stay in the informal risk sharing contract or revert to autarky. In other words, the choice of production technology to be used next period depends on the state of the world today. Below I construct two related examples, where at the constrained-efficient solution, a household prefers to use the same technology in all states of the world. The only switching, which is costless for simplicity, may occur when the household leaves the risk sharing contract, and stays in autarky thereafter. Even without solving the general model with possible switching at any time and state, switching has to be allowed when the threatpoints are computed.

The timing is as follows. At time 0, each household chooses a technology. Note that each technology takes one period to yield some income (one may have agricultural production in mind, for example). At time 1, the state of the world, and incomes are realized first, according to the technology chosen at time 0. Then each household may decide to stay in the risk sharing arrangement, or deviate. In the first case, each household makes a payment to the other household as specified by the contract, consumption takes place, and finally, each household also decides which technology to use. In case one of the households deviates, no payments are made, each household consumes her income generated by the technology she chose one period before, and finally, each household chooses a technology, knowing that she will be in autarky in all future periods. At time 2 and thereafter, the same sequence of events follows as at time 1.

Let us now turn to the numerical examples. Suppose that two technologies are available in the economy, a safer one with lower expected income, which we call the “old” technology, and a riskier, “new” technology with higher profits in expectation. Both technologies yield an independently and identically distributed (iid) income process, with equal probabilities for each state. Once again, income of a household takes two values,  $y^l$  (low) or  $y^h$  (high), thus there are four income states. The old technology has the following payoffs:  $y^l = 1.4$  or  $y^h = 2.5$ . The new technology yields  $y^l = 1.2$  or  $y^h = 2.9$ , in each period. Households discount the future at the rate  $\delta = 0.95$ , and they both have a utility function of the CRRA form.

Table 1: The welfare of the poor (no wealth) in autarky

income \ technology	<b>old</b>	new
low	<b>-29.765</b>	-30.327
high	<b>-29.340</b>	-29.676

Table 2: The welfare of the rich (some wealth) in autarky

income \ technology	old	<b>new</b>
low	-20.346	<b>-20.323</b>
high	-20.204	<b>-20.108</b>

Now, let us look at two examples with different kind of heterogeneity among households. In example 1, the rich household has some exogenous wealth that yields a fixed income every period, which is in addition to the stochastic income process from production, while the poor has no wealth. In example 2, households differ in their coefficient of relative risk aversion. I will call the less risk averse household the rich, and the more risk averse the poor, abusing terminology.

*Example 1.* Suppose that both households' coefficient of relative risk aversion  $\sigma = 1.5$ . The poor household has no wealth, while the rich household possesses some assets that yield a sure revenue  $w = 2$  each period. So the poor household's income is  $y(s_t)$ , and the rich has  $w + y(s_t)$ . The social planner's objective is

$$\max_{\{c_1(s_t), c_2(s_t)\}} \sum_{t=0}^{\infty} \sum_{s_t} 0.95^t \pi(s_t) \left( \frac{c_1(s_t)^{-0.5}}{-0.5} + x_0 \frac{c_2(s_t)^{-0.5}}{-0.5} \right), \quad (20)$$

with  $\pi(s_t) = 0.25, \forall s_t$ , subject to resource and enforceability constraints. I set  $x_0 = 3$  in the social planner's objective (20).

With these parameter values, in autarky the poor prefers to use the old technology, while the rich chooses the new technology. Tables 1 and 2 show the autarky values, or lifetime utilities, discounted to time 1, that the poor and the rich get, respectively. In these tables the higher values by row, and the resulting technology choice is marked bold.

The values for the old technology are indeed higher for the poor household, and the lifetime utility the new technology gives is higher for the rich, for both low and high

Table 3: The welfare of the poor (no wealth) with informal insurance

state\technology	( <b>new</b> ,new)	(old,new)
(low,low)	<b>-29.093</b>	-29.293
(low,high)	<b>-28.840</b>	-29.081
(high,low)	<b>-28.840</b>	-29.126
(high,high)	<b>-28.676</b>	-28.946

Table 4: The welfare of the rich (some wealth) with informal insurance

state\technology	(new, <b>new</b> )	(new,old)
(low,low)	<b>-20.172</b>	-20.178
(low,high)	<b>-19.997</b>	-20.046
(high,low)	<b>-19.997</b>	-20.022
(high,high)	<b>-19.883</b>	-19.938

income today. From these values the threatpoints can be computed. If a household chose the technology optimal in autarky yesterday, just take the values from tables 1 and 2. When the household chose the other technology before, still participating in the risk sharing arrangement, but deviates to autarky today, she consumes the income realization from the other technology today, while tomorrow she receives the optimal autarky values above.

Now we can look at the limited commitment solution, using these threatpoints. I find a simple subgame perfect equilibrium (SPE) of this infinite game, supported by reversion to autarky, where both households choose the new technology in all periods and states. I compare the payoffs of a given technology choice, described by (technology choice of the poor, technology choice of the rich) with the payoffs of a one-sided deviation. Table 3 shows the lifetime utilities for the poor at the limited commitment solution, in the four income states, described by (income of the poor, income of the rich). Similarly, table 4 shows the values for the rich.

Table 3 tells us that (new,new) is preferred by the poor to (old,new). That is, the poor chooses the new technology, given that the rich does so as well, and given the possibility of an informal risk sharing arrangement between the two households.

Table 5: The welfare of the poor (high risk-aversion) in autarky

income\technology	<b>old</b>	new
low	<b>-5.828</b>	-6.608
high	<b>-5.594</b>	-6.235

Table 6: The welfare of the rich (low risk-aversion) in autarky

income\technology	<b>old</b>	new
low	<b>-55.695</b>	-56.145
high	<b>-55.214</b>	-55.411

Table 4 shows that the rich would still rather use the new technology. This example shows that (i) the availability of insurance to smooth consumption may indeed affect the choice of production technology, and (ii) the poor household may forego higher expected income to avoid facing more risk, because of lack of wealth coupled with lack of insurance. Without insurance, wealth inequality causes inequality in expected incomes, or, in other words, the availability of the new technology reinforces inequality, since it allows the wealthy to have higher expected income from production than the poor.

*Example 2.* Now, neither household has any wealth, but the poor household is more risk averse than the rich. The poor household's coefficient of relative risk aversion  $\sigma_1 = 2.5$ , while for the rich  $\sigma_2 = 1.3$ . The social planner's objective is

$$\max_{\{c_1(s_t), c_2(s_t)\}} \sum_{t=0}^{\infty} \sum_{s_t} 0.95^t \pi(s_t) \left( \frac{c_1(s_t)^{-1.5}}{-1.5} + x_0 \frac{c_2(s_t)^{-0.3}}{-0.3} \right), \quad (21)$$

with  $\pi(s_t) = 0.25$  for all  $s_t$ , as before, subject to resource and enforceability constraints. I now set  $x_0 = 0.5$  in the social planner's objective (21).

With these parameter values, in autarky both households prefer the old technology. The autarky values for the poor household are shown in table 5, and for the rich in table 6.

The values for the old technology are indeed higher for both households for both low and high income. From these values we can calculate the threatpoints, similarly as for example 1. If a household chose the old technology in the previous period, just take

Table 7: The welfare of the poor (high risk-aversion) with informal insurance

state \ technology	(old,new)	(new,new)
(low,low)	<b>-5.497</b>	-5.564
(low,high)	<b>-5.339</b>	-5.382
(high,low)	<b>-5.377</b>	-5.382
(high,high)	<b>-5.292</b>	-5.311

Table 8: The welfare of the rich (low risk-aversion) with informal insurance

state \ technology	(old,new)	(old,old)
(low,low)	<b>-55.116</b>	-55.122
(low,high)	<b>-54.540</b>	-54.741
(high,low)	<b>-54.697</b>	-54.741
(high,high)	<b>-54.322</b>	-54.493

the values for the old technology from the tables. When the household chose the new technology before, still participating in the risk sharing arrangement, but deviates to autarky today, she gets the payoff from the new technology today, while tomorrow she receives the old technology values above.

These threatpoints are used to find the constrained-efficient solution. Given the constrained-efficient informal risk sharing contract, the poor household chooses the old technology in all periods and states, while the rich produces using the new technology. Once again, I compare the payoffs of a given technology choice, described by (technology choice of the poor, technology choice of the rich) with the payoffs of a one-sided deviation. Tables 7 and 8 show the lifetime utility for the poor and the rich, respectively, in the four income states, described by (income of the poor, income of the rich), allowing households to enter into an informal risk sharing arrangement.

We see that the poor would rather use the old technology given that the rich uses the new, and that (old,new) is preferred by the rich to (old,old). This second example shows as well that the availability of insurance to smooth consumption may indeed affect the choice of production technology. It also demonstrates that higher risk aversion may cause more income smoothing, thus lower expected incomes. Further, we see a

perverse affect of informal insurance in that it actually causes inequality in expected incomes. Notice that in autarky both households choose the same technology, and informal insurance allows the less risk averse household to become the “rich”, that is, to have higher expected income. In terms of welfare, however, both households are better off if they share risk. This is trivially true, since, by definition, in the constrained-efficient solution both households must be at least as well off as in autarky, and they are strictly better off if some transfers occur in any state, which is the case here.

As mentioned already, the general solution of the model of risk sharing with limited commitment and technology choice is an interesting and difficult task for future research. The difficulty comes from the fact that, in any period and any state of the world, a household may decide to switch between the available technologies, based on the expected lifetime utility they provide, given that the risk sharing contract is constrained-efficient. But the constrained-efficient transfers depend also on future technology choices. So we have to find the decision on technologies and the informal insurance contract simultaneously.

## 5 Concluding remarks

Empirical evidence from low income rural communities suggests the existence of informal insurance arrangements that achieve partial insurance. This paper has shown a way to model the observed partial insurance. In particular, risk sharing contracts were required to be self-enforcing. The numerical techniques developed to solve the model allow one to compute the allocation in parametrized economies. In this paper I have used these techniques to examine some issues related to income inequality. The importance of the possible effects shown by the examples, coming from the interaction of inequality and informal insurance contracts, is an empirical question.

However, the results warn of the possible adverse consequences of inequality on the welfare of the poor, even if an increase in inequality only means that the rich get richer. Further, we have seen that inequality may be reinforced without insurance, when the wealthy choose a more profitable technology, while the poor prefer the less risky, less efficient technology.

One direction for future theoretical work is to develop the model with more than

one technologies. A first attempt is made in section 4.3 here. This extension would be a very important step, since in low-income economies, production and consumption decisions are often intertwined, because of incomplete markets. It is not optimal for risk-averse households to maximize their expected income, when financial instruments or insurance contracts are not available to smooth consumption inter-temporally or across states of nature.

The framework and methods discussed in this paper may also be useful for examining the impact of some policy intervention in future work, for example a micro-insurance program, taking into account existing informal arrangements to share risk.

## Appendix - Computation

The aim is to solve for the decision variables, that is, consumption  $c_i(s_t, x_{t-1})$ ,  $\forall i$ , and the relative weight of household 2  $x_t(s_t, x_{t-1})$ <sup>12</sup>, and for the lifetime utility household  $i$  gets from her consumption stream being in the informal risk sharing arrangement  $V_i(s_t, x_{t-1})$ ,  $\forall i$ , given the state of the world today  $(s_t, x_{t-1})$ .

Define a grid over the continuous variable  $x$  for each value of  $s_t$ . Denote  $X$  the set of gridpoints (I define the same points for all  $s_t$ ). Guess a solution for the value functions, that is, guess  $V_i^0(s_t, x_{t-1})$ ,  $\forall i$  and each gridpoint. Unfortunately, the algorithm does not converge from any initial guess for the value functions, but the value of the perfect risk sharing case will do.<sup>13</sup>

Now proceed to update the guess. Suppose we are at the  $n^{\text{th}}$  iteration. Let us look at gridpoint  $(\tilde{s}_t, \tilde{x}_{t-1})$ . Three cases have to be distinguished: (a) neither enforcement constraint binds, (b) the enforcement constraint for household 1 binds, or (c) the enforcement constraint for household 2 binds (the two constraints cannot bind at the same time, since only one of the two households has to make a transfer, and obviously the resource constraint always binds). We first suppose that neither enforcement constraint binds, that is, we try to keep  $x$  constant, then we see if we can do that or not.

(a) *Neither enforcement constraint binds.* This is the easy case, since  $x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1}$ . So we only have to find  $c_1(\tilde{s}_t, \tilde{x}_{t-1})$  and  $c_2(\tilde{s}_t, \tilde{x}_{t-1})$ , and we have two conditions:  $u'(c_1(\tilde{s}_t, \tilde{x}_{t-1}))/u'(c_2(\tilde{s}_t, \tilde{x}_{t-1})) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1}$  and the resource constraint  $c_1(\tilde{s}_t, \tilde{x}_{t-1}) + c_2(\tilde{s}_t, \tilde{x}_{t-1}) = y_1(\tilde{s}_t) + y_2(\tilde{s}_t)$ . Replacing for  $c_2(\tilde{s}_t, \tilde{x}_{t-1})$  from the resource constraint we have

$$u'(c_1(\tilde{s}_t, \tilde{x}_{t-1}))/u'(y_1(\tilde{s}_t) + y_2(\tilde{s}_t) - c_1(\tilde{s}_t, \tilde{x}_{t-1})) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1}.$$

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<sup>12</sup>Here I outline the algorithm for two households. There is no difficulty in extending the algorithm to  $n$  households theoretically. The state space has to include the vector of relative weights of length  $n-1$ . However, in terms of computation time we face the curse of dimensionality, and the computation time for  $n$  large, while obtaining the allocation and utilities with acceptable precision, is prohibitive.

<sup>13</sup>Characterizing the convergence properties of the algorithm is left for future research. However, we know that the algorithm does not converge to the constrained-efficient solution from any initial guess for the value functions. For example, if we set the initial guess equal to the autarkic values, every iteration yields these same autarkic values. This is natural, since autarky is also a subgame perfect equilibrium (SPE).

Supposing logarithmic utility we can easily find closed form solutions. In sum, with  $u(c_i) = \log(c_i)$ , we have the following updated policy functions:

$$(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1}$$

$$c_1(\tilde{s}_t, \tilde{x}_{t-1}) = \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}$$

$$c_2(\tilde{s}_t, \tilde{x}_{t-1}) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}.$$

The next step is to check whether either of the enforcement constraints is violated. We can do this by verifying the weak inequality

$$u(c_i(\tilde{s}_t, \tilde{x}_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_i^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})) \geq U_i^{aut}(\tilde{s}_t). \quad (22)$$

Notice that we use  $V_i^{n-1}()$ . If (22) is satisfied  $\forall i$ , we set  $V_i^n(\tilde{s}_t, \tilde{x}_{t-1})$  equal to the left hand side of (22), and we are done with gridpoint  $(\tilde{s}_t, \tilde{x}_{t-1})$ . If it is violated for household 1, we have to proceed to (b). If (22) is violated for household 2, we proceed to (c).

(b) *The enforcement constraint for household 1 binds.* Now we want to find  $c_1(\tilde{s}_t, \tilde{x}_{t-1})$ ,  $c_2(\tilde{s}_t, \tilde{x}_{t-1})$ , and  $x_t(\tilde{s}_t, \tilde{x}_{t-1})$ , and we have three conditions:  $u'(c_1(\tilde{s}_t, \tilde{x}_{t-1})) / u'(c_2(\tilde{s}_t, \tilde{x}_{t-1})) = x_t(\tilde{s}_t, \tilde{x}_{t-1})$ , the resource constraint  $c_1(\tilde{s}_t, \tilde{x}_{t-1}) + c_2(\tilde{s}_t, \tilde{x}_{t-1}) = y_1(\tilde{s}_t) + y_2(\tilde{s}_t)$ , and we know that household 1's enforcement constraint is satisfied with equality, that is,  $u(c_1(\tilde{s}_t, \tilde{x}_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_1^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})) = U_1^{aut}(\tilde{s}_t)$ . In practice, since  $x$  is discretized, in general this equality will only be satisfied approximatively.

Let us look at the case  $u(c_i) = \log(c_i)$  once again. Now we can write an equation with only one unknown,  $x_t(\tilde{s}_t, \tilde{x}_{t-1})$ :

$$\log\left(\frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}\right) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_1^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})) = U_1^{aut}(\tilde{s}_t). \quad (23)$$

Once again we use  $V_1^{n-1}()$ , but we evaluate it at the gridpoints  $(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1}))$ , where the economy may end up next period. Since  $x$  is discrete, one cannot use standard

techniques to find  $x_t(\tilde{s}_t, \tilde{x}_{t-1})$ . Instead, we look for  $x_t(\tilde{s}_t, \tilde{x}_{t-1}) \in X$  such that the left hand side of (23) is as close as possible to  $U_1^{aut}(\tilde{s}_t)$ , provided that it is weakly greater. Once we have  $x_t(\tilde{s}_t, \tilde{x}_{t-1})$ , we can easily obtain the rest of the policies. In sum, for logarithmic utility, we have the policy updates

$$x_t(\tilde{s}_t, \tilde{x}_{t-1}) = (\text{the solution of (23)})$$

$$c_1(\tilde{s}_t, \tilde{x}_{t-1}) = \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}$$

$$c_2(\tilde{s}_t, \tilde{x}_{t-1}) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}.$$

Finally, we can compute

$$V_1^n(\tilde{s}_t, \tilde{x}_{t-1}) = \log(c_1(\tilde{s}_t, \tilde{x}_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_1^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})),$$

or the left hand side of (23), and

$$V_2^n(\tilde{s}_t, \tilde{x}_{t-1}) = \log(c_2(\tilde{s}_t, \tilde{x}_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_2^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})).$$

Notice once again that we use  $V_i^{n-1}()$  on the right hand side.

(c) *The enforcement constraint for household 2 binds.* We proceed symmetrically as in (b).

Now we are done with gridpoint  $(\tilde{s}_t, \tilde{x}_{t-1})$ . We have to do the above steps at all other gridpoints as well. Then the  $n^{\text{th}}$  iteration is complete. We continue iterating until the policy and value functions converge given some convergence criterion. For example we stop iterating when  $|V_i^n(\tilde{s}_t, \tilde{x}_{t-1}) - V_i^{n-1}(\tilde{s}_t, \tilde{x}_{t-1})| < \epsilon, \forall i$ , for some small  $\epsilon$ .

To obtain actual numbers for the consumption allocation and the value functions, we have to specify the utility functions for the two households, their discount factor, the initial relative weight in the social planner's objective, as well as the income processes. Using appropriate household survey data, all these can be estimated<sup>14</sup>, which allows structural testing of the model.

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<sup>14</sup>Except for the initial relative weight of household 2 in the social planner's objective. But remember that the distribution of the consumption allocation is independent of the initial relative weight with probability 1, given that we are in the case of partial insurance, and the economy has been running for a sufficient number of periods.

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