The more we know, the less we agree: Higher-order expectations and public announcements

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Abstract

Polarization of opinions after public announcement is widely observed, but often considered to be inconsistent with Bayesian learning. I show that this is not the case in environments where higher-order expectations play a role. I characterize informational structures where public announcement leads to polarization in all higher-order expectations, but not in first-order expectations. To illustrate the economic consequences, I modify two workhorse models of asymmetric information. I show in a dynamic Grossman-Stiglitz(1980) model that hectic trading around public announcements is consistent with common priors and Bayesian learning. I show in a Morris-Shin(1998) model that more disclosure of public information might increase the chance of successful currency attacks.

1 Introduction

It is widely observed that new public information frequently polarize decision makers' opinions about a particular issue instead of decreasing their disagreement. This phenomenon was highlighted in political economy in the context of voters reaction to news, in finance in the context of increasing trading volume around public announcements, and in behavioral economics in relation to experiments where subjects are asked to evaluate new evidence about socially sensitive issues. As the most standard Gaussian information structures with Bayesian learning are inconsistent with polarization, economists proposed various ways to depart from the standard framework including cognitive biases, bimodal preferences and heterogenous priors. In this paper, I focus on environments where decision makers worry about the expectation of other agents. I show that in this context the Gaussian-Bayesian framework with common priors is consistent with polarization. To illustrate the economic relevance of this observation, I modify two standard workhorse models of informational economics. First, I show that public announcement can significantly increase trading volume in a Grossman-Stiglitz-type asset market. Second, I show that in a Morris-Shin-type speculative attack model the central bank's commitment to release new information can increase the ex ante probability of speculative attacks.

The main result of this paper is to characterize those informational environments under which higher-order expectations (expectations about the expectation of others) are polarized by public announcement in a Gaussian-Bayesian common prior environment. Importantly, the same environments are inconsistent with polarization in first-order expectations (expectations about a particular issue). In particular, I consider N pairs of agents. Each agent observes private and public signals about the value of a fundamental. Agents of the first pair form expectations about the fundamental value. Agents of the n-th pair, $n = 2, 3..N$, form expectations about the expectation of one of the agents of pair $n - 1$. All signals and the fundamental value are jointly normal and agents share the same prior. I consider the effect of a new public signal on the distance between the expectations of agents of the same pair n, that is, the possibility of polarization in expectations of order n. It is a well-known result that the disagreement in the first pair will decrease after the announcement under general conditions. In contrast, disagreement in any of the n-th pair, $n > 1$, will increase due the public announcement if the pairwise correlation between the private signals of agents of different pairs is sufficiently low and the new public signal is not independent of the private signals.

It is easy to see the mathematical intuition behind the result by considering the extreme case when private signals are independent across pairs of agents. Before the public announcement, this implies no disagreement between agents in any of the pairs n, $n > 1$. The reason is that the only difference between agents of the same pair is their private signal. If the private signals of pair n do not reveal any information about the private signals of pair $n - 1$, then agents of pair n will agree in their expectation of expectation of agents of pair $n - 1$. However, there will be disagreement after the public announcement. As the public signal is correlated to any private signal, private signals across pairs cease to be independent after the observation of the public signal. Intuitively, the public announcement connects the private information sets of agents of different pairs, thus it gives relevance to the private signals in the judgement of other private signals.

To illustrate that the result has economically important consequences, I modify two standard models with learning. First, I consider a Grossman-Stiglitz model of asset pricing under asymmetric information. Similarly to Allen et al. (2006), I use a version with overlapping groups of traders. There are early traders and late-comers and each trader receives a private signal. In the first and second periods early traders trade with each other. Between these periods there might be a public announcement. In the third period late-comers arrive, buy all the assets of early traders and trade among each other. At the end of the third period late-comers liquidate their assets for the fundamental value. This structure implies that the decision of early traders depends on their expectation of the liquidation price in the third period, which, in turn, depends on the expectation of late comers. Thus, second-order expectations matter.

An implication of the main result of the paper is that if the correlation between the private signals of early traders and late traders is sufficiently low, then the public announcement will increase trading volume. This is a success given the difficulty of Grossman-Stiglitz-type of models to explain the well established stylized fact of large trading volume around announcements. The inclusion of second-order expectations helps because in a Grossman-Stiglitz set-up a public announcement affects traders positions through two opposite forces. First, a public announcement increases the precision of the estimation of the trader which would lead to more aggressive trading. In the same time, disagreement among traders in first-order expectations decrease which lead to less aggressive trading. In the original Grossman-Stiglitz (1980) model these two forces exactly cancel out and public announcements do not have any effect on trading volume. However, in my version, early traders are interested in the third-period price which depends on the expectation of late-comers. Thus, the public announcement can increase the disagreement among early traders. In this case, the two forces go to the same direction leading to large trading volume around announcements.

As a second application, I modify the speculative currency attack model of Morris and Shin (1998). As in the original version, speculators receive private signals about the state of the economy. Based on their information, they decide whether to attack the fixed exchange rate system of the central bank. If the size of the attack is sufficiently high compared to the state of the economy, the central bank abandons the peg. As attacking is costly for speculators, they profit from an attack only if the central bank abandons the peg. The main difference between the modified version and the original one is that I do not allow the central bank to observe the state of the economy. Instead, central bank receives a signal about the state of the economy and abandons the peg if its posterior is low relative to the size of the attack. Thus, speculators have to second guess the expectation of the central bank. That is, speculators' second-order expectations matter. I ask the question whether more public information increases or decreases the ex ante probability of successful speculative attacks.

The second implication of the main result of the paper is that the correlation between the private signals of speculators and the private signal of the central bank is sufficiently low, more public information will polarize speculators' posterior estimation of the probability of devaluation. As speculators with the most extreme opinion attack, polarization might lead to more speculative attacks and, eventually, larger ex ante chance of devaluation. I show that this is always the case, if the public announcement is relatively imprecise. The result illustrates the interaction of two different notions of the transparency of the central bank. Svennson and Faust(2001) defines transparency as the correlation between the private signals of private agents and the private signal of the central bank. In contrast, it is also common to describe a transparent regime in terms of disclosure of more public information. The result argues that if the central bank is not transparent in the first sense, more transparency in the second sense might increase the probability of successful currency attack. To the extent that higher probability of a currency crisis decreases welfare, this result sheds more light on the debate whether transparency is always beneficial or not.

[Literature review to be completed]

The structure of this paper is as follows. In the next section I connect disagreement increasing public announcements to higher-order expectations. In section 3, I present the first application... In section 4. Finally I conclude.

2 Increasing disagreement in higher-order expectations

In this section, I analyze the effect of public announcements to expectations of different order in a general Bayesian-Gaussian model of learning. This is a purely statistical model where the only action of agents is to form expectations and their only objective is to make these expectations as accurate as possible. I leave the economic intuition to the rest of the paper where I will build up two separate economic models to analyze the implications of the statistical observations we make in this section.

Let us consider a continuum of agents divided in T groups. In each group there is a unit mass of agents. There is a fundamental variable of interest, θ. Each agent in any of the groups observes the same realization of M public signals y_m $m = 1, .., M$ on θ. Agent i in group t observes a realization of N_t group specific private signals $x_{i,t}^n$ $n = 1, ..., N_t$..Let us denote the information set of agent i in group t by $S_{i,t}$. All signals and θ are jointly normally distributed, all signals are positively correlated to θ, and the pairwise correlation between any two signals is non-negative.

I am interested in the effect of an additional public signal y on the dispersion of expectations of different order. For this purpose, I require agents in group 1 to form expectation on θ,

$$\begin{aligned} \hat{f}_1 &\equiv E(\theta|S_{i,t}) \\ f_1 &\equiv E(\theta|S_{i,t}, y) \end{aligned}$$

before and after the observing the public announcement, respectively. Throughout the paper, whenever it is important to distinguish between variables with and without announcement, I denote variables corresponding to the case of no announcement with a hat. f_1 and $\hat{f}_1$ are first-order expectations. I require agents in group $t > 1$ to form expectations on the expectation of an agent in group $t-1$,

$$\begin{aligned} \hat{f}_t &\equiv E\left(\hat{f}_{t-1}|S_{i,t}\right) \\ f_t &\equiv E(f_{t-1}|S_{i,t}, y). \end{aligned}$$

Thus, an agent in group t forms an expectation of order t.

I say that for the given information structure public announcement increases disagreement or causes polarization in the t-th expectation, if

$$Var\left(\hat{f}_t\right) < Var(f_t|y),$$

that is, if the dispersion of forecasts in group t increases after the announcement.

It is a well known result that under weak conditions, there cannot be polarization in first-order expectations in a Gaussian information structure. It is a general property of distributions with the monotone likelihood ratio introduced by Milgrom (1980).[1] For completeness, in the next proposition I provide a version of the proof for the informational structure used in this paper.

[1] See, for example, Dixit and Weibull (2007) for the argument.

Proposition 1 (Milgrom) *Let*

$$\frac{\partial \hat{f}_1}{\partial x_{i,1}^n}, \frac{\partial f_1}{\partial x_{i,1}^n}, \frac{\partial f_1}{\partial y} > 0 \quad (1)$$

$$\frac{\partial E(y|S_{i,1})}{\partial x_{i,1}^n} \geq 0 \quad (2)$$

for all n, then the public signal causes polarization in the first order of expectations.

Proof. Using the Projection Theorem[2] gives

$$f_1 = E(\theta|S_{i,1}, y_{m+1}) = E(\theta|S_{i,1}) + \frac{Cov(y, \theta|S_{i,1})}{Var(y|S_{i,1})}(y - E(y|S_{i,1}))$$

which, together with the assumptions of the proposition, implies

$$\frac{\partial f_1}{\partial x_{i,1}^n} = \frac{\partial E(\theta|S_{i,1}, y_{m+1})}{\partial x_{i,1}^n} = \frac{\partial E(\theta|S)}{\partial x_{i,1}^n} - \frac{\partial f_1}{\partial y}\frac{\partial E(y|S_{i,1})}{\partial x_{i,1}^n} < \frac{\partial E(\theta|S_{i,1})}{\partial x_{i,1}^n} = \frac{\partial \hat{f}_1}{\partial x_{i,1}^n}$$

for all n. Thus, the linearity of expectations gives the result. ■

Conditions in Proposition 1 are intuitive. Conditions in (1) require that private and public signals remain positively related to the fundamental even conditonally on the rest of the information structure. This is an almost trivial consequence of the concept of a signal. Condition (2) requires that private signals are not inversely related to the public signal. This automatically holds if the information set of agents contain a private signal only ($N_1 = 1, M = 0$) or if the public has the usual form of $y = \theta + \varepsilon$,where ε is an independent noise term. Even if $S_{i,1}$ contains many private and public signals, condition (2) is a natural assumption in most economic environments.

In the remaining of this section I show that polarization in higher-order expectations is consistent with a Gaussian information structure. Intuitively, polarization in higher-order expectations differs from polarization in first-order expectations, because of the critical role of the strength of connection between private signals of agents in different groups. The forecasts of agents in group t vary only to the extent that their forecast on the private signals of an agent in

[2] The Projection Theorem states that if $\mathbf{v}_\theta$ and $\mathbf{v}_s$ are vectors of variables which are jointly normally distributed with the vector of expected values $\boldsymbol{\mu}_\theta, \boldsymbol{\mu}_s$, respectively and covariance matrix

$$\begin{bmatrix} \Sigma_\theta & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_s \end{bmatrix},$$

then

$$\mathbf{v}_\theta|\mathbf{v}_s\tilde{}N\left(\boldsymbol{\mu}_\theta + \Sigma_{\theta,s}\Sigma_s^{-1}(\mathbf{v}_s - \boldsymbol{\mu}_s), \Sigma_\theta - \Sigma_{\theta,s}\Sigma_s^{-1}\Sigma_{s,\theta}\right).$$

group $t-1$, $x_{t-1,n}$ is sensitive to their private signals $x_{t,n}$.The next proposition shows that if the connection between private signals across groups is relatively weak, the public announcement will increase the dispersion of forecasts in all groups. The first part of the proposition gives a sufficient condition for the general case, while the second part gives a necessary and sufficient condition for the case where the information set of each agent contains a single private signal of the same quality.

Proposition 2 *Suppose that the correlation between y and any of the private signals is positive and suppose that condition in (1) hold.*

1. *Let $\sigma_{x,x'}$ the maximum of the covariances between a private signal of an agent in group $t-1$ and any of the signals of the information set of an agent in group t for all $t>1$.*

$$\sigma_{x,x'} = \max_{t\in(1,..T)} \max_{\substack{n\in(1,..,N_{t-1}) \\ v\in S_{i,t}}} cov\left(x_{i,t-1}^{n}, v\right).$$

   Then there is a $T_{\sigma_{x,x'}}>0$ threshold, that for any $\sigma_{x,x'} < T_{\sigma_{x,x'}}$, there is polarization in expectations of any order of $t>1$.

2. *Suppose that $M=0$ and $N_t=1$ for all t, and the covariance structure of signals is symmetric in the sense that*

$$\begin{aligned} var\left(x_{i,t}\right) &= \sigma_x^2 \\ corr\left(x_{i,t}, y\right) &= \rho_{x,y} \\ corr\left(x_{i,t}, x_{j,t}\right) &= \rho_{x,x} \\ corr\left(x_{i,t}, x_{j,u}\right) &= \rho_{x,x'} \end{aligned}$$

   for all t and $u\neq t$ and $i\neq j$ and let

$$\begin{aligned} a_\theta &= \frac{\partial f_1}{\partial x_{i,1}} \\ \hat{a}_\theta &= \frac{\partial \hat{f}_1}{\partial x_{i,1}} \end{aligned}$$

   If and only if

$$\rho_{x,x'} < \frac{\rho_{x,y}^2 a_\theta}{\left(\hat{a}_\theta\left(1-\rho_{x,y}^2\right)+a_\theta\right)}$$

   then there is polarization in expectations of any order of $t>1$.

Proof.

1. Using the Projection Theorem

$$E\left(x_{t-1,n}|S_{i,t},y\right)=E\left(x_{t-1,n}|S_{i,t}\right)+\frac{Cov\left(y,x_{t-1}|S_{i,t}\right)}{Var\left(y|S_{i,t}\right)}\left(y-E\left(y|S_{i,t}\right)\right)$$

consequently

$$\frac{\partial E\left(x_{t-1,n}|S_{i,1},y_{m+1}\right)}{\partial x_{t,m}}=\frac{\partial E\left(x_{t-1,n}|S_{i,t}\right)}{\partial x_{t,m}}-\frac{\partial E\left(x_{t-1,n}|S_{i,t},y\right)}{\partial y}\frac{\partial E\left(y|S_{i,1}\right)}{\partial x_{t,m}}.$$

Thus,

$$\left\|\frac{\partial E\left(x_{t-1,n}|S_{i,1},y_{m+1}\right)}{\partial x_{t,m}}\right\|>\left\|\frac{\partial E\left(x_{t-1,n}|S_{i,t}\right)}{\partial x_{t,m}}\right\|$$

holds, if

$$\frac{1}{2}\frac{\partial E\left(x_{t-1,n}|S_{i,t},y\right)}{\partial y}\frac{\partial E\left(y|S_{i,1}\right)}{\partial x_{t,m}}>\frac{\partial E\left(x_{t-1,n}|S_{i,t}\right)}{\partial x_{t,m}}=\left[\Sigma_{x_{t-1,n},S_{i,t}}\Sigma_{S_{i,t}}^{-1}\right]_m \tag{3}$$

where $\Sigma_{x_{t-1,n},S_{i,t}}$ is the vector of the covariances of $x_{t-1,n}$ and the each signal in $S_{i,t}$, $\Sigma_{S_{i,t}}$ is the covariance matrix of the signals in $S_{i,t}$ and the operator $[\cdot]_m$ denotes the m-th row of the given matrix. Observe that under the conditions of the proposition, as $\sigma_{x,x'}\to 0$, the right hand side of (3) goes to zero while the right hand side converges to a positive constant. So $T_{\sigma_{x,x'}}$ of the proposition exists.

2. Let

$$\begin{aligned}a_x &\equiv \frac{\partial E\left(x_{t-1}|x_t,y\right)}{\partial x_t}=\frac{\sigma_{x,x'}\sigma_y^2-\sigma_{x,y}^2}{\sigma_x^2\sigma_y^2-\sigma_{x,y}^2}\\ \hat{a}_x &\equiv \frac{\partial E\left(x_{t-1}|x_t\right)}{\partial x_t}=\frac{\sigma_{x,x'}}{\sigma_x^2}.\end{aligned}$$

Note that

$$\left|\frac{\partial \hat{f}_t}{\partial x_t}\right|=\left|\left(\hat{a}_x\right)^{t-1}\frac{\partial \hat{f}_1}{\partial x_1}\right| \text{ and } \left|\frac{\partial f_t}{\partial x_t}\right|=\left|\left(a_x\right)^{t-1}\frac{\partial f_1}{\partial x_1}\right|$$

As $\frac{\partial \hat{f}_1}{\partial x_1}>\frac{\partial f_1}{\partial x_1}>0$, $\left|\hat{a}_x\frac{\partial \hat{f}_1}{\partial x_1}\right|<\left|a_x\frac{\partial f_1}{\partial x_1}\right|$ implies $|\hat{a}_x|<|a_x|$ and $\left|\left(\hat{a}_x\right)^{t-1}\frac{\partial \hat{f}_1}{\partial x_1}\right|<\left|\left(a_x\right)^{t-1}\frac{\partial f_1}{\partial x_1}\right|$ for all t. Note also that $\left|\frac{\partial \hat{f}_t}{\partial \hat{x}_t}\right|<\left|\frac{\partial f_t}{\partial x_t}\right|$ implies polarization in the t-th order of higher-order expectations. Substituting in the definition of $\hat{a}_x$ and a_x into $\left|\hat{a}_x\frac{\partial \hat{f}_1}{\partial x_1}\right|<\left|a_x\frac{\partial f_1}{\partial x_1}\right|$ and simple algebraic manipulation gives $\rho_{x,x'}<\frac{\rho_{x,y}^2 a_\theta}{\left(\frac{\sigma_\theta\rho_{\theta,x}}{\sigma_x}\left(1-\rho_{x,y}^2\right)+a_\theta\right)}$, which concludes the proof.

■

For simplicity, in the rest of the paper I focus on the case where each agent observes only one private signal and there are only two groups of agents: $T = 2$, $N_1 = N_2 = 1$, $M = 0$. It is easy to see that in this case polarization of the forecasts of the second group depends only on the effect of the public announcement on the term $|a_{x_1} a_\theta|$ as

$$\begin{aligned} Var\left(E\left(E\left(\theta|x_{j,1},y\right)\right)|x_{i,2},y\right) &= \left(a_{x_1}a_\theta\right)^2\sigma^2_{x_{j,1}} \\ Var\left(E\left(E\left(\theta|x_{j,1}\right)\right)|x_{i,2}\right) &= \left(\hat{a}_{x_1}\hat{a}_\theta\right)^2\sigma^2_{x_{j,1}} \end{aligned} \tag{4}$$

This term is the sensitivity of a second group agent's forecast on her private signal and depends crucially on the correlation between the private signals of agents in different groups. To see better the intuition why, let us consider the following extreme example where this correlation is zero.

Example 1 *Suppose θ stands for the performance of the economy and it has two components: $\theta = \theta_1 + \theta_2$, where θ_1 is the performance of the public sector and θ_2 is the performance of the private sector. Suppose that θ_1 and θ_2 are independent and have zero mean. Assume that group 1 contains only one agent, the central bank. The central bank observes a private signal on the performance of the public sector: $x_1 = \theta_1 + \varepsilon$. Group 2 contains a unit mass of speculators, each observing a private signal about the performance of the private sector, $x_2 = \theta_2 + \varepsilon_i$. We are interested in the change in the dispersion of the forecasts of group 2 after the announcement of the public signal $y = \theta + \eta$.*

Observe that before the public announcement, speculators have no private information on the private information of the central bank, so their forecast of the forecast of the central bank is

$$\hat{f}_2 = E\left(E\left(\theta|x_1\right)|x_2\right) = E_j\left(\hat{a}_\theta x_1|x_2\right) = 0.$$

Thus, there is no dispersion in the forecast of the second group at all:

$$Var\left(\hat{f}_2\right) = 0.$$

However, after the announcement of y, a speculators' forecast, f_2^a changes to

$$\begin{aligned} E\left(E\left(\theta|x_1,y\right)|x_2,y\right) &= E_j\left(a_\theta x_1 + c_\theta y|x_2,y\right) = \\ &= a_\theta E_j\left(x_1|x_2,y\right) + c_\theta y = a_\theta a_{x_1} x_2 + c_\theta y, \end{aligned}$$

where $a_\theta = \frac{\partial f_1}{\partial x_1}, c_\theta = \frac{\partial f_1}{\partial y}$ *and* $a_{x_1} = \frac{\partial E(x_1|x_2,y)}{\partial x_2}$. *It is easy to check that* $a_{x_1} a_\theta \neq 0$ *Thus, the dispersion of speculators' forecasts changes to*

$$Var\left(f_2|y\right) = \left(a_{x_1} a_\theta\right)^2 \left(\sigma^2_{\theta_1} + \sigma^2_{\varepsilon_i}\right) > 0.$$

In the example, the public announcement increases the disagreement among speculators because the public announcement connects the private signals of speculators with the private signal of the central bank. Before the public announcement, a speculator with a high private signal on the performance of the private sector has the same forecast of the performance of the public sector than a speculator with a low private signal. Hence, the two speculators have the same forecast of the central bank's forecast. After the public announcement, a speculator who is pessimistic about the performance of the private sector has a higher estimation on the performance of the public sector than a speculator who is optimistic about the performance of the private sector. The reason is that the public signal gives information on the sum of the two sectors. Thus, the dispersion of the forecasts of the two speculators increases after the announcement. Proposition 1 showed that the sensitivity of first-order expectations to private signals decreases after an announcement. Proposition 2 shows that this do not have to be the case with higher-order expectations.

One might wonder why this property has not got any attention in the literature. This statistical property might have been overlooked in the literature, because of two possible reasons. The first one is that interest in models where higher-order expectations play an important role is relatively recent. The second one is that even in such models the information structure is almost always assumed to be the form where both private and public signals are noisy observations of the fundamental: $x^i_t = \theta + \varepsilon_i, y = \theta + \eta$. This structure imposes a rigid structure on the correlation structure of signals and θ. In particular,

$$cov\left(x_t, x_u\right) = cov\left(x^i_t, x^j_t\right) = cov\left(x_t, y\right) = var\left(\theta\right)$$

for any agent. It is easy to see that this structure is inconsistent with the assumptions of Proposition 2.

In this section, I highlighted a statistical property of normally distributed variables. In the next section, I argue that the statistical property highlighted in this section has important economic consequences by modifying two standard workhorse models of economics with learning.

3 Application I: trading volume in a Grossman-Stiglitz model

In rational expectation models of asset pricing under asymmetric information has difficulties to generate the well established empirical fact that trading volume increases around public announcements. To see the reason, let us consider the static Grossman-Stiglitz model[3] where the optimal demand function of a trader i is given by

$$d^i = \frac{E(\theta|S_i) - p}{\gamma Var(\theta|S_i)} \tag{5}$$

where θ is the true value of the asset, S_i is the information set of trader i including the equilibrium price p and γ is the absolute risk aversion of the trader. Given the demand functions and the normally distributed random supply, the equilibrium price is given by the market clearing condition. Thus, the equilibrium price is a noisy average of the best guesses of traders. Consequently, the numerator of (5) is a noisy measure of disagreement among traders. Hence, traders' aggregate position, $\int_i |d^i| di$, positively depends both on the average disagreement among traders and the precision of their estimation, $\frac{1}{Var(\theta|I_i)}$. A public announcement related to θ, will unambiguously increase the precision of traders' estimation. As we established in the previous section, disagreement about θ decreases after the announcement under general assumptions. Thus, the two effects work in the opposite direction and the aggregate change in position and the resulting trading volume tend to be small. In particular, in the basic Grossman-Stiglitz model where traders receive the private signal $x_i = \theta + \varepsilon_i$ and the public announcement is $y = \theta + \eta$ where $\theta, \varepsilon_i, \eta$ are normally and independently distributed with zero mean, the two effects exactly cancel out and there is no trading volume at all after the announcement.

In the rest of this section, I analyze a version of the dynamic Grossman-Stiglitz model where some traders liquidate has to liquidate their position early. This modification was introduced by Allen, Morris and Shin (2006). This early traders are interested in the liquidation price instead of the fundamental value. As these early traders are interested in the expectation of other traders, the disagreement among them might increase due to the announcement as Proposition 2 stated. In this case, the two effects work in the same direction which leads to large trading volume around announcements.

[3] In the discussed form, this model was developed in steps in Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981).

3.1 The set-up

I modify a standard, dynamic, CARA-Normal, rational expectations model with differential information (e.g. He and Wang, 1995, Brown and Jennings, 1989). Preferences of traders are given by $U_i(W_i) = -e^{-\gamma W_i}$, where W_i is monetary wealth at the time of the exit, γ is the absolute risk-aversion parameter and in each period traders submit demand curves to an auctioneer to buy up the random supply of assets:

$$u_t \tilde{} N\left(0, \frac{1}{\delta_t^2}\right).$$

Traders base their portfolio decision on the private signal which they receive at the moment of their entry and all available public signals, i.e., past and present prices and public announcements. They update their beliefs by Bayes' Rule. Prices, p_t, are determined by market clearing in each period.

I introduce two non-standard assumption. First, I assume overlapping groups of traders similarly to Allen, Morris and Shin (2006). In particular, I will have two groups of traders with continuum of traders in each group, who trade in 2+1 periods ($t = 0, 1, 2$). Traders in the first group trade among themselves in period 0 and 1 and sell all of their remaining assets in period 2. Traders in the second group take all positions of the first group, trade among each other in period 2 and liquidate for the uncertain value of θ at the end of the game. For purely expositional purposes, let us interpret the presented model in terms of a 24-hour day in the USD/GBP market with the first group representing traders based in London, while the second group representing traders in New York. Period 0 and 1 are daylight periods in London, so Londoners trade among themselves twice, and then they go to sleep, so they sell all their holdings to New Yorkers. They do not hold positions overnight[4]. Period 2 is daylight in New York, so New Yorkers trade among themselves and get θ in the evening. I assume that if there is a public announcement, y, then it will be released at the beginning of period 1. Hence, I will focus on the differences in trading patterns of Londoners (early traders) with and without an announcement.

The second departure from the standard assumptions is a more general information structure. I assume that the fundamental value of the asset – the exchange rate in this interpretation–

[4] Although I use the interpretation of a 24-hour FX market only for expositional reasons, it is a stylized fact among FX dealers that they do not hold positions overnight (see Lyons, 2001).

is given by

$$\theta = \theta_s + \theta_k + \theta_w$$

where $\theta_s, \theta_k, \theta_w$ are the US factor, the UK factor and the world factor respectively. I assume that the private signal that Londoners receive contains noisy information on the UK factor and the world factor, but does not contain information on the US factor: $x_i = \theta_k + \theta_w + \varepsilon_i$, while the private signals of New Yorkers contain information on the US factor and the world factor, but not on the UK factor: $z_j = \theta_s + \theta_w + \varepsilon_j$. Hence, the world factor simply represents the common element in the information set of agents in different groups, while the US factor and the UK factor represent group-specific information. The public signal contains information on fundamental value: $y = \theta + \eta$. I assume that all factors and noise terms are i.i.d. and normally distributed:

$$\theta_k, \theta_s \sim N\left(0, \frac{1}{\kappa}\right), \theta_w \sim N\left(0, \frac{1}{\omega}\right), \varepsilon_i, \varepsilon_j \sim N\left(0, \frac{1}{\alpha}\right), \eta \sim N\left(0, \frac{1}{\beta}\right).$$

The presented model nests the information structures of many versions of the Grossman-Stiglitz model. The next table summarizes the connections.

our parameter	*private*	*public*	*fundamental*	$p_2(\cdot)$	model
$\kappa, \delta_2 \to \infty$	$\theta_w + \varepsilon_i$	$\theta_w + \eta$	θ_w	$\approx \theta_w$	Brown-Jennings(1989), Kim-Verrecchia(1991,1994)
$\kappa \to \infty$	$\theta_w + \varepsilon_i$	$\theta_w + \eta$	θ_w	$\approx \theta_w + u_2$	He-Wang (1995)
$\omega \to \infty$	$\theta_k + \varepsilon_i$	$\theta_s + \eta$	$\theta_k + \theta_s$	$\approx \theta_s + u_2$	independent information sets

When $\kappa \to \infty$, the group-specific factors, θ_s, θ_k lose their importance and we end up in a one-factor structure with θ_w only. When also $\delta_2 \to \infty$, second period price, p_2, is fully revealing, so Londoners behave as if they could liquidate for θ_w, which is the only relevant factor remaining. Hence, in the case of $\kappa, \delta_2 \to \infty$, effectively we have a two period model with one factor as in Brown and Jennings. When δ_2 is finite, the model resembles to that of He and Wang (1995) as the liquidation value for Londoners, p_2, will contain the random term u_2 as well. However, when only $\omega \to \infty$, we have a different structure. Very similarly to Example 2, in this structure the information sets of Londoners and New Yorkers get separated. As I will show in the next section: the larger ω, the larger the effect of increasing disagreement.

3.2 Equilibrium and existence

Finding the linear rational expectation equilibrium consists of three fairly standard steps. First, let us conjecture that prices are given by the functions

$$\begin{aligned} p_2 &= a_2(\theta_s+\theta_w)+c_2y+f_2q_1+g_2q_0-e_2u_2 \\ p_1 &= a_1(\theta_k+\theta_w)+c_1y+f_1q_0-e_1u_1 \\ p_0 &= a_0(\theta_k+\theta_w)-e_0u_0, \end{aligned} \tag{6}$$

where $c_t, b_t, e_t, f_1, f_2, g_2$ are undetermined coefficients, while q_1, q_0 are specified as follows. Prices together with past prices and the public information are informationally equivalent with the following price signals

$$\begin{aligned} q_2 &= \frac{1}{a_2}(p_2-c_2y-f_2q_1-g_2q_0)=(\theta_s+\theta_w)-\frac{e_2}{a_2}u_2 \\ q_1 &= \frac{1}{a_1}(p_1-c_1y-f_1q_0)=(\theta_k+\theta_w)-\frac{e_1}{a_1}u_1 \\ q_0 &= \frac{1}{a_0}p_0=(\theta_k+\theta_w)-\frac{e_0}{a_0}u_0. \end{aligned} \tag{7}$$

Let us define τ_t^2 as the precision of q_t:

$$\frac{1}{\tau_t^2}\equiv\frac{e_t^2}{a_t^2\delta_t^2} \text{ or } \frac{\tau_t}{\delta_t}=\frac{a_t}{e_t}.$$

Second, given the conjectured price functions, let us derive the optimal strategies of traders. To ease on the notation, I define the following coefficients of variables in traders' information sets in different conditional expectations of New Yorkers and Londoners:

$$E(\theta|z_j,y,q_2,q_1,q_0) = a_\theta z_j+b_\theta q_2+c_\theta y+h_\theta q_1+g_\theta q_0 \tag{8}$$

$$E(\theta_s+\theta_w|x_i,y,q_1,q_0) = a_sx_i+b_sq_1+c_sy+h_sq_0. \tag{9}$$

From the first order conditions of the maximization problem of New Yorkers in period 2 and Londoners in period 1 are

$$d_2^j = \frac{E(\theta|z_j,y,q_2,q_1,q_0)-p_2}{\gamma var(\theta|z_j,y,q_2,q_1,q_0)} \tag{10}$$

$$d_1^i = \frac{E(p_2|x_i,y,q_1,q_0)-p_1}{\gamma var(p_2|x_i,y,q_1,q_0)}, \tag{11}$$

respectively. In period 0, Londoners maximize the following expected utility :

$$\max_{d_0^i} E\left(-\exp\left(-\alpha\left(p_1-p_0\right)d_0^i-d_1^i\left(p_2-p_1\right)\right)|x_i,q_0\right).$$

Deriving the period 0 demand function is a bit more work as there are two random variables in this expression, p_1 (or q_1) and y. In the appendix, I show that the resulting demand function in period 0 has the form of

$$d_0^i=\frac{E\left(p_1|x_i,q_0\right)-p_0}{\gamma V_1}+\frac{E\left(d_1^i|x_i,q_0\right)}{\gamma V_2} \tag{12}$$

where V_1 and V_2 are functions of the variances and expectations of θ, y, q_1, q_2 conditional on the information set of Londoners in the first period. Intuitively, the first part in the nominator of expression (12) represents the short-term demand component, while the second part represents the hedging component for demand in period 1.

Third, let us impose the market clearing conditions and find the undetermined coefficients which simultaneously satisfy all three market clearing conditions. From (10) and (8), the market clearing condition in period 2 is

$$D_2=\frac{a_\theta\left(\theta_s+\theta_w\right)+b_\theta q_2+c_\theta y+h_\theta q_1+g_\theta q_0-p_2}{\gamma var\left(\theta|z_j,y,p_2,q_1,q_0\right)}=u_2.$$

Using (7) gives

$$a_\theta\left(\theta_s+\theta_w\right)+b_\theta q_2+c_\theta y+h_\theta q_1+g_\theta q_0-\gamma var\left(\theta|z_j,y,q_2,q_1,q_0\right)u_2=a_2q_2+c_2y+h_2q_1+g_2q_0.$$

Setting the coefficients of random variables on the two sides of the equation to equal gives

$$c_2=c_\theta,\quad h_2=h_\theta,\ \text{and}\quad g_2=g_\theta \tag{13}$$

and

$$a_\theta\left(\theta_s+\theta_w\right)-\gamma var\left(\theta|z_j,y,q_2,q_1,q_0\right)u_2=\left(a_2-b_\theta\right)q_2.$$

As $q_2=\theta_s+\theta_w-\frac{e_2}{a_2}u_2$, this gives

$$a_2=a_\theta+b_\theta \tag{14}$$

and

$$\frac{a_\theta}{\gamma var\left(\theta|z_j,y,q_2,q_1,q_0\right)}=\frac{a_2}{e_2}.$$

We can write the latter as

$$\frac{a_\theta}{\gamma var\left(\theta|z_j,y,q_2,q_1,q_0\right)}=\frac{\tau_2}{\delta_2}. \tag{15}$$

Note that the undetermined coefficients enter the left hand side of (15) only through the precision of the price signals, τ_0, τ_1, τ_2. Thus, the market clearing condition in period 2 implies that the undetermined coefficients e_t, a_t, $t = 0, 1, 2$ has to satisfy

$$\tau_2 = F_2(\tau_2, \tau_1, \tau_0), \tag{16}$$

where $F_2(\cdot)$ is a function determined by equation (15).

Turning to the market clearing condition in period 1

$$E(p_2|x_i, y, q_1, q_0) - p_1 = \gamma var(p_2|x_i, y, q_1, q_0) u_1,$$

we get

$$a_2 E(\theta_s + \theta_w | x_i = \theta_k + \theta_w, y, q_1, q_0) + c_2 y + h_2 q_1 + g_2 q_0 - e_2 u_2 - (a_1 q_1 + c_1 y + h_1 q_0) = \gamma var(p_2|x_i, y, q_1, q_0) u_1$$

which implies

$$c_1 = a_2 c_s + c_2 = (a_\theta + b_\theta) c_s + c_\theta, ..h_1 = a_2 g_s + g_2 = (a_\theta + b_\theta) g_s + g_\theta$$

and, after substituting in the definition of q_1,

$$a_1 = a_2(a_s + b_s) + h_2 = (a_\theta + b_\theta)(a_s + b_s) + h_\theta$$

and

$$\frac{\tau_1}{\delta_1} = \frac{a_s}{\frac{e_2}{a_2} + \gamma \frac{1}{a_2} var(p_2|x_i, y, q_1, q_0)} = \frac{a_s}{\frac{1}{\tau_2^2} + \gamma \frac{1}{a_\theta + b_\theta} var(p_2|x_i, y, q_1, q_0)}. \tag{17}$$

As

$$var(p_2|x_i, y, q_1, q_0) = (a_\theta + b_\theta)^2 var(\theta_s + \theta_w | x_i = \theta_k + \theta_w, y, q_1, q_0)$$

undetermined coefficients enter the left side of equation (17) only through the noise of the price signals, $\tau_t = 0, 1, 2$. Thus, the market clearing condition in period 1 imposes the condition that $\tau_t = 0, 1, 2$ has to satisfy the equation

$$\tau_1 = F_1(\tau_2, \tau_1, \tau_0) \tag{18}$$

where $F_1(\cdot)$ is a function determined by equation (17).

In the appendix, I show by very similar steps that the market clearing condition in period 0 imposes a third restriction on the noise term of the price signals, $\tau_t = 0, 1, 2$ which has the form of

$$\tau_0 = F_0(\tau_2, \tau_1, \tau_0). \tag{19}$$

Thus, the existence of the equilibrium is equivalent to the existence of a fixed point of the system described by equations (16) ,(18) and 19. It is easy to check that the corresponding equilibrium intensities when there is no announcement will be given by the same equations by letting the variance of the public signal to go to infinity, that is, setting $\beta = 0$. As before, when it is necessary, I distinguish between variables with and without announcement by denoting the no announcement case with a hat. The following proposition states that the equilibrium exists.

Theorem 1 (Existence) *An equilibrium exists both for the announcement and the no-announcement cases.*

Proof. The proof is in the Appendix. ■

3.2.1 Trading volume

A side result of the derivation of the existence is that the demand functions turn out to have very simple forms:

$$d_2^j = \frac{\tau_2}{\delta_2}(z_j - q_2) \tag{20}$$

$$d_1^i = \frac{\tau_1}{\delta_1}(x_i - q_1) \tag{21}$$

$$d_0^i = \frac{\tau_0}{\delta_0}(x_i - q_0). \tag{22}$$

We get expression (20) by substituting (8), (13), (6) and (15) into (10). Similarly, substituting (9), (6), (**??**), (**??**) and (17) into (11) gives (20). I show in the appendix that (22) also holds. Expressions (20)-(22) show that traders' positions are determined as a product of the coefficient $\frac{\tau_t}{\delta_t}$ and the difference between the private information of the trader and a signal about the about the average valuation of all traders revealed by the price. Thus, $\frac{\tau_t}{\delta t}$ determines how intensively the trader uses her private information to bet against the others. I will label the fraction $\frac{\tau_t}{\delta_t}$ as the trading intensity in period t.

Using the definition of τ_t, we can rewrite expressions (20)-(22) as

$$d_2^j = \frac{\tau_2}{\delta_2}\varepsilon_j + u_2 \tag{23}$$

$$d_1^i = \frac{\tau_1}{\delta_1}\varepsilon_i + u_1 \tag{24}$$

$$d_0^i = \frac{\tau_0}{\delta_0}\varepsilon_i + u_0. \tag{25}$$

The expressions show that in each period, total positions consist of two parts. There is a risk-sharing part, u_t, which is purchased by each agent regardless of her information, and there is a speculative part, $\frac{\tau_t}{\delta t}\varepsilon_i$, which depends on the difference between the agent's signal and the true value of the factor, ε_i or ε_j, and the coefficient $\frac{\tau_t}{\delta t}$. So trading activity and volume will depend how these coefficients change across periods.

As the focus of this paper is the effect of announcement on trading volume, we are interested in the change of volume in period 1 due to the announcement. From equations (21) and (22), the amount of trading of trader i in period 1 is given by

$$v_1^i = d_1^i - d_0^i = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i - u_0 + u_1.$$

Just as total positions, the total amount of trade of individual i consists of two parts. There is an information-independent risk-sharing part, $u_1 - u_0$, and there is a speculative part $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i$. The latter is determined by the difference of trading intensities in the two periods and the private information of the trader. If we aggregate across traders, we get the following expression for total volume in period 1:

$$\begin{aligned} V_1^{total} &= \frac{1}{2}\int \left|d_1^i - d_0^i\right| di = \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1}-\frac{\tau_0}{\delta_0}\right)\varepsilon_i > u_0 - u_1} \left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi\left(\alpha\varepsilon_i\right) d\varepsilon_i - \\ &- \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1}-\frac{\tau_0}{\delta_0}\right)\varepsilon_i < u_0 - u_1} \left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi\left(\alpha\varepsilon_i\right) d\varepsilon_i = \\ &= \left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right| \frac{1}{\sqrt{\alpha}}\phi\left(T\right) + sgn\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\left(u_1 - u_0\right)\frac{1}{2}\left(1 - 2\Phi\left(K\right)\right) \end{aligned}$$

with $K = \alpha\frac{u_0 - u_1}{\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}}$,where I used the result that if $\zeta \sim N\left(\mu, \sigma^2\right)$, then

$$\int_{\zeta > L} \zeta \frac{\phi\left(\frac{\zeta-\mu}{\sigma}\right)}{\Phi\left(\alpha\right)} d\zeta = E\left(\zeta|\zeta > L\right) = \mu + \sigma\lambda\left(\alpha\right)$$

with $\lambda\left(\alpha\right) = \frac{\phi(\alpha)}{1-\Phi(\alpha)}$ and $\alpha = \frac{L-\mu}{\sigma}$.

Hence, aggregate volume depends only on the realization of $u_1 - u_0$, the precision of the private signals α, and the distance between trading intensities, $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$. As the first one is unrelated to information or announcements, I focus on the speculative volume, which I define as the volume when there is no risk-sharing trade:

$$V_1 = V_1^{total}|_{u_0 = u_1} = \left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right| \frac{1}{\sqrt{\alpha 2\pi}}. \tag{26}$$

Hence, the effect of announcement on speculative volume, we only have to compare the change in trading intensities in the announcement case, $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$, and the no-announcement case, $\left|\frac{\hat{\tau}_1}{\delta_1} - \frac{\hat{\tau}_0}{\delta_0}\right|$. The main result of this paper is that the outcome of this comparison will depend heavily and systematically on the information structure, i.e., on the relative importance of the common factor, θ_w, and the individual factors θ_s, θ_k. The following proposition shows that as individual factors become less important and the common factor becomes more important, volume disappears. This result is in line with our earlier observation that in a rational model with one factor trading volume around announcements is small, because the effects of increasing precision of opinions and decreasing disagreement cancel out.

Proposition 3 *As $\delta_2, \kappa \to \infty$*

$$\frac{\hat{\tau}_2}{\delta_2} = \frac{\hat{\tau}_1}{\delta_1} = \frac{\hat{\tau}_0}{\delta_0} = \frac{\tau_2}{\delta_2} = \frac{\tau_1}{\delta_1} = \frac{\tau_0}{\delta_0} = \frac{\alpha}{\gamma},$$

hence $\hat{V}_1 = V_1 = 0$ in this limit.

Proof. The proof is in the appendix. ■

In contrast, the next proposition shows that if the importance of the common information element, θ_w, is small enough, the position of Londoners is larger with public announcement. Furthermore, the trading volume is also larger if there is an announcement in period 1. If the effect of announcement is measured by the proportion of volume in the announcement case to volume in the no-announcement case, this proportion will be arbitrary large as $\omega \to \infty$. The same is true for the proportion of speculative positions in both periods.

Proposition 4 *If ω is large enough $D_1 = \frac{1}{2}\int \left|\frac{\tau_1}{\delta_0}\varepsilon_i\right| di > \frac{1}{2}\int \left|\frac{\hat{\tau}_1}{\delta_0}\varepsilon_i\right| di = D_1$ and $V_1 > \hat{V}_1$ and as $\omega \to \infty$,$\frac{D_1}{\hat{D}_1} \to \infty$,$\frac{D_0}{\hat{D}_0} \to \infty$. Furthermore, $\frac{V_1}{\hat{V}_1} \to \infty$ (for almost all parameters).*

Proof. The proof is in the appendix. ■

This result is based on the same intuition as the one illustrated by the Example in section 2. In the limit, when ω is very large, the importance of the world factor, θ_w, diminishes and there is no correlation between the private information of any Londoner and of any New Yorker Traders speculate only on the value of those variables that are not part of the public information set. From the Londoners point of view in period 1 (the second period when they trade), the only variable in p_2 which is not part of the public information set – apart from the noise, u_2 – is θ_s. But first period traders have no information on θ_s, they have information only on θ_k. Hence, each traders' guess on θ_s will be the a priori mean, and traders will not take speculative positions against each other. But if Londoners do not trade on their private information, their private information cannot be channeled into prices, so p_1 will be very noisy. Let us turn our attention to period 0. Londoners should bet on p_1 and p_2, but they do not have any information either on p_1, as it is vert noisy, or on p_2, because they do not know anything about θ_s. Hence, there will be no speculative trade in period 0 either. It means that speculative volume, the difference between individual speculative positions in period 0 and period 1, will also be zero. However, the situation changes with public announcement. Public announcement is $y \approx \theta_s + \theta_k + \eta$, if θ_w is insignificant. So Londoners will have some information on the sum of θ_s and θ_k. But together with their private information on θ_k, it gives them some information on θ_s. What is more, as they have different guesses on θ_k due to their different private signals, their guesses on θ_s will also be different. Therefore, public announcement increases disagreement. In particular, an early trader with a high private signal, will overestimate θ_k, so for a given $y \approx \theta_s + \theta_k + \eta$, will underestimate θ_s. This is why, the observation of the public signal will make her to hold a negative position, i.e., to sell assets.[5] With the opposite logic as in the no-announcement case, there will be trade in all periods and there will be volume.

4 Application II: Speculative currency attacks

In this application, I modify the currency attack model of Morris and Shin (1998). I use elements of variants of this model introduced by Angeletos and Werning (2006), Hellwig, Mukherji and Tsyvinski (2006) and Goldstein, Ozdenoren and Yuan.(2008). The main point of departure from Morris and Shin (1998) is that I do not allow the central bank either to directly observe the state of the economy or to perfectly infer it from the size of the speculative attack. Instead, the central bank observes a private signal about the state and observes the size of the attack with noise. Thus, speculators have to guess the central banks' guess about the economy when they assess the chance of devaluation.[6].

[5] The fact that after the public announcement, if ω is large, traders with high private signal will sell the asset is the symptom of second-order expectations moving against first order expectations. Thus, in this case, traders have a high estimation of the fundamental value, but still they expect a low interim price. This phenomenon is not in the focus of this paper, but discussed in detail in Kondor (2004).

[6] Goldstein, Ozdenoren and Yuan (2008) also assumes that the central bank has imperfect information of the fundamental. The main difference between my model and theirs is that in their model the central bank cost's

An additional novel element in the version analyzed here is that the central bank cost of defending the peg depends non-linearly on the size of the attack In Morris and Shin (1998), the central bank devalues when the fundamental of the economy is week relatively to the proportion of speculators, A, who decide to attack the peg. In my variant, the central bank devalues the currency when her estimate of the state of the economy is low relative to a monotonic transformation of the size of the attack $\Phi^{-1}(A)$ where $\Phi^{-1}(\cdot)$ is the standard cumulative normal density. This modification keeps the model analytically very tractable, despite the increased complexity due to the assumption of imperfect knowledge of the central bank. As a side effect, this modification changes the properties of the model in terms of the uniqueness of the equilibrium. Given that the uniqueness of equilibrium is in the very focus of the literature of speculative currency attacks, I discuss this issue in a separate part at the end of this section.

4.1 The set up

There is a unit mass of risk-neutral speculators decide whether to attack the currency regime for cost $1 > \kappa > \frac{1}{2}$. Attacking is advantageous if and only if the central bank decide to devalue the currency. Their pay-off is summarized by the following table.

	devaluation	no devaluation
attack	$1-\kappa$	$-\kappa$
do not attack	0	0

Risk-neutrality implies that a speculator attacks only if she estimates the probability of devaluation larger than κ.

Defending the exchange rate regime is costly for the central bank. If A fraction of speculators attack,the cost of defence is $\Phi^{-1}(A)$ where Φ^{-1} is the inverse of the standard normal cumulative distribution function. In addition there are some noise traders who might buy or sell the domestic currency. The cost of defending the peg from the attack of the noise-traders is $u\tilde{~}N\left(0,\frac{1}{\delta}\right)$. Thus, the total cost of defending the peg is $C \equiv \Phi^{-1}(A) + u$.

The Central Bank abandons the peg if the cost,C is large relative to the central bank's estimate of the fundamentals

$$\lambda C > E(\theta|S_{cb}), \tag{27}$$

function of defending the peg does not depend directly on the size of the attack.

where θ is the fundamental of the economy and S_{cb} is the information set of the central bank and λ is a constant controlling for the trade-off.

The information structure is as follows. The fundamental of the economy contains three factors, $\theta = \theta_a + \theta_b + \theta_c$, where θ_a is a factor specific to the speculators information set, θ_b is a factor specific to the central bank's information set, while θ_c is a common factor. Consistently with this notation, the central bank observes a private signal $z = \theta_b + \theta_c + \varepsilon_{cb}$, and each speculator observes a private signal $x_i = \theta_a + \theta_c + \varepsilon_i$. There might be a public announcement of $y = \theta + \eta$. The cost of defence C is also public information. I assume that all factors and noise terms are i.i.d. and normally distributed:

$$\begin{aligned}\theta_a, \theta_b &\sim N\left(0, \frac{1}{\kappa}\right), \theta_c \sim N\left(0, \frac{1}{\omega}\right), \varepsilon_i \sim N\left(0, \frac{1}{\alpha}\right) \\ \varepsilon_{cb} &\sim N\left(0, \frac{1}{\upsilon}\right), \eta \sim N\left(0, \frac{1}{\beta}\right)\end{aligned}$$

Our main question of interest is that whether the existence of a public signal increases or decreases the unconditional probability of a successful attack.

4.2 Equilibrium and existence

As it is usual in the literature, I am looking for monotone (threshold) equilibria where the central bank chooses the same action as long as her signal is below a certain threshold, z^*, and chooses the other action otherwise. Similarly, each speculator chooses the same action as long as her signal is below a certain threshold x^* and chooses the other action otherwise. Both thresholds, z^*, x^* can depend on public information only. If z^* and x^* is finite, I call the equilibrium a finite monotone equilibrium. I derive all expressions for the case when the public signal is released.

Solving for the equilibrium consists of four steps.

First, we conjecture the form of the optimal strategy and derive the information content of the cost of the attack under the conjecture. Let us conjecture that there is a threshold z^* that the central bank devalues if and only if $z < z^*$ and there is a threshold x^* that the speculators attack if and only if $x_i < x^*(y, q)$. The later implies that the aggregate demand for dollars is

$$A = \Phi\left(\psi\left(x^* - \theta_s - \theta_c\right)\right) \tag{28}$$

where $\psi \equiv \sqrt{\frac{1}{\frac{1}{\omega}+\frac{1}{\kappa}+\frac{1}{\alpha}}}$ is the standard deviation of the private signal of each speculator. Using the assumption that the cost of the attack, C is $C = \Phi^{-1}(A) + u$, the cost of the attack is informationally equivalent to the cost signal, q, where

$$q \equiv x^* - \frac{1}{\psi}C = \theta_s + \theta_c - \frac{1}{\psi}u. \tag{29}$$

Second, by definition, if the central bank has a signal z^*, the central bank must be indifferent between abandoning the exchange regime or defending it. Thus, z^* must satisfy

$$\lambda C = E\left(\theta | z = z^*, q, y\right). \tag{30}$$

Third, by definition, if a speculator has a signal of x^*, she must be indifferent between attacking the exchange rate regime or not attacking. The probability of devaluation under the information set of speculator i is

$$\Pr\left(z < z^* | x_i, y, q\right) = \Phi\left(\frac{z^* - E\left(z|x_i, y, q\right)}{\sigma_{z|x_i,y,q}}\right) \tag{31}$$

where $\sigma_{z|x_i,y,q} = \sqrt{Var\left(z|x_i, y, q\right)}$.Consequently, x^* must satisfy

$$\Phi\left(\frac{z^* - E\left(z|x_i = x^*, y, q\right)}{\sigma_{z|x_i,y,q}}\right) = \kappa \tag{32}$$

Finally, substituting in the definition of q into (30) and (32) and some simple algebra, equations (30) and (32) give a system of two equations with the two unknowns z^*, x^* of the following form

$$z^* = \frac{\lambda\psi x^* - \left(\left(b_\theta + \psi\lambda\right)q + c_\theta y\right)}{a_\theta} \tag{33}$$

$$x^* = \frac{z^* - \Phi^{-1}\left(\kappa\right)\sigma_{z|x_i,y,q} - c_z y - b_z q}{a_z} \tag{34}$$

where I used the notation of

$$\begin{aligned} E\left(\theta|x_i, q, y\right) &= a_\theta x_i + b_\theta q + c_\theta y \\ E\left(z|x_i, q, y\right) &= a_z x_i + b_z q + c_z y. \end{aligned}$$

Note, that both equations (33) and (34) are linear in x^* and z^*. Hence, there is a unique intercept of the two lines as long as $\frac{\lambda\psi}{a_\theta} \neq a_z$.

The derived equilibrium is consistent with our conjecture that a speculator attacks if $x < x^*$ only if expression (31) is increasing in x_i. This is true if and only if $a_z > 0$. Otherwise, we should conjecture that speculators attack if and only if $x > x^*$. In this case, (28) and (29) modify to

$$\begin{aligned} A &= \Phi\left(-\psi\left(x^* - \theta_s - \theta_c\right)\right) && (35) \\ q &\equiv x^* + \frac{1}{\psi}C = \theta_s + \theta_c + \frac{1}{\psi}u, && (36) \end{aligned}$$

respectively. Note that the difference between expressions (28)-(29) and (35)-(36) is the change of the sign before the constant ψ. Hence, the derivation of the equilibrium is incorporates both cases if we substitute

$$\tilde{\psi} = \left\{ \begin{array}{ccc} \psi & if & a_z \geq 0 \\ -\psi & otherwise & \end{array} \right\}$$

instead of ψ in expressions (28), (29) and (33).

The following proposition summarizes this subsection.

Proposition 5 *As long as* $\frac{\lambda\tilde{\psi}}{a_\theta} \neq a_z$, *there is a unique finite monotone equilibrium described by*

$$\begin{aligned} x^* &= \frac{a_\theta \Phi^{-1}(\kappa)\,\sigma_{z|x_i,y,q} + (a_\theta c_z + c_\theta)\,y + \left(a_\theta b_z + \left(b_\theta + \tilde{\psi}\lambda\right)\right)q}{\lambda\tilde{\psi} - a_z a_\theta} \\ z^* &= \frac{\lambda\tilde{\psi}x^* - \left(\left(b_\theta + \tilde{\psi}\tilde{\lambda}\right)q + c_\theta y\right)}{a_\theta}. \end{aligned}$$

The central bank abandons the peg if and only if $z \leq z^*$ *and the speculators attack*

1. *if and only if* $x < x^*$ *as long as* $a_z \geq 0$ *and*

2. *if and only if* $x > x^*$ *as long as* $a_z < 0$..

4.3 Probability of successful currency attacks

I am interested in the effect of public information on the probability of a successful attack. For a given information structure, the probability of a successful attack is the unconditional probability that the private signal of the central bank is smaller than her threshold, z^*. If the public signal is released, this is

$$\Pr(z < z^*) = \Pr\left(z < \Phi^{-1}(\kappa)\,\sigma_{z|x_i,y,q} + a_z x^* + \left(b_\theta + \tilde{\psi}\lambda\right) q + c_z y\right)\right) =$$

$$= \Pr\left(z - \left(\frac{a_z b_\theta + (a_z + b_z)\,\tilde{\psi}\lambda}{\lambda\tilde{\psi} - a_z a_\theta}\right) q - \left(\frac{a_z c_\theta + c_z \lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right) y < \frac{\Phi^{-1}(\kappa)\,\sigma_{z|x_i,y,q}\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right) =$$

$$= \Phi\left(\Phi^{-1}(\kappa)\,\frac{\lambda\psi}{\lambda\psi - |a_z|\,a_\theta}\,\frac{\sigma_{z|x_i,y,q}}{\sigma_{z-\left(\frac{a_z b_\theta + (a_z+b_z)\tilde{\psi}\lambda}{(\lambda\tilde{\psi} - a_z a_\theta)}\right)q - \left(\frac{a_z c_\theta + c_z\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right)y}}\right) \equiv \Phi\left(\Phi^{-1}(\kappa)\,K\right).$$

where

$$K \equiv \frac{\lambda\psi}{\lambda\psi - |a_z|\,a_\theta}\,\frac{\sigma_{z|x_i,y,q}}{\sigma_{z-\left(\frac{a_z b_\theta + (a_z+b_z)\tilde{\psi}\lambda}{(\lambda\tilde{\psi} - a_z a_\theta)}\right)q - \left(\frac{a_z c_\theta + c_z\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right)y}}$$

is a coefficient which depends only on the parameters of the model. Observe that the probability of a successful attack is increasing in K. Similarly, if there is no announcement the probability of a successful attack is

$$\Pr(z < \hat{z}^*) = \Phi\left(\Phi^{-1}(\kappa)\,\frac{\lambda\psi}{\lambda\psi - |\hat{a}_z|\,\hat{a}_\theta}\,\frac{\sigma_{z|x_i,q}}{\sigma_{z-\left(\frac{\hat{a}_z\hat{b}_\theta + (\hat{a}_z+\hat{b}_z)\tilde{\psi}\lambda}{(\lambda\tilde{\psi} - \hat{a}_z\hat{a}_\theta)}\right)q}}\right) \equiv \Phi\left(\Phi^{-1}(\kappa)\,\hat{K}\right).$$

Thus, if $K > \hat{K}$ then more public information increases the probability of a successful attack. Observe, that both K and $\hat{K}$ are products of two components. From our observation (4) it apparent that the first term is highly related to polarization. If and only if the release of the public signal increases polarization in second-order expectations, the term, $\frac{\lambda\psi}{\lambda\psi - |a_z|a_\theta}$, will increase due to the announcement. From section 1, we already know that this happens when the connection between the private signals of speculators and the private signal of the central bank is week. That is, when ω is large. The second term is harder to interpret. It is the fraction of the precision of speculators' estimate of the central bank's signal and the standard deviation of a linear combination of z.and the public signals. The effect of an announcement on this component is ambiguous. Still, in the next proposition I show that if ω is large and the precision of the public signal is relatively low, the announcement will increase the probability of a successful speculative attack.

Proposition 6 *There are finite positive thresholds T_ω, T_β, that if $\omega > T_\omega$, and $\beta < T_\beta$ then*

$$K > \hat{K}$$

and the release of the public information increases the probability of a successful speculative attack.

Proof. [The proof is to be completed] ■

Why polarization is connected to the probability of successful currency attacks? The intuition is based on the fact that always those speculators attack who think that the probability of devaluation is high relative to the cost of the attack κ. If there is more dispersion in speculators' assessment about the probability of devaluation, there will be more mass on the tails and more speculators with an assessment larger than κ. The form of central bank's cost function, (27), implies that the dispersion in speculators' assessment on the probability of devaluation is proportional to the dispersion in speculators' assessment on the forecast of the central bank. Thus, if the public announcement polarizes speculators second-order expectation regardless of the content of y, then the ex ante probability of a sufficiently large attack is higher in a regime with public announcement. Although the exact mechanism is more complex as the cost of the attack reveals information for all agents about all other agents' information and all agent reacts to the optimal strategy of other agents, Proposition 6 shows that the intuition still goes through if the quality of the public announcement is sufficiently low.

4.4 Uniqueness

Compared to Morris and Shin (1998), the modification of the form of central bank's cost function changes the properties of the equilibrium regarding the issue of uniqueness versus multiplicity. The changes are two fold. First, as it is stated in proposition .5., there is a unique finite monotone equilibrium regardless of the precision of private and public signals. This is in contrast to the standard Morris and Shin (1998) model where there are multiple finite monotone equilibria if the public signal is too precise relative to the private signal. This source of multiplicity is absent in my framework. The reason is mainly technical: in the original Morris and Shin model the best response functions corresponding to equations (33)-(34) are non-linear and might intersect more than once. In my framework, there is always exactly one intersection.

Second, even if in the current framework there is always a unique finite monotone equilibrium, there is never a unique monotone equilibrium. In particular, both of the following two strategy profiles form equilibria: (1) all speculators attack regardless of their signals and the central bank devalues regardless of her signal (2) none of the speculators attack regardless of their signal and the central bank keeps the peg regardless of her signal. In Morris and Shin (1998), none of these strategy profiles forms an equilibrium. The reason is that in Morris and

Shin, the cost function of the central bank maps the size of the attack to the unit interval, while in my version the cost function maps the size of the attack to the real line. Thus, in my version the cost of defending the peg might be unboundedly low or unboundedly high which validates the new equilibria.

In the rest of this section, I show that only the finite monotone equilibrium is robust to a simple perturbation of the system. Let us suppose that the cost of defending the peg from speculators is bounded from above and from below. In particular, let us suppose that the cost of defence when A proportion of speculators attack is

$$\tilde{C} = \left\{ \begin{array}{ccc} \Phi^{-1}(\xi) + u & if & \Phi^{-1}(A) \leq \Phi^{-1}(\xi) \\ \Phi^{-1}(A) + u & if & \Phi^{-1}(\xi) < \Phi^{-1}(A) < \Phi^{-1}(1-\xi) \\ \Phi^{-1}(1-\xi) + u & if & \Phi^{-1}(1-\xi) \leq \Phi^{-1}(A) \end{array} \right\} \tag{37}$$

instead of C, where ξ is a small positive constant. It is apparent that as ξ goes to zero, the system converges to the original framework. In the next proposition, I show that as long as ξ is sufficiently small, this perturbation eliminates all monotone equilibria, but the one presented in proposition 5.

Proposition 7 *Under perturbed system characterized by (37), there is a T_ξ threshold that if $\xi < T_\xi$, there is a unique monotone equilibrium. Furthermore, as $\xi \to 0$, the equilibrium converges to the one defined in Proposition 5.*

Proof. Note that under (37), devaluation for any z_i is not a best response if each speculator attacks for any x_i as

$$\lim_{z_i \to \infty} E\left(\theta | z_i, y, \Phi^{-1}(\xi) + u\right) > \lambda\left(\Phi^{-1}(\xi) + u\right)$$

for any realized u. Similarly, keeping the peg for any z_i is not a best response if none of the speculator attack for any x_i Thus, all monotone equilibria must be finite monotone equilibria.

Following the derivation of the equilibrium in subsection 5, a monotone equilibrium of the perturbed system must satisfy equations

$$\lambda \tilde{C} = E\left(\theta | z = z^*, \tilde{C}, y\right) \tag{38}$$

$$\kappa = \Pr\left(z < z^* | x_i = x^*, \tilde{C}, y\right). \tag{39}$$

Note that

$$E\left(\theta|z_i,\tilde{C},y\right)=$$
$$=E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(\xi\right)>u>\tilde{C}-\Phi^{-1}\left(1-\xi\right)\right)\Pr\left(\tilde{C}-\Phi^{-1}\left(\xi\right)>u>\tilde{C}-\Phi^{-1}\left(1-\xi\right)|z_i,\tilde{C},y\right)+$$
$$+E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(\xi\right)<u\right)\Pr\left(\tilde{C}-\Phi^{-1}\left(\xi\right)<u|z_i,\tilde{C},y\right)+$$
$$+E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(1-\xi\right)>u\right)\Pr\left(\tilde{C}-\Phi^{-1}\left(1-\xi\right)>u|z_i,\tilde{C},y\right)$$

and

$$\Pr\left(z<z^*|x_i,\tilde{C},y\right)=$$
$$=\Pr\left(z<z^*|x_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(\xi\right)>u>\tilde{C}-\Phi^{-1}\left(1-\xi\right)\right)\Pr\left(\tilde{C}-\Phi^{-1}\left(\xi\right)>u>\tilde{C}-\Phi^{-1}\left(1-\xi\right)|x_i,\tilde{C},y\right)$$
$$+\Pr\left(z<z^*|x_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(\xi\right)<u\right)\Pr\left(\tilde{C}-\Phi^{-1}\left(\xi\right)<u|x_i,\tilde{C},y\right)+$$
$$+\Pr\left(z<z^*|x,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(1-\xi\right)>u\right)\Pr\left(\tilde{C}-\Phi^{-1}\left(1-\xi\right)>u|x_i,\tilde{C},y\right).$$

As

$$\begin{aligned}\lim_{\xi\to0}\Pr\left(\tilde{C}-\Phi^{-1}\left(\xi\right)<u|z_i,\tilde{C},y\right)&=\lim_{\xi\to0}\Pr\left(\tilde{C}-\Phi^{-1}\left(\xi\right)<u|x_i,\tilde{C},y\right)=0\\\lim_{\xi\to0}\Pr\left(\tilde{C}-\Phi^{-1}\left(1-\xi\right)>u|z_i,\tilde{C},y\right)&=\lim_{\xi\to0}\Pr\left(\tilde{C}-\Phi^{-1}\left(1-\xi\right)>u|x_i,\tilde{C},y\right)=0\end{aligned}$$

and

$$\begin{aligned}\lim_{\xi\to0}E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(\xi\right)>u>\tilde{C}-\Phi^{-1}\left(1-\xi\right)\right)&=E\left(\theta|z_i,q,y\right)\\\lim_{\xi\to0}\Pr\left(z<z^*|x_i,\tilde{C},y,\tilde{C}-\Phi^{-1}\left(\xi\right)>u>\tilde{C}-\Phi^{-1}\left(1-\xi\right)\right)&=\Pr\left(z<z^*|x_i,q,y\right),\end{aligned}$$

if $\xi\to0$, equations (39) and (38) converges to (32) and (30) respectively. Thus, for a sufficiently small ξ, the best response functions defined by (39) and (38) are arbitrary close to the straight lines in the (x^*,z^*) space which are given by (33) and (34). This proves the statement. ■

5 Conclusion

[to be completed]

References

[1] Adam, K., 2003. *Optimal monetary policy with imperfect common knowledge*, mimeo.

[2] Allen, F., Morris, S., Shin, H.S., 2004. *Beauty contests, bubbles and iterated expectations in asset markets*, Yale, mimeo.

[3] Amato, J. D.,Shin, H. S., 2003. *Public and private information in monetary policy models*, LSE, mimeo.

[4] Bacchetta, P., Wincoop, E. van, 2004. *Higher order expectations in asset pricing*, University of Lausanne, mimeo.

[5] Bamber, L. S., Barron, O. E., Stober, T. L. 1997. Trading volume and different aspects of disagreement coincident with earnings announcements, *Accounting Review*, 72(4), 575-597.

[6] Banerjee, S., Kaniel, R., Kremer, I., 2006. Momentum as an outcome of differences in higher-order beleifs, Stanford, mimeo.

[7] Brown, D. P., Jennings, R. H., 1989. On technical analysis, *Review of Financial Studies*, 2(4), 527-551.

[8] Brunnermeier, M., 2001. *Asset-pricing under asymmetric information - Bubbles, crashes, technical analysis and herding*, Oxford University Press.

[9] Diamond, D., Verrecchia, R. E., 1981. Information aggregation in a noisy rational expectations economy, *Journal of Financial Economics*, 9, 221-235.

[10] Evans, M. D. D., Lyons, R. K., 2001. *Why order flow explains exchange rates*, Berkeley, mimeo.

[11] Evans, M. D. D., Lyons, R. K., 2003. *How is macro news transmitted to exchange rates?*, Berkeley, mimeo.

[12] Fleming, M., Remolona, E., 1999. Price formation and liquidity in the US treasury market, *Journal of Finance*, 54, 1901-1915.

[13] Foster, D. F., Viswanathan, S., 1996. Strategic trading when agents forecast the forecasts of others, *Journal of Finance,* 51(4), 1437-1478.

[14] Grossman, S., Stiglitz, J., 1980. On the impossibility of informationally efficient markets, *American Economic Review*, 70, 393-408.

[15] Hamilton, J. D., 1994. *Time Series Analysis*, Princeton University Press, Princeton.

[16] Harris, M., Raviv, A., 1993. Differences of opinion make a horse race, *Review of Financial Studies*, 6, 473-506.

[17] He, H., Wang, J., 1995. Differential information and dynamic behavior of stock trading volume, *Review of Financial Studies*, 8(4), 919-972.

[18] Hellwig, C., 2002. *Public announcements, adjustment delays and the business cycle*, UCLA, mimeo.

[19] Hellwig, M. F., 1980. On the aggregation of information in competitive markets, *Journal of Economic Theory*, 22, 477-498.

[20] Jewitt, I., 2004. *Information acquisition and disclosure in auctions*, Oxford, mimeo.

[21] Kandel, E., Pearson, N. D., 1995. Differential interpretation of public signals and trade in speculative markets, *Journal of Political Economy*, 103(4), 831-872.

[22] Kim, O., Verrecchia, R. E., 1991. Trading volume and price reactions to public announcements, *Journal of Accounting Research*, 29(2), 302-321.

[23] Kim, O., Verrecchia, R. E., 1994. Liquidity and volume around earning announcements, *Journal of Accounting and Economics*, 17(1-2), January, 41-67.

[24] Kim, O., Verrecchia, R. E., 1997. Pre-announcement and event-period private information. *Journal of Accounting and Economics* 24 (3), 395-420

[25] Kondor, P. 2004. *Rational Trader Risk*, LSE, mimeo.

[26] Love, R., 2004. *First and second moment effects of macroeconomics news in high frequency foreign exchange data*, LSE, mimeo.

[27] Love, R., Payne, R., 2003. *Macroeconomic news, order flows, and exchange rates*, Discussion Paper 475, Financial Markets Group, London School of Economics.

[28] Lyons, R. K., 2001. *The Microstructure Approach to Exchange Rates*, MIT Press, Boston.

[29] Makarov, I., Rytchkov, O., 2006. *Forecasting the forecasts of others: Implications for asset pricing*, MIT, mimeo.

[30] Milgrom, P. R., 1981. Good news and bad news: Representation theorems and applications, *Bell Journal of Economics*, 12(2), 380-391.

[31] Morris, S., Shin, H. S., 1998. Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks, *American Economic Review*, 88, 587-597.

[32] Varian, H. R., 1989. Differences of opinion in financial markets in C. Stone (eds): *Financial risk: theory, evidence and implications, proceedings of the eleventh annual economic policy conference of the Federal Reserve Bank of St. Louis*, Kluwer, Boston pp. 3-37.

[33] Woodford, M., 2003. Imperfect common knowledge and the effects of monetary policy, in P. Aghion, R. Frydman, J. Stiglitz and M. Woodford (eds) *Knowledge, information and expectations in modern macroeconomics: In honor of Edmund S. Phelps*, Princeton University Press, Princeton, pp 25-58.

Appendix

[The appendix is under revision]

A.1 Demand in period 0

In period zero traders maximize the expected utility

$$E\left(-\exp\left(-\gamma\left(p_1-p_0\right)d_0^i-\frac{E\left(p_2|q_1,y,x_i,q_0\right)-p_1}{var\left(p_2|q_1,y,x_i,q_0\right)}\left(p_2-p_1\right)\right)|x_i,q_0\right)=$$
$$=E\left(E\left(-\exp\left(-\gamma\left(p_1-p_0\right)d_0^i-\frac{E\left(p_2|q_1,y,x_i,q_0\right)-p_1}{var\left(p_2|q_1,y,x_i,q_0\right)}\left(p_2-p_1\right)\right)q_0,y,x_i,q_1\right)|x_i,q_0\right)=$$
$$=E\left(-\exp\left(-\gamma\left(p_1-p_0\right)d_0^i-\frac{\left(E\left(p_2|q_1,y,x_i,q_0\right)-p_1\right)^2}{2var\left(p_2|q_1,y,x_j,q_0\right)}\right)|x_i,q_0\right)=$$
$$=E\left(-\exp\left(-\gamma\left(a_1q_1+c_1y+f_1q_0-p_0\right)d_0^i-\frac{1}{2}s\left(x_i-q_1\right)^2\right)|x_i,q_0\right)$$

where

$$s=\frac{a_s^2}{\left(var\left(\theta_s+\theta_w|q_1,y,x_j,q_0\right)+\frac{1}{\tau_2^2}\right)}=a_2\gamma\frac{\tau_1}{\delta_1}a_s.$$

If we write the expression in the inner bracket into matrix form and we use the standard result for the expectation of exponentials with quadratic forms[7], we get

$$E\left(-\exp\left(\begin{array}{c}-\frac{1}{2}sx_i^2-\gamma\left(f_1q_0-p_0\right)d_0^i+\left(\begin{array}{cc}-\gamma c_1d_0^i & \left(-\gamma a_1d_0^i+sx_i\right)\end{array}\right)\left(\begin{array}{c}y\\ q_1\end{array}\right)\\ -\left(\begin{array}{cc}y & q_1\end{array}\right)\left(\frac{1}{2}s\right)\left(\begin{array}{cc}0&0\\0&1\end{array}\right)\left(\begin{array}{c}y\\ q_1\end{array}\right)\end{array}\right)|x_i,q_0\right)=$$

[7] If c is constant scalar, L is a $nx1$ constant vector, N is an nxn constant matrix and M is an $nx1$ stochastic matrix and I is an information set, then

$$E\left(-\exp\left(c+L'M-M'NM'\right)|I\right)=$$
$$-\left|W\right|^{-1/2}\left|2N+W^{-1}\right|^{-1/2}\exp\left(c+L'Q-Q'NQ+\frac{1}{2}\left(L'-2Q'N\right)\right)\left(2N+W^{-1}\right)^{-1}\left(L-2NQ\right)\right)$$

where $Q=E\left(M|I\right)$ and $W=var\left(M|I\right)$ (see Brunnermeier, 2001, page 110).

$$
= -\exp\left(\begin{array}{c}
-\frac{1}{2}sx_i^2-\gamma\left(f_1q_0-p_0\right)d_0^i+\left(\begin{array}{cc}-\gamma c_1d_0^i & \left(-\gamma a_1d_0^i+sx_i\right)\end{array}\right)\left(\begin{array}{c}\mu_y\\ \mu_q\end{array}\right)+\\
\frac{1}{2}\left(\left(\begin{array}{cc}-\gamma c_1d_0^i & \left(-\gamma a_1d_0^i+sx_i\right)\end{array}\right)-2\left(\begin{array}{cc}\mu_y & \mu_q\end{array}\right)\left(\begin{array}{cc}0 & 0\\ 0 & \frac{1}{2}s\end{array}\right)\right)\\
\left(\left(\begin{array}{cc}0 & 0\\ 0 & s\end{array}\right)+\left(\begin{array}{cc}\sigma_y & \sigma_{yq}\\ \sigma_{yq} & \sigma_q\end{array}\right)^{-1}\right)^{-1}\left(\left(\begin{array}{c}-\gamma c_1d_0^i\\ -\gamma a_1d_0^i+sx_i\end{array}\right)-2\left(\begin{array}{cc}0 & 0\\ 0 & \frac{1}{2}s\end{array}\right)\left(\begin{array}{c}\mu_y\\ \mu_q\end{array}\right)\right)\\
-\left(\begin{array}{cc}\mu_y & \mu_q\end{array}\right)\left(\begin{array}{cc}0 & 0\\ 0 & \frac{1}{2}s\end{array}\right)\left(\begin{array}{c}\mu_y\\ \mu_q\end{array}\right)
\end{array}\right)
$$

where

$$
\begin{aligned}
\mu_q &= E\left(q_1|x_i,q_0\right)\\
\mu_y &= E\left(y|x_i,q_0\right).
\end{aligned}
$$

and

$$
\left(\begin{array}{cc}\sigma_y & \sigma_{yq}\\ \sigma_{yq} & \sigma_q\end{array}\right)=var\left(\left(\begin{array}{c}y\\ q_1\end{array}\right)|x_i,q_0\right).
$$

Maximizing the term in the bracket with respect to d_0^i gives the demand function

$$
d_0^i=\frac{\left(c_1\mu_y+\mu_qa_1+q_0f_1-p_0\right)\left(\sigma_qs+1\right)}{\gamma\left(c_1^2\sigma_y+c_1^2s\sigma_y\sigma_q-c_1^2s\sigma_{yq}^2+\sigma_qa_1^2+2c_1\sigma_{yq}a_1\right)}+\frac{s\left(x_i-\mu_q\right)\left(c_1\sigma_{yq}+a_1\sigma_q\right)}{\gamma\left(c_1^2\sigma_y+c_1^2s\sigma_y\sigma_q-c_1^2s\sigma_{yq}^2+\sigma_qa_1^2+2c_1\sigma_{yq}a_1\right)}.
$$

The second order condition of the maximization is

$$
\left(c_1^2\sigma_y+c_1^2s\sigma_y\sigma_q-c_1^2s\sigma_{yq}^2+\sigma_qa_1^2+2c_1\sigma_{yq}a_1\right)>0.
$$

A.2 Expectations, variances and coefficients in the price-functions

I give here the conditional expectations and variances obtained by standard results on normal variables (see, e.g., Brunnermeier, 2001, p12). I also give the equilibrium expressions for coefficients in the price function. The method to obtain the latter is described in the text.

$$
\begin{aligned}
\sigma_y &= \frac{2\kappa\omega\beta+\kappa^2\beta+\beta\alpha\omega+\beta\alpha\kappa+\beta\tau_0^2\omega+\beta\tau_0^2\kappa+\kappa\alpha\omega+\kappa^2\alpha+\kappa\tau_0^2\omega+\kappa^2\tau_0^2+\kappa^2\omega}{\left(\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega\right)\kappa\beta}\\
\sigma_q &= \frac{\tau_1^2\omega+\tau_1^2\kappa+\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega}{\left(\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega\right)\tau_1^2}\\
\sigma_{yq} &= \frac{\omega+\kappa}{\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega}\\
b_y &= \left(\omega+\kappa\right)\frac{\alpha}{\alpha\omega+\alpha\kappa+\tau_0^2\omega+\tau_0^2\kappa+\kappa\omega}\\
c_1 &= \left(b_2c_s+\bar{c}\right)\\
b_1 &= \left(b_2\left(b_s+e_s\right)+\bar{f}\right)
\end{aligned}
$$

where

$b_2 = \frac{\left(\alpha\omega\tau_0^2+\alpha\omega\tau_1^2+\kappa\omega\alpha+\kappa\alpha\tau_0^2+\kappa\alpha\tau_1^2+\alpha\kappa^2+\tau_0^2\omega\tau_2^2+\tau_1^2\omega\tau_2^2+\kappa\omega\tau_2^2+\kappa\tau_0^2\tau_2^2+\kappa\tau_1^2\tau_2^2+\tau_2^2\kappa^2\right)}{B}$

with

$$B = \left(\begin{array}{c} \alpha\omega\beta + \kappa^2\beta + \tau_1^2\kappa^2 + \tau_2^2\kappa^2 + \alpha\kappa^2 + \kappa^2\omega + \tau_0^2\kappa^2 + \alpha\kappa\beta + \kappa\omega\tau_1^2 + \tau_1^2\omega\tau_2^2 + \\ \kappa\omega\tau_2^2 + 2\kappa\tau_0^2\tau_2^2 + \tau_0^2\omega\tau_2^2 + 2\kappa\tau_1^2\tau_2^2 + \kappa\omega\tau_0^2 + \kappa\omega\alpha + \\ +\tau_0^2\kappa\beta + \tau_1^2\omega\beta + \tau_1^2\kappa\beta + \tau_0^2\omega\beta + \tau_2^2\beta\kappa + \tau_1^2\tau_2^2\beta + \tau_0^2\tau_2^2\beta + \tau_2^2\omega\beta + 2\kappa\omega\beta \\ +2\kappa\alpha\tau_0^2 + 2\kappa\alpha\tau_1^2 + \alpha\omega\tau_1^2 + \alpha\tau_1^2\beta + \alpha\omega\tau_0^2 + \alpha\tau_0^2\beta \end{array} \right)$$

and

$\bar{f} = \frac{1}{B}\tau_1^2\left(\alpha + \tau_2^2 + \kappa\right)(\omega + \kappa)$

$\bar{c} = \frac{\beta\left(\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa\right)}{B}$

$c_s = \frac{\beta\left(\kappa+\tau_1^2+\alpha+\tau_0^2\right)(\omega+\kappa)}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$e_s = \frac{\tau_1^2\kappa^2-\tau_1^2\omega\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$b_s = \frac{\alpha\kappa^2-\alpha\omega\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$var\left(\theta_s + \theta_w | x_i, q_1, q_0, y\right) =$

$= \frac{\tau_1^2\omega+2\tau_1^2\kappa+\omega\beta+\kappa^2+\kappa\omega+\alpha\omega+2\alpha\kappa+\tau_1^2\beta+\alpha\beta+2\tau_0^2\kappa+\tau_0^2\omega+\beta\kappa+\tau_0^2\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$

$var\left(\theta | z_j, q_2, q_1, q_0, y\right) = \frac{\left(\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa\right)}{B}$

A.3 Proof of existence

The equilibrium is given by the fixed point of the system

$$\begin{aligned} \tau_2 &= f_2\left(\tau_2, \tau_1, \tau_0\right) = \\ &= \delta_2 \frac{1}{a}\alpha \frac{\tau_0^2\omega+\tau_1^2\omega+\kappa\omega+\tau_0^2\kappa+\tau_1^2\kappa+\kappa^2}{\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa} \\ \tau_1 &= f_1\left(\tau_2, \tau_1, \tau_0\right) = \\ &= \delta_1 \tau_2^2 \alpha \frac{\kappa^2-\omega\beta}{a\left(\kappa\tau_0^2\tau_2^2+\kappa\omega\tau_2^2+\tau_2^2\kappa^2+\omega\tau_1^2\tau_2^2+2\alpha\omega\tau_2^2+\kappa\tau_1^2\tau_2^2+2\kappa\alpha\tau_2^2+\tau_0^2\omega\tau_2^2+\alpha\kappa^2+\kappa\omega\alpha\right)} \\ \tau_0 &= f_0\left(\tau_2, \tau_1, \tau_0\right) = \frac{\delta_0(\sigma_q s+1)(c_1+b_1)b_y+s(1-b_y)(c_1\sigma_{yq}+b_1\sigma_q)}{a\left(c_1^2\sigma_y+c_1^2 s\sigma_y\sigma_q-c_1^2 s\sigma_{yq}^2+\sigma_q b_1^2+2c_1\sigma_{yq}b_1\right)} \end{aligned}$$

I show the existence in three steps.

Lemma 1 *Let us fix $\tau_0 = \bar{\tau}_0$ at any arbitrary level. The system $\tau_2 = f_2\left(\tau_2, \tau_1, \bar{\tau}_0\right)$, $\tau_1 = f_1\left(\tau_2, \tau_1, \bar{\tau}_0\right)$ will have at least one fix point, (τ_1^*, τ_2^*). Additionally, $\tau_2^{\min} \leq \tau_2^* < \delta_2 \frac{1}{a}\alpha$ where*

$\tau_2^{\min}$ is the single root of $\delta_2 \frac{1}{a} \alpha \frac{\kappa\omega+\kappa^2}{\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2} = \tau_2$ *and* $\frac{(\kappa^2-\omega\beta)}{a2\kappa} < (>) \tau_1^* \leq (\geq) 0$ *if and only if* $\kappa^2 < (\geq)\omega\beta$*. Furthermore, let* $\tau_1^*(\tau_0)$ *and* $\tau_2^*(\tau_0)$ *are given as the fixed points with the smallest absolute value corresponding to* $\bar{\tau}_0 = \tau_0$*. Then these functions will be continuous.*

Proof. Notice first, that $\tau_1 = f_1(\tau_2, \tau_1, \bar{\tau}_0)$ determines a third degree polynom in τ_1, which is monotone increasing so it gives a single root for every τ_0 and τ_2. Similarly,$\tau_2 = f_2(\tau_2, \tau_1, \bar{\tau}_0)$ also determines a monotone increasing third degree polynom in τ_2 which gives a single unique root for every τ_0 and τ_1. It is also apparent that a marginal change in τ_0 or τ_2 in the first polynom or a marginal change in τ_0 or τ_1 in the second polynom will cause only a marginal change in the roots. This gives the continuity of $\tau_1^*(\tau_0)$ and $\tau_2^*(\tau_0)$.

For the existence, note that from the root of the polynom $\tau_1 = f_1(\tau_2, \tau_1, \bar{\tau}_0)$, $\tau_1^2(\tau_2^2)$ is a well defined continuous function. Therefore, the equilibrium is given by the fixed point of

$$\tau_2 = \frac{1}{a}\alpha\delta_2 g_2(\tau_2) = f_2\left(\tau_2, \tau_1^2\left(\tau_2^2\right), \bar{\tau}_0\right)$$

where $g(\cdot)$ continuously maps τ_2 to the unite interval. As

$$\lim_{\tau_2 \to 0} \frac{1}{a}\alpha\delta_2 g(\tau_2) = \delta_2 \frac{1}{a}\alpha \frac{\kappa\omega+\kappa^2}{\tau_1^2(0)\kappa+\kappa^2+\tau_1^2(0)\omega+\tau_0^2\kappa+\alpha\kappa+\tau_0^2\omega+\alpha\omega+2\kappa\omega} > 0,$$

where $\tau_1^2(0)$ is finite and

$$0 < \frac{1}{a}\alpha\delta_2 g(\tau_2) \leq \delta_2 \frac{1}{a}\alpha,$$

there has to be a fixed point. The rest of the lemma comes from simple observation. ■

Lemma 2 *The second order condition of the maximization problem in period 0 always holds, so the denominator of* $f_0(\tau_1, \tau_2, \tau_0)$

$$a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0$$

Proof. Note that the matrix

$$Q = \left(\begin{pmatrix} 0 & 0 \\ 0 & s \end{pmatrix} + \begin{pmatrix} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{pmatrix}^{-1}\right)^{-1}$$

is positive definite as $s > 0$. Consequently

$$0 < xQx^T$$

for all x. The lemma comes from the choice of

$$x = \left(\begin{pmatrix} -ac_1d_1 & (-ab_1d_1 + sx_i) \end{pmatrix} - 2 \begin{pmatrix} \mu_y & \mu_q \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}s \end{pmatrix} \right)$$

as then

$$0 < xQx^T = \begin{pmatrix} \frac{a^2\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}{\sigma_q s+1} d_1^2 - \\ -\frac{2ac_1\sigma_{yq}sx_i - 2\sigma_q ab_1\mu_q s - 2ac_1\sigma_{yq}\mu_q s + 2\sigma_q ab_1 sx_i}{\sigma_q s+1} d_1 \\ + \frac{s^2\sigma_q\left(x_i-\mu_q\right)^2}{\sigma_q s+1} \end{pmatrix},$$

which is possible for all d_1 only if

$$a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0.$$

■

Proposition 8 *An equilibrium exists.*

Proof. From Lemma 1, I have to show that the expression $\tau_0 = g_0(\tau_0) = f_0(\tau_2^*(\tau_0), \tau_1^*(\tau_0), \tau_0)$ has at least one fix point. I proceed in 4 steps.

1. Note, that $g_0(\tau_0) = g_0(-\tau_0)$ for all τ_0. It is so, because τ_0 enters as τ_0^2 to all building-blocks of $g_0(\tau_0)$.

2. I show that $\lim_{\tau_0\to\infty} g_0(\tau_0) = 0$. Let us check the building-blocks separately. As $\tau_0 \to 0$, τ_2^* goes to a constant, τ_1^* goes to 0 by the order of $\frac{1}{\tau_0^2}$, hence c_1, σ_y, s goes to constants, σ_{yq} and b_y goes to 0 by the order of $\frac{1}{\tau_0^2}$, σ_q goes to infinity by the order of τ_0^4 and b_1 goes to 0 by the order of $\frac{1}{\tau_0^2}$. So the numerator of $g_0(\tau_0)$, $(\sigma_q s + 1)(c_1 + b_1) b_y + s(1 - b_y)(c_1\sigma_{yq} + b_1\sigma_q)$, goes to infinity by the order of τ_0^2 from the speed of convergence of the term $\sigma_q s(c_1 b_y + b_1)$. The denominator, $a\left(c_1^2\sigma_y + 2c_1\sigma_{yq}b_1 + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2\right)$ also goes to infinity but by the order of τ_0^4 given by the term of $c_1^2 s\sigma_y\sigma_q$. Hence, $\lim_{\tau_0\to\infty} g_0(\tau_0) = 0$.

3. The function $g_0(\tau_0)$ is continuous. It comes by the positivity of the denominator, which holds because of Lemma 2, and the continuity of all building-blocks.

4. Hence, $g_0(\tau_0)$ will cross the 45° line necessarily, because it is symmetric, continuous and goes to 0 as τ_0 increases without bound. Therefore, there will be a fixed point with $\tau_0^* \geq (<)\, 0$ if $g_0(0) \geq (<)\, 0$.

■

A.4 Other proofs

Proof of Proposition 3. For $\frac{\tau_2}{\delta_2}, \frac{\tau_1}{\delta_1}, \frac{\tau_2^n}{\delta_2}$ and $\frac{\tau_1^n}{\delta_1}$, the result comes from the simple observation that the ordered limit

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f_2\left(\tau_2, \tau_1, \tau_0\right) = \frac{\alpha}{a}$$

and after substitution of $\tau_2 = \frac{\alpha}{a}$

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f_1\left(\tau_2, \tau_1, \tau_0\right) = \frac{\alpha}{a}.$$

The result for $\frac{\tau_0}{\delta_0}$ and $\frac{\tau_0^n}{\delta_0}$ can be obtained in a similar, but much more tedious way, if we take the limit of all the building-blocks of $f_0\left(\tau_2, \tau_1, \tau_0\right)$ separately. ■

Proof of Proposition 4. The second half of the statement implies the first half as both the aggregate holdings and the volume are continuous functions of ω. For the second half of the statement, it is sufficient to show that $\lim_{\omega \to \infty} \left| \frac{\tau_0^n}{\delta_0} - \frac{\tau_1^n}{\delta_1} \right| = \lim_{\omega \to \infty} \left| \frac{\tau_0^n}{\delta_0} \right| = \lim_{\omega \to \infty} \left| \frac{\tau_1^n}{\delta_1} \right| \to 0$ while $\left| \frac{\tau_0}{\delta_0} - \frac{\tau_1}{\delta_1} \right| \to C_1, \left| \frac{\tau_1}{\delta_1} \right| \to C_2$ and $\left| \frac{\tau_0}{\delta_0} \right| \to C_3$, where C_1, C_2 and C_3 are non zero constants. In the no-announcement case, the equilibrium is characterized by the following equations:

$$\begin{aligned} \tau_2^n &= f_2\left(\tau_2^n, \tau_1^n, \tau_0^n\right) = \\ &= \frac{\delta_2 \frac{1}{a} \alpha \left(\tau_0^{n2} \omega + \tau_1^{n2} \omega + \kappa\omega + \tau_0^{n2} \kappa + \tau_1^2 \kappa + \kappa^2\right)}{\tau_0^{n2}\kappa + \tau_0^{n2}\tau_2^2 + \tau_0^{n2}\omega + \alpha\tau_0^{n2} + \alpha\tau_1^{n2} + \alpha\omega + \kappa\alpha + \kappa^2 + 2\kappa\omega + \kappa\tau_2^{n2} + \omega\tau_2^{n2} + \tau_1^{n2}\tau_2^{n2} + \tau_1^{n2}\omega + \tau_1^{n2}\kappa} \end{aligned}$$

$$\begin{aligned} \tau_1^n &= f_1\left(\tau_2^n, \tau_1^n, \tau_0^n\right) = \\ &= \delta_1 \tau_2^2 \alpha \frac{\kappa^2}{a\left(\kappa\tau_0^{n2}\tau_2^2 + \kappa\omega\tau_2^2 + \tau_2^2\kappa^2 + \omega\tau_1^{n2}\tau_2^2 + 2\alpha\omega\tau_2^2 + \kappa\tau_1^{n2}\tau_2^2 + 2\kappa\alpha\tau_2^2 + \tau_0^{n2}\omega\tau_2^{n2} + \alpha\kappa^2 + \kappa\omega\alpha\right)} \end{aligned}$$

$$\tau_0^n = f_0\left(\tau_2^n, \tau_1^n, \tau_0^n\right) = \delta_0 \frac{b_y^n + \sigma_q^n s^n}{b_1^n a \sigma_q^n}.$$

It is apparent, that for any τ_1^n, τ_0^n, $\lim_{\omega \to \infty} \tau_2^n$ is a finite, positive constant, and for any finite, positive τ_2^n and any τ_0^n, $\lim_{\omega \to \infty} \tau_1^n = 0$. Consequently,

$$\lim_{\omega \to 0} \sigma_q^n = \lim_{\tau_1 \to 0} \sigma_q^n = \infty.$$

Hence,

$$\lim_{\omega \to \infty} \delta_0 \frac{b_y^n + \sigma_q^n s^n}{b_1^n a \sigma_q^n} = \lim_{\omega \to \infty} \frac{s^n}{b_1^n} = \lim_{\omega \to \infty} \frac{\frac{b_s^n}{\left(\sigma_s^n + \frac{1}{\tau_2^2}\right)}}{b_2^n \frac{\left(b_s^n + e_s^n\right)}{b_s^n} + \frac{\bar{f}}{b_s}} = 0$$

, which holds because $\lim_{\omega \to \infty} b_s^n = 0$, but $\lim_{\omega \to \infty} b_2^n, \lim_{\omega \to \infty} \sigma_s^n$ and $\lim_{\omega \to \infty} \frac{(b_s + e_s)}{b_s} = \lim_{\omega \to \infty} \frac{\tau_1^2 \kappa^2 + \alpha\kappa^2}{\alpha\kappa^2} = 1$ are non-zero constants.

In the announcement case, our equilibrium determining equations converge to the following expressions as $\omega \to \infty$:

$$
\begin{aligned}
\tau_2 &= \delta_2 \frac{1}{a} \alpha \frac{\left(\kappa + \tau_0^2 + \tau_1^2\right)}{\left(\tau_0^2 + 2\kappa + \tau_2^2 + \tau_1^2 + \alpha\right)} \\
\tau_1 &= -\delta_1 \tau_2^2 \alpha \frac{\beta}{a\left(\tau_1^2 \tau_2^2 + \kappa \tau_2^2 + \tau_0^2 \tau_2^2 + 2\alpha \tau_2^2 + \kappa \alpha\right)} \\
\tau_0 &= \frac{\delta_0 \left(\sigma_q s + 1\right)\left(c_1 + b_1\right) b_y + s\left(1 - b_y\right)\left(c_1 \sigma_{yq} + b_1 \sigma_q\right)}{a\left(c_1^2 \sigma_y + c_1^2 s \sigma_y \sigma_q - c_1^2 s \sigma_{yq}^2 + \sigma_q b_1^2 + 2 c_1 \sigma_{yq} b_1\right)},
\end{aligned}
$$

where the building-blocks of the last equations are all of the corresponding limiting functions. By the observation of expressions for the building-blocks, it is apparent that all of them are going to finite, non-zero constants as $\omega \to \infty$. Hence, just by the same reasoning as in the existence theorem, there must be at least one equilibrium where all τ_2, τ_1, τ_0 are finite and non-zero. If $\lim_{\omega \to \infty} \frac{\tau_1}{\delta_1}$ and $\lim_{\omega \to \infty} \frac{\tau_0}{\delta_1}$ is equal for certain combinations of the parameters, a small perturbation on the parameters (for example perturbing δ_0) would unambiguously make $\lim_{\omega \to \infty} \left| \frac{\tau_0}{\delta_0} - \frac{\tau_1}{\delta_1} \right| > 0$. ■