Stock market expectations and portfolio choice of American households

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Abstract

Using survey data on expectations and the composition of household savings, this paper aims at explaining the stockholding puzzle: the low stock market participation despite high historical performance of stocks. We estimate a joint model of stockholding and survey answers, both based on stock market expectations. The estimated level of risk tolerance that links subjective beliefs to stockholding is moderate, supporting to the validity of our measures of subjective expectations. Heterogeneity in expectations leads to heterogeneity in stockholding, and low average expectations, high uncertainty, and large heterogeneity in expectations explain much of the stockholder puzzle.

JEL Codes: D12, D8

1 Introduction[1]

Despite the superior historical performance of stocks over alternative bonds or bank accounts, many American households have been reluctant to hold stocks and stock-market based assets. This phenomenon is sometimes called the “stockholding puzzle” or the “stock market participation puzzle” (Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Campbell, 2006; Poterba, Rauh, Venti, and Wise, 2006). A related phenomenon is the "equity premium puzzle", the fact that high historical returns on stocks cannot be rationalized by choices of a representative consumer with "sensible" risk preferences (Mehra and Prescott, 1985, Kocherlakota, 1996). The behavior of American households is puzzling in light of historical returns. But actual expectations of decision-makers may be different from the

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estimates based on historical series. In fact, structural uncertainty may lead to posterior expectations that are very different from least squares estimates (Weitzman, 2007). The recent financial turmoil underlines the room for structural uncertainty.

In this paper, we relate the portfolios of households to their stock market expectations in a 55 to 65 year old sample of the Health and Retirement Study (HRS) of 2002. We jointly model portfolio choice and survey response behavior relating subjective beliefs about the probability distribution of stock returns to survey questions about probabilities. The relationship of expectations and household portfolio choice is kept simple in the Samuelson-Merton framework (Merton, 1969). The relationship of expectations and survey response to subjective probability questions is also modeled in a direct way. We estimate relevant heterogeneity in the location (mean) and uncertainty (variance) of one-year returns, and relate those to household portfolio choices.

The substantive contribution of our analysis is in showing that people's expectations of the performance of the stock market go a long way toward explaining the stockholding puzzle. On average, people expect the mean of future returns to be significantly lower and the variance higher than what historical returns would imply, and heterogeneity in expectations is a strong predictor of heterogeneity in stockholding. We also show that a general tendency to be optimistic is strongly related to optimism about stock returns and in turn increases stockholding. Optimism/pessimism is viewed as a fixed personality trait, and we proxy it, using data from earlier observations, by positive errors in forecasting sunshine, optimism about economic growth, and symptoms of clinical depression. Conversely, a general tendency to be uncertain about future events, proxied by the tendency to give fifty-fifty answers to other probability questions, is strongly related to uncertainty about stock market returns and in turn decreases stockholding. We find evidence about expectations tracking recent events on the stock market. The results also imply that a significant part of stockholding differences among demographic groups is explained by differences in expectations. Lower stockholding by single women, single men, African Americans, lower educated and those with lower cognitive capacity is in a significant part due to their lower and often more uncertain expectations.

Our methodological contribution is in developing a measurement model for joint estimation of the effect of expectations on portfolio choice on the one hand and survey answers on the other hand. The model is consistent with survey response being a result of individual behavior under circumstances that differ from circumstances when making an actual investment decision. The model is shown to be consistent with documented features of measurement error. The results validate the use of survey measures of expectations formulated as probability questions. Once survey noise is properly accounted for, these measures provide substantial and potentially unbiased information about expectations relevant for economic behavior. Validation is achieved by showing that the level of risk tolerance that links subjective beliefs to stockholding is moderate.

Our approach to household investment decisions, is that of demand analysis, as is usual in household finance research (Campbell, 2006). While equilibrium issues are not addressed, the results have implications for prices and turnover as well. High uncertainty on average and substantial heterogeneity in uncertainty support Weitzman's (2007) argument for the indefinite nature of the posterior variance of stock-market returns, and its potential role in explaining the equity premium. Moreover, the substantial heterogeneity of expectations is in line with the basic argument of many disagreement models (Hong and Stein, 2007) that postulate that heterogeneity in beliefs may be essential for trade.

Most of the empirical literature on the stockholding puzzle has focused on reduced-form evidence on the effects of wealth and education (Guiso, Haliassos, and Jappelli, 2002; Ameriks and Zeldes, 2004), cognitive capacity (Christelis, Jappelli and Padula, 2006), health (Rosen and Wu, 2003), or social interactions (Guiso, Sapienza and Zingales, 2004; Hong, Kubik and Stein, 2004). The causal channels have usually been interpreted as some kind of transaction costs (Vissing-Jorgensen, 2004). While transaction costs are outside standard theoretical models, the role of expectations in stockholding is a central element of them. Our results imply that a significant part of the reduced-form correlations operate through expectations.

Investigating survey measures of subjective probabilities is a relatively new line of research (see Manski, 2004). Within this literature, stock market expectations are analyzed by Dominitz and Manski (2006) and Winter et. al. (2006). Our approach differs from theirs in two ways. First, we connect expectations to investment behavior in a structural way. Second, we directly address the noise of measured subjective probabilities (in a spirit close to Hill, Perry and Willis, 2006). These two contributions together enable us to provide validation of survey measures of expectations as important and, if properly treated, potentially unbiased measures of relevant heterogeneity.

2 Data

We use data from the Health and Retirement Study (HRS, see e.g. Juster and Suzman, 1995). HRS has had a number of probability questions from 1992 on. It added questions on stock price expectations in the 2002 wave. HRS is representative of American population 50 years of age or older, and their households. Besides subjective probabilities, HRS collects data on the amount and structure of savings, including 401(k) accounts, a rich set of demographic variables, and measures of cognitive functioning.

In order to focus on households that are nearing the end of the wealth accumulation phase of the life cycle but have not started decumulating their wealth yet, we restricted our sample to the younger part of the survey. We kept people who were parts of the original HRS and War Babies study cohorts and were 55 to 65 years old in 2002.

While expectations are defined at the individual level, saving behavior is at the household level. HRS respondents are either couples or individuals without spouses (expectations of other possible members of the households are not elicited). In order to focus on our main question, and because of these data limitations, we simplify the problem of how households make financial decisions by picking one member per household. The one person we pick is the designated "financial respondent" of the household, the person who is the most knowledgeable about the savings and assets of the household, and who is therefore selected by the household to answer the asset questions. We also drop households with missing savings information in either 2002 or 2004 (less than 2 per cent of households). The sample for our main analysis consists of 3715 individuals, each representing a household.

Households are asked whether they have investments in stocks or mutual funds. If “yes,” we call these people “direct stockholders”. HRS also asks about retirement accounts and their composition (the latter in a very simplified way). Persons who are not direct stockholders, but are in households with some stocks or mutual fund investments in retirement accounts are called “indirect stockholders.” Stockholding is low (see Table 1). 56 per cent of the households with financial respondents between 55 and 65 years of age were neither direct nor indirect stockholders in 2002. The table also shows that stockholding status is relatively stable over time. Similarly to other papers on stock market participation, the likelihood of being a stockholder is monotonically increasing in financial wealth in the sample, and so is the fraction of stock-market based assets for stockholders (evidence is presented in Appendix Tables 1 and 2).

In order to analyze the role of households’ expectations in their savings behavior, one needs to measure statistics that are sufficient for making the portfolio choice decision. In order to keep things simple and focus on the first two moments of returns expectations, we assume that people believe that yearly returns are i.i.d. and normally distributed. These assumptions are close to what we see in historical data. They imply that the mean and the standard deviation are sufficient statistics for the returns distribution. Moreover, as we shall see, it is exactly those statistics that are needed in simple portfolio choice theory (see later for more details).

The historical density of nominal yearly log returns on the Dow Jones Industrial Average between 1946 and 2002 is depicted by the histogram in Figure 1. (We used closing prices in the first trading day of June each year as June is the modal survey response month in HRS 2002.) The distribution is very close to normal and passes tests for independence; the mean is $\mu = 0.07$, and standard deviation is $\sigma = 0.15$. Series with different time spans yield somewhat different moments: including pre-war returns from 1929 yields $\mu = 0.05$, $\sigma = 0.24$; the last twenty years before the 2002 interview give $\mu = 0.12$, $\sigma = 0.15$; while the last five years give $\mu = 0.04$, $\sigma = 0.14$. The expectations of a person whose beliefs are based on the historical record may be visualized by the density in Figure 1. Although not necessary for the analysis, it may sometimes help to assume that people represent those expectations by a mental image of the density function.

Although the mean and the standard deviation are sufficient statistics under the maintained assumption of i.i.d. normality, they are not straightforward to elicit in surveys. While it certainly makes sense to ask about expected returns, the same is not true for the standard deviation: most people don’t know what a standard deviation is, let alone have the ability to estimate it. Asking for specific probabilities is therefore a more promising alternative.

The survey we use asks two such probabilities: the probability that returns will be positive, and the probability that they will be larger than 10 per cent. If, based on the density in their mind, respondents can calculate the appropriate probabilities, two probabilities exactly identify the distribution under the normality assumption. The corresponding probability of positive returns (p_0) is 68 per cent, and the probability of returns at least 10% (p_{10}) is 42 per cent.

Identifying the mean and standard deviation from two probabilities is relatively straightforward, as we show later. Intuitively, the level of the probabilities (e.g. the average of the two) is informative about the expected value: the higher the probabilities, the higher the expectations. At the same time, the difference between the probabilities is informative about the spread of the distribution: the higher the difference the smaller the standard deviation (the steeper the c.d.f., or alternatively, the more probability mass is concentrated on the same support segment of the p.d.f.).

The main questionnaire of HRS 2002 contained two questions about the respondents' expectations of future performance of the U.S. stock market. One (p_0) asked what the respondent thought the probability is that the market will go up at all, and another one (p_{10}) about the probability that it will go up by at least 10 per cent. The questions themselves were phrased the following way.

> We are interested in how well you think the economy will do in the next year.
>
> p_0 question: By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?
>
> p_{10} question: By next year at this time, what is the chance they will have grown by 10 percent or more?

Of the 3642 respondents in the sample, 3004 (82%) answered both the p_0 and the p_{10} question.[2] Of the 18 per cent whose answers are missing, the vast majority come from "I don't know" answers as opposed to refusals, and most people who said "I don't know" to one of the questions said the same to the other.[3] It seems, therefore, that the missing answers reflect genuine ignorance. Missing answers to other probability questions in the HRS are much less frequent, but they vary somewhat with the "difficulty" of the question (1 per cent for whether tomorrow will be a sunny day, and 4 per cent for whether the respondent's

[2] One half of the respondents were asked the two probability questions in a different order: first the p_{10}, then the p_0 question. Respondents were randomly assigned to the two sequences. The purpose of the different ordering was to explore the potential effects of anchoring. The distributions of the two answers are slightly different (answers to the question that is asked first tend to have a somewhat lower mean and a smaller standard deviation), but the differences are very small and often statistically insignificant. In what follows, we shall ignore the ordering of the questions.

[3] If one gave a non-valid answer to the first question, the second one was skipped. But of those who gave a 0 to 100 per cent probability answer to the first question, very few said "I don't know" on the second question. The rate is the same for those who got the p_0 question first and those who got the p_{10} question first.

income will keep up with inflation or whether one would live to be a given age). It seems that, for many respondents, the stock market questions are just too difficult to answer. We shall further explore the nature of the missing answers later.

Besides the questions in the core questionnaire, HRS 2002 asked other stock market expectation questions from about 5 per cent of its respondents in an experimental module added to the core questionnaire. The module questions included the same questions as the core questionnaire, and some more. Typically, the experimental module questions followed the core stock market probability questions by 20-30 minutes and a hundred questions.

3 Information and measurement problems in reported subjective probabilities

Survey responses to subjective probability questions have been shown to contain a significant amount of information in general (see, for example, Hurd and McGarry, 1995; Hurd and Smith, 2002; Van der Klaauw and Wolpin, 2002; Dominitz and Manski, 2004, Finkelstein and McGarry, 2006). Table 2 shows mean responses to the core stock market expectation questions in HRS 2002, 2004 and 2006, by stock ownership. The p_0 question was asked in all three years while the p_{10} question was specific to 2002. The first thing to notice is the very low mean answers: a 49 per cent chance of positive returns (p_0), and 39 per cent chance of at least 10% returns (p_{10}) in 2002. Mean p_0 is somewhat higher in 2004 and 2006 but barely above fifty per cent. Recall that historical series imply $p_0 = 68$ per cent and $p_{10} = 49$ per cent.[4] The second observation is that, on average, reported probabilities of gains are significantly higher for stockholders (both direct and indirect) in all years. Reported p_0 probabilities are also above fifty per cent. At the same time, reported probabilities of non-holders are not only lower but are always below fifty per cent. The third observation is that the average distance between p_0 and p_{10} in 2002 is larger for stockholders (both direct and indirect). Since the distance between the two probability answers is a measure of the dispersion of the distribution (a larger distance is related to a smaller dispersion because more probability mass is concentrated on the same support segment), the larger distance among stockholders indicates smaller subjective variance. These results suggest that reported probabilities are meaningful predictors of investment behavior.

Year 2002 saw dramatic drops in stock prices (as well as some bouncing back). If expectations reflect recent movements of stock prices while stockholding has more inertia, one may worry that our results are driven by unusually low expectations in 2002. It is therefore important to note that cross-year variation in reported probabilities is modest.[5] There

[4] The use of nominal returns for comparison serves an illustrative purpose. For investment behavior, real after tax returns are the relevant concept. The wording of the survey question ("will be worth more") may also be interpreted as real after tax returns. Those, however, implicitly depend on expectations of nominal returns, inflationary expectations and important tax considerations. In this section, we ignore those issues.

[5] Our structural estimates confirm these results: latent relevant expectations track recent changes to a

is remarkable stability in stock market expectations among stockholders (both direct and indirect), and the overall variation is driven by answers of non-holders.

While reported probabilities seem to contain important information about relevant expectations, they exhibit serious problems. The most important problems are focal and other round answers, apparent violations of the law of probability, and test-retest noise, measured by the propensity to give a different answer to the same question if asked twice. We also revisit the problem of missing probability answers. We present evidence on the extent of the problems and explore evidence on their nature: whether they reflect problems in relevant expectations or measurement errors due to the survey situation. The latter question is explored by estimating the relationship of each phenomenon to stockholding and other covariates.

Figure 2 shows the histogram (empirical density) of each of the four probability answers (two from the core and two from the experimental module questionnaire). The pictures are typical for survey probability answers; see Manski (2004) for examples. Virtually all answers are at some round numbers, including 0 and 100 per cent. Focal values at 50 per cent account for an especially large part of all answers. In the American context, the answer "fifty-fifty" to such a probability question may be interpreted as a synonym for "I don't know." At the same time, 50 per cent is a frequent response to probability questions in Europe as well (Hurd, Rohwedder and Winter, 2005). The model of survey response that we explore in this paper will exhibit the feature that large uncertainty leads to an answer at 50 per cent.

Table 3 documents that quite a few respondents give answers that apparently violate the laws of probability. 14% of the respondents give a larger probability answer to the p_{10} question than the p_0 question, whereas the former set of events is a proper subset of the latter. Yet another 43% give the same answer to the two questions, implying a zero density between the no change and 10% increase. Under the normality assumption, this would imply an infinite variance. It turns out that there is no clear relationship between the propensity to give focal or other round answers and the propensity to give zero mass or negative mass answers.

Table 4 looks at differences between the core and experimental module questions. Recall that the experimental module included the same stock market probability questions as the core questionnaire. The repeated questions give us a unique opportunity to analyze differences in answers given to identical question within the same survey, typically 20-30 minutes and a hundred questions apart from each other. Such an exercise is sometimes called a test-retest analysis. The results show that the overall distribution of the module answers ($p_{0'}$ and $p_{10'}$) are very similar to those in the core survey (p_0 and p_{10}, respectively). In what follows, we are going to treat the core and module answers as drawn from the same distribution.

While the distributions are close, there are significant differences in the individual answers. Of the 179 respondents in the estimation sample who answered all four questions,

modest extent. (The effect will be identified from within-2002 variation of stock prices and interview dates.)

only 50 (28 per cent) gave the same answer to p_0 and $p_{0'}$, 33 (18 per cent) to p_{10} and $p_{10'}$, and only 19 (11 per cent) gave the same answer to both. The correlation of the core and module answers is low, $Corr\left(p_0, p_{0'}\right) = 0.49$ and $Corr\left(p_{10}, p_{10'}\right) = 0.37$.

Table 5 shows how stock ownership is related to the different kinds of survey noise. The estimates in the table are from regressions with each measurement problem on the left-hand side and broad stockholder status on the right-hand side. For most measurement problems these are average partial effects from probit models, while for the absolute value of the test-retest difference these are OLS coefficients. Estimates from two models are presented: the models of the first type include dummies for the interval of the p_0 answers, while the models of the second type contain demographic covariates well (see the notes to the table for the specific variables). The full table with coefficient estimates can be found in Appendix Table 3, together with estimates without any covariates. Dummies for the p_0 intervals provide a crude control for the level of expectations. Many of the measurement problems are correlated with the level of expectations for mechanical reasons: e.g. answering $p_0 = 0$ is related to low expectations, or high p_0 answers give less room for answering $p_{10} > p_0$. A failure to control for the level of expectations would cause us attribute the estimated coefficient on stockholding to a relationship to measurement problem itself while it may be that the relationship is to the level of expectations. Note that controlling for the p_0 intervals means restricting the sample in the case of the focal (0, 50, 100) answers to the neighboring intervals.

Missing answers are strongly negatively related to stockholding. 6 per cent of stockholders give missing answers to the stock market expectation questions compared to 26 per cent of non-stockholders. Women, singles and racial minorities are more likely to give missing answers, while such answers are negatively related to education and cognitive capacity.

At the same time, the rest of the measurement problems do not seem to be related to stockholding once the level of expectations is controlled. A closer inspection of the tables in Appendix Table 3 reveals that demographic characteristics are also at most very weakly related to the measurement problems, race being the only potential exception (Hispanics are more likely to give $p_0 < p_{10}$ while test-retest noise is smaller for African Americans). Remarkably, the absolute test-retest difference is not related to stockholding even in raw correlations, and there is no apparent relationship to education or the cognitive measures (of which memory is an important element, see later).

The results imply that nonresponse is strongly related to relevant heterogeneity in stock market expectations in a way that is consistent with genuine ignorance. At the same time, all of the other measurement problems seem specific to the survey situation. The strongest evidence for pure survey error is the result on large but independent test-retest noise. The presence of pure survey error precludes the identification of relevant expectations at the individual level but opens the possibility to identify their conditional moments. Measurement will be based on a structural econometric model derived from a model of investment behavior and a model of survey response.

4 Investment behavior

Our model of investment behavior model is the simple and very intuitive model of Merton (1969). Besides its simplicity it is logically consistent with our maintained assumption of i.i.d. normal returns. Consider an individual who saves for retirement. For simplicity, assume that at time 0 she has wealth W_0 to invest and she wants to maximize the expected utility of W_T, her wealth when she retires at some predetermined time T. Assume that the only thing she cares about is her wealth at retirement (W_T), and that she has a conventional constant relative risk aversion (CRRA) power utility function over W_T with parameter of relative risk aversion $1/\alpha$ (α denoting the coefficient of risk tolerance):

$$\max_{s_t} E_t \frac{W_T^{1-1/\alpha}}{1-1/\alpha}. \tag{1}$$

The investor can choose between investing in two assets: a risk-free asset with known rate of return r ("bank account") and one risky asset ("stocks") with an uncertain return. The instantaneous rate of return of the risky asset, denoted by dS/S, is assumed to follow a Brownian motion. The investment decision consists of choosing an optimal fraction of wealth invested into the risky asset for each time t between 0 and T, which we denote by s_t^*.

The equation of motion for the instantaneous return to the risky asset is given by

$$\frac{dS}{S} = \mu\ dt + \sigma\ dz \tag{2}$$

where dz is the increment to a standard Wiener process. This is a continuous time version of a random walk with drift, where the drift is μ and the variance is σ^2 (both normalized to the unit time-interval). Throughout the analysis we assume that the investor knows the random walk nature of the process and that its parameters are constant. The well-known solution to this problem is a constant fraction of wealth invested into the risky asset

$$s_t^* = s^* = \alpha \frac{\mu - r}{\sigma^2}. \tag{3}$$

The optimal share invested into stocks is increasing in its mean return, decreasing in the return of the risk-free asset, and decreasing in the variance and the degree of risk aversion. $(\mu - r)/\sigma^2$ is also known as the Sharpe ratio. The Merton model's implication is that the optimal share of the risky asset is proportional to the Sharpe ratio, and the proportionality coefficient is the parameter of risk tolerance, the inverse of risk aversion. This model is extremely simplistic and implies behavior that is not supported by the data (such as frequent rebalancing). At the same time, the investment rule in (3) is very intuitive and can be thought of as a parsimonious way of relating higher (lower) expectations or lower (higher) uncertainty to the desire to have a larger (smaller) share of stocks in savings. Moreover, Poterba, Rauh, Venti and Wise (2005) show that the constant share investment rule performs surprisingly well against alternative, more sophisticated ones in a more realistic, simulation-based analysis.

An important observation is that as a model of demand for the risky asset, there is nothing in this model that requires μ and σ^2 to take any specific value, such as their historical estimates. Heterogeneous beliefs in the parameters are also consistent with the model with each individual having beliefs of μ_i and σ_i. Those who believe $\mu_i < r$ will hold a zero fraction of their savings in stocks (assuming no short sales), and those who believe σ_i is larger will hold less stocks. In this paper, we focus on heterogeneity in expectations: μ_i and σ_i. We always keep r constant, but in some of our estimates we make use of heterogeneity in α estimated by Kimball, Sahm and Shapiro (2007).

Some more notation will be helpful for the next section. Let R_i denote annual returns on the stock market as perceived by individual i. If beliefs of individual i are identical to the historical record, the distribution is like the histogram in Figure 1. We maintain the assumption that individual i believes that yearly returns are *i.i.d.* normally distributed random variables. Let $R_{i(t+1)}$ denote the return for next year $(t+1)$ as viewed by individual i at time t. We model this by writing

$$\begin{aligned} R_{i(t+1)} &= \mu_{it} + \eta_{it} \\ \eta_{it}|\mu_{it} &\sim N\left(0, \sigma_{it}^2\right) \end{aligned} \tag{4}$$

where μ_{it} is individual i's subjective expected value at time t, and η_{it} is the way she perceives, at time t, possible deviations from the expected value. She has subjective moments $E\left(R_{i(t+1)}\right) = \mu_{it}$ and $V\left(R_{i(t+1)}|\mu_{it}\right) = \sigma_{it}^2$: at time t, individual i perceives next year's returns as a random variable with mean μ_{it} and standard deviation σ_{it}. This is an atheoretical way of representing individual i's expectations, one that puts no restrictions on either μ or σ. In what follows, we will refer to heterogeneity in R as *relevant heterogeneity.*

5 Survey response behavior

When the individual is approached by an interviewer and confronted with a question about her fundamental expectations, we assume that her answer is based on a possibly different (but hopefully related) object $\tilde{R}_{i(t+1)j}$ (where $j = 0, 10, 0', 10'$ denotes the particular question on the survey). The idea here is that of survey noise. We assume that questions on the same survey are related to the same t. Sometimes we shall refer to $\tilde{R}$ as *noisy expectations.* The goal of the paper is to establish and estimate the relationship between measured heterogeneity and relevant heterogeneity. In other words, we seek to estimate the relationship between $\tilde{R}_{i(t+1)j}$ and $R_{i(t+1)}$.

We are interested in expectations that would be relevant in an investment situation. At the same time, the survey situation is very different from an investment situation. There is considerably less time allowed, and there are practically no incentives to get the answers right. We assume that when confronted with probability question j on the survey, individual

i retrieves (or constructs) a noisy version of the fundamental random variable:

$$\tilde{R}_{i(t+1)j} = \mu_{it} + \eta_{it} + v_{itj}, \tag{5}$$

where

$$v_{itj} | (\mu_{it}, \eta_{it}) \sim N\left(0, \sigma_v^2\right).$$

This noise is classical measurement error in the sense that it is independent of everything. It is also additive to μ but it will not be additive to the measured subjective probability variables. We assume that relevant expectations do not change. That is, within a survey $\mu_{it} = \mu_i$ and $\sigma_{it} = \sigma_i$. t therefore refers to the date of the survey (as opposed to the time of the question within the survey). To simplify notation, we omit the t subscript from now on.

We assume that the answer to each probability question j is based on a possibly different draw of v and thus a different noisy expectation, $\tilde{R}_{i(t+1)j}$. Noise components in the in core and module answers are assumed to be independent. At the same time, it would make sense for adjacent answers (i.e., p_0 and p_{10}, or $p_{0'}$ and $p_{10'}$) to be affected by the same noise (the same draw of v). However, the evidence of negative probability mass answers indicates that some people respond as if they forgot their previous answer. We assume that it is purely due to lack of attention on the survey, rather than the inability to think in terms of probabilities. Technically, we assume that for each question j, there is a new draw of survey noise v_{itj}, such that $Corr\left(v_0, v_{0'}\right) = Corr\left(v_{10}, v_{10'}\right) = 0$, but $Corr\left(v_0, v_{10}\right) = Corr\left(v_{0'}, v_{10'}\right) = \rho_v$.

A key assumption in our model is that the noise component v has the same variance across people while the fundamental density is larger among people who are truly more uncertain about future returns in the stock market. This assumption is consistent with evidence presented earlier that test-retest differences between answers to the probability questions in the HRS 2002 core and experimental module, which we attribute to survey noise, are unrelated to observable characteristics of respondents.

In the survey people are asked to answer probability questions. Throughout this paper, we assume that whatever way people form that answer, it is based on the noisy expectations $\tilde{R}$ introduced above. A helpful story may be that people first form a mental image of the density function of $\tilde{R}$ in their head, and then they try to calculate the probability in question. The benchmark answer to each probability question j is the proper integral, which we shall call the precise probability and is denoted by p_j^* :

$$p_{ij}^* = \Pr\left(\tilde{R}_{ij} > \tau_j | \mu_i, v_{ij}\right) = \Pr\left(\frac{\eta_i}{\sigma_i} > \frac{\tau_j - \mu_i - v_{ij}}{\sigma_i}\right) = \Phi\left(\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i}\right) \tag{6}$$

so that $p_{i0}^* = \Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right)$ and $p_{i10}^* = \Phi\left(\frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right)$. The precise probability p_{ij}^* maps the individual mean (μ_i), the individual variance (σ_i) and the noise draw at the given probability question (v_{ij}) to a proper probability (τ_j is fixed by the probability question). The higher the individual mean or the noise draw, the higher the precise probability.

An important feature of the precise probability is that a mean-preserving spread in fundamental uncertainty (σ_i) pushes it towards 0.5. Mechanically, this is because given μ_i and

v_{ij}, an increase in σ_i moves the index towards zero, for any value of τ_j. One consequence of this phenomenon is that in a population that is heterogenous in μ_i and has high enough uncertainty σ_i, the average precise probability would be biased towards 0.5 relative to the precise probability based on the average μ. As fundamental uncertainty approaches infinity $(\sigma_i \to \infty)$, the index approaches zero, making the precise probability 0.5 $\left(p^*_{ij} \to 0.5\right)$. This is very much in line with the casual interpretation of a "fifty-fifty" answer reflecting ignorance. Infinitely large uncertainty can be interpreted as ignorance (returns can be anything with approximately equal likelihood), for example as an uninformative prior that was not sharpened by learning. Therefore, a person who is completely ignorant about the stock market would answer fifty per cent for both p_0 and p_{10} if she were to give the precise probability for an answer.

Assuming that people give exactly the precise probability as an answer is problematic for two reasons. First, in he spirit of our survey response models, it would not be rational for a respondent to put in the effort necessary for the calculation. Calculating probabilities is a difficult task, no matter what density one has in mind. Second, while high uncertainty may explain the large fraction of 50-50 answers, the prevalence of 0, 100, and other rounded answers is incompatible with answers reflecting the precise probability itself.

A more realistic (and perhaps more rational) model would assume that respondents make a guess of what that probability could be. In this paper we do not model that "guessing" process but simply allow for actual answers to be round numbers (or, in fact any other integers) close to the appropriate precise probabilities. In particular, we assume that an answer within a pre-specified interval can correspond to any precise probability within that interval. The admissible intervals are exogenously given and are the same for everyone. Formally, if the reported probability (p_{ij}) is in a pre-specified interval or 'bin' $\left[\underline{b}, \bar{b}\right]$, then the precise probability $\left(p^*_{ij}\right)$ implied by the parameters of the (noisy) density is also in this interval. In formulae:

$$p_{ij} \in \left[\underline{b}, \bar{b}\right] \Leftrightarrow p^*_{ij} \in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i}\right) < \bar{b} \tag{7}$$

When we implement the model, the bins will be defined (in percentage terms) as $[0, 5)$, $[5, 15)$, $[15, 25)$,..., $[95, 100]$. This way the bins allow for rounding to the nearest ten, and treat all other numbers not round (including 25 and 75 per cent).

For this model, rounding to 0 or 100 is no different from rounding to, say, 10 or 90. Rounding to 50 has a special role, not because of rounding itself (that is, again, assumed to be governed by the same mechanism), but because increasing fundamental uncertainty pushes precise probabilities $\left(p^*_{ij}\right)$ towards 0.5, that is inside the $[45, 55)$ interval. For a probability questions characterized by τ_j, and for a given mean μ_i and noise draw v_{ij}, there is always a large enough fundamental uncertainty σ_i that leads to an answer in the $[45, 55)$ interval.

This model is admittedly atheoretical: it is more of a statistical as opposed to an economic model of survey response behavior. The length and location of the intervals are exogenous, and the model is silent about why some people round while others don't. In its atheoretical way, however, the model is compatible with the difficulty of calculating the integral, and it allows people to pick the round number they feel closest to where the precise probability should be. It also allows for people to report an erroneous not-round probability as long as it belongs to the same interval as the precise probability. Since no information about rounding is used, this model is consistent with the fact that giving round answers is only weakly related to stockholding and demographics. And, last but not least, this is probably the simplest model that can deliver those results.[6]

18 per cent of our sample of financial respondents did not answer the stock market probability questions but answered "don't know" instead. Correlations with observables (including stockholding) support the hypothesis that most of these "don't know" answers reflect genuine ignorance about the stock market. A straightforward way to model complete ignorance in our framework is to assume that people give missing answers if their relevant uncertainty is prohibitively high, i.e.

$$\sigma_i \to \infty$$

Such expectations automatically result in $s^* \to 0$. Recall that 6 per cent of stockholders gave missing answers to the stock market expectation questions compared to 26 per cent of non stockholders. In the remainder of the analysis we ignore people with missing stock market probability answers.

6 Estimation

This is an empirical paper, with several objectives. In logical order, the first objective is methodological. We would like to show that answers to the probability questions considered here can be used to extract useful information about relevant heterogeneity in stock-market expectations after survey noise is properly accounted for. This can be thought of as a validation exercise: validity of survey measures is provided by their relationship with observed stockholding.

We test the validity of the extracted information against observed stockholding by establishing that the Sharpe ratio estimated from survey measures of expectations is a good predictor of observed stockholding. Besides sign and statistical significance, the magnitude

[6]The modal response hypothesis put forward by Hill, Perry and Willis (2007) is less atheoretical but more complicated alternative. In their setup, uncertainty about the underlying variable (here σ_i) induces a distribution of potential probability answers given noisy expectations (here $\tilde{R}$), and people are assumed to give the mode of that distribution for an asnwer (therefore the *modal response* hypothesis). As uncertainty grows, the distribution of potential probability answers becomes skewed so that the mode shifts towards zero or hundred percent. At extreme uncertatinty, however, the distribution becomes virtually bimodal with two "arms" at zero and hundred percent. Hill, Perry and Willis hypothetize that in such cases people give 50 per cent answers. Recall that in our setup, increasing uncertainty shifts answers towards 50%. In the modal response hypothesis, the shift is first towards zero or hundred percent and only then to 50%.

of the coefficient is also informative: it is estimated risk tolerance. Expectation measures can therefore be validated in the spirit of Mehra and Prescott (1985). The magnitude of the estimated risk tolerance tells whether, on average, the risk tolerance that connects people's stockholding to their actual expectations is sensible. In an extension we allow for heterogeneity in risk tolerance and estimate it as a coefficient multiplied by the risk tolerance proxy constructed by Kimball, Sahm and Shapiro (2007). The proxy is estimated using answers to hypothetical gambles in the HRS from 1992 through 2002. The estimated coefficient tells to what extent the proxy strikes a balance between people's stock market expectations and their stockholding.

One substantive goal of the analysis is to estimate moments of the distribution of relevant (as opposed to measured) stock-market expectations in the population represented by our sample. We estimate the expected value of both μ_i and σ_i as well as the standard deviation of μ_i. We do not estimate the heterogeneity in σ_i as we cannot estimate the variance of unobserved heterogeneity in σ in a robust way, as we discuss later. Besides unconditional moments, we are interested in the variation of expectations with observable individual characteristics. The other substantive goal, of course, is to establish the extent to which relevant heterogeneity in expectations causes heterogeneity in stockholding. This latter relationship is estimated by the same parameter that validates the expectation measures: we need to show that the Sharpe ratio does vary with stockholding in a statistically significant way, and its coefficient, estimated risk tolerance, is in the sensible range.

The objectives listed above can be achieved by estimating a structural model of stockholding and answers to probability questions, with stock-market expectations being latent variables behind both. We estimate a joint model of stockholding and answers to the probability questions asked in the core survey. The left-hand side variables of the model are the share of wealth held in stocks (s_i) and the probability answers (p_{0i}, p_{10i}). These left-hand side variables are modelled as functions of observable variables, in part through latent variables (μ_i, σ_i), as well as unobservables and survey noise.

In the richest version of the model, the uncensored optimal share of stocks in the portfolio (s_i^*) depends on latent expectations characterized by μ_i and σ_i, risk tolerance α, as well as other factors x_i (observables) and u_{si} (unobservables). The latent level of expectations μ_i, depends on the same set of covariates x_i as well as μ-specific variables (instruments) z_μ as well as unobservables $u_{\mu i}$. The latent uncertainty of expectations σ_i, depends again on the same set of covariates x_i and σ-specific instruments $z_{\sigma i}$.

$$s_i^* = \beta_s' x_i + \alpha \frac{(\mu_i - r)}{\sigma_i^2} + u_{si}. \tag{8}$$

$$\mu_i = \beta_\mu' x_i + \gamma_\mu' z_{\mu i} + u_{\mu i} \tag{9}$$

$$\log(\sigma_i) = \beta_\sigma' x_i + \gamma_\sigma' z_{\sigma i} \tag{10}$$

Note that the equation for σ is modeled as log-linear and contains no unobserved heterogeneity. Log-linearity is specified in order to achieve a positive support without the parameter restrictions. Unobserved heterogeneity is omitted because it would severely weaken the estimation (log-normal variable in the denominator). This specification is analogous to the heteroskedastic probit or tobit model. Unobservables u_s and u_μ are assumed to be normally distributed. Simpler versions of the model are specified without the x and/or the z right-hand side variables.

The latent variables s_i^*, μ_i and σ_i are related to three observed left-hand side variables: the share of stocks in the portfolio (s_i) and the two probability answers (p_{0i}, p_{10i}). The optimal share in stocks (s_i^*) translates in to observed stockholding (s_i) with potential corner solutions. Since we assume normal distribution for unobservables, this leads to a two-way censored Tobit model.

$$s_i = \left\{ \begin{array}{lll} 0 & if & s_i^* \leq 0 \\ s_i^* & if & 0 < s_i^* < 1 \\ 1 & if & s_i^* \geq 1 \end{array} \right\} \tag{11}$$

Survey answers to probability questions (p_{0i} and p_{10i}) depend on expectation parameters μ_i and σ_i, and question-specific survey noise v_{0i} and v_{10i} in the way we derived above.

$$\begin{aligned} p_{i0} &\in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right) < \bar{b} \\ p_{i10} &\in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right) < \bar{b} \end{aligned} \tag{12}$$

The bins $\left[\underline{b}, \bar{b}\right]$ are defined (in percentage terms) as $[0, 5)$, $[5, 15)$, $[15, 25)$,..., $[95, 100]$. The survey noise variables v_0 and v_{10} are assumed to be jointly normal and correlated, but independent of everything else. This independence reflects their role as pure survey noise.

Vector x_i contains the right-hand side variables that, when used, enter the equation of s^* in a direct way and enter the equations of μ and σ as well. These variables include demographics, education, cognitive capacity, and detailed wealth measures. Recall that in the survey, all assets are defined at the household level whereas expectations are asked from all individuals. Households are either one-member or two-member units (singles or couples). The sample used in this analysis contains one individual per household, the financial respondent who is the most knowledgeable about the savings and assets of the household.

Demographic variables include the gender of the respondent interacted with whether the household is single or a couple, race (African American or Hispanic), education, and a measure of cognitive capacity. The cognitive measure is based on four short tests measured in HRS between 1992 and 2000 (immediate word recall, delayed word recall, successively subtracting seven from one hundred, and dementia control questions). The measure used in this analysis is the first factor of the four aggregate scores for each individual, where aggregate scores are sums of the yearly scores (missing values filled in with their cross-sectional mean). McArdle, Fisher and Kadlec (2007) argue that the first factor of these tests measure episodic memory. Indeed, our first factor is most highly correlated with the word recall variables ($\rho > 0.95$). It also explains the vast majority of the correlation across the four tests.

The wealth measures are splines in log net total wealth and the log value of specific assets. The spline is taken over the following intervals: negative wealth (in which case the log of the negative wealth is taken); below \$1k; between \$1k and \$10k; between \$10k and \$100k; above \$100k. The individual wealth items considered are value of home, mortgage outstanding, value of financial assets, value of retirement accounts.

Vector $z_{\mu i}$ consists of four variables that are specific to the level of expectations (μ) and are excluded from the other equations. Three of these are proxies for general optimism/pessimism as a time-invariant personality trait, and the fourth variables captures reactions of expectations to recent events on the stock market. All three optimism/pessimism variables are constructed from answers to earlier surveys.

The first variable in z_μ is a dummy denoting positive errors in predicting sunny weather. HRS 1994 and 2000 included a "warm-up" question to the series of subjective probability questions about the probability that the day following the interview would be sunny. We obtained realized weather data for the day in question at the zip-code location of the interview, and we regressed the probability answer on sunny hours (their fraction to hours of daylight). The residual of this regression can be interpreted as a forecast error. The variable entered here is a dummy indicating whether the respondent's average forecast error was positive on both of the two surveys.

The second variable is also a measure of optimism/pessimism: it is the individual's subjective probability answer to the likelihood of a major recession that would the near future. The question was asked in HRS 1992, 1996 and 1998, and the measure we use is the average of those answers (missing values filled by cross-sectional means again). The content of the question is closely related to stock market performance so this variable may reflect some knowledge about the economy besides optimism/pessimism. Those who expected a major recession between 1992 and 1998 were probably not very familiar with economic conditions then, or they were pessimistic by personality, or both. We prefer the optimism/pessimism interpretation because current expectations over economic growth may be incorporated into current asset prices and thus would not affect returns.

The third variable is the score created from the nine-item psychological depression tests administered to the respondents in all waves of the HRS between 1992 and 2000. This test lists symptoms of clinical depression, and we use the score as a measure of time-invariant general pessimism.

The fourth variable that appears exclusively in the equation of μ is the level of stock prices before the interview, approximated by the closing Dow Jones Industrial Average on the first day of the month of the interview. HRS 2002 took place between April 2002 and February 2003, with most interviews completed by September. The time of the interview is not entirely random, but it is exogenous to the stock market index. The stock market had a roller-coaster drive during that period, with the Dow Jones index dropping from 9,900 in April to 7600 in September, and back to 10,000 in the next February. Figure 3 demonstrates systematic co-movement of the monthly average answer to the p_0 question and the stock market index (except for the two dozen interviews completed in the last month). Note however that the magnitude of the changes in p_0 is modest: on average, it takes a 500 point change in the index to change p_0 by one percentage point.

The instrument for the subjective variance ($\ln \sigma$) is a single variable, the fraction of fifty per cent answers to all probability questions (except for p_0 and p_{10}) given by the individual in all of the surveys from year 1992 to 2002 (normalized to be in the $[0, 1]$ range). The idea behind using the instrument is the effect general uncertainty on uncertainty about stock market returns. General uncertainty is assumed to affect uncertainty in many domains, and fifty-fifty answers are related to high uncertainty (see above, at equation (6)). This instrument is very similar to the one used in Hill, Perry and Willis (2006): they included the fraction 0 and 100 per cent answers as well in their survey response model, because in their model, such answers may also be the result of imprecise beliefs.

In an extension, we make use of a proxy for risk tolerance for HRS respondents estimated by Kimball, Sahm and Shapiro (2007) from answers to hypothetical gambles over lifetime earnings in HRS 1992 to 2002. Using these measures, Sahm (2007) established a significant positive relationship of risk tolerance and stockholding in a larger sample of HRS respondents. We denote the proxy as a_i, and we enter it for the risk preference parameter as $\alpha_i = \beta_a a_i$. The interpretation of β_α is how much a_i, the Kimball-Sahm-Shapiro proxy, captures risk tolerance in our model. If it did so perfectly, we would expect $\beta_\alpha = 1$. Deviations from these values would indicate a discrepancy of the measure and our model.

The summary statistics of all variables are in the Appendix Table 4. The statistics are shown for the estimation sample: respondents of HRS 2002 who were either singles of age 55 to 65 or financial respondents from couples with average age 55 to 65. Individuals in the estimation sample are those who answered the two core stock market expectation questions.

The level (μ) and uncertainty (σ) of stock market expectations is identified from the two probability answers (p_0 and p_{10}) and the instruments (z_μ and z_σ). The first and perhaps most important identification problem is separating survey noise from relevant heterogeneity. Mechanically, the problem lies in the fact that in the mapping from expectations to probability answers, the role of μ and v is interchangeable (see (6)). Intuitively, inter-personal differences in observed probability answers can be a result of either differences in relevant expectations (μ) or survey noise (v). We have three sources of identification in our models.

One identification source is the fact that survey noise does not enter the stockholding decision, by definition. Joint estimation of stockholding with probability answers therefore helps identification. Another source is the presence of instruments in the equation of μ and of $\log(\sigma)$ excluded from the equation of $\log(\sigma)$ and μ, respectively.[7]

The third and possibly most valuable source of identification is the fact that for a subset of respondents, we observe answers to the same probability questions in the experimental

[7] The first set of instruments affect μ and thus μ/σ but not v/σ by assumption, while the second set affects σ and thus both μ/σ and v/σ. These help separating variation in μ/σ and v/σ and, with the help of the distributional assumptions, the variation in μ and v.

module ($p_{0'}$ and $p_{10'}$) besides the core questionnaire answers (p_0 and p_{10}). By assumption (which is supported by evidence), module answers are based on the same relevant expectations parameters μ and σ but different and, most importantly, independent noise variables v. Because of small samples we did not incorporate the other experimental module probability answers into the estimation model in a direct way. Instead, we estimated the most important moments of the noise distribution in a separate analysis, using the four probability answers and nothing else. These estimates were then used as calibrated values in the main estimation. Details of the identification and estimation of moments of the noise distribution are summarized in Appendix A. The results imply $\sigma_v^2 \in (0.14, 0.26)$ and $\rho \in (0.5, 0.7)$. The estimates presented in this paper assume $\sigma_v^2 = 0.16$ ($\sigma_v = 0.4$) and $\rho = 0.7$. We experimented with models when both σ_v and ρ were estimated and received very similar results for all parameters.

Due to the presence of survey noise and interval response, relevant heterogeneity in stock market expectations are not identified at the individual level. Instead, conditional expected values and (in the case of μ) the population standard deviation are identified. Separating the effect of the level of expectations (μ) from uncertainty about returns (σ) is possible because of the two probability answers and also because of the exclusion restrictions. Given calibrated values for the noise parameters, any of these two sources are sufficient for identification. The variance of the two unobservables (u_s and u_μ) is identified in the Tobit models. It turns out that identification of the covariance of u_s and u_μ is very poor, and therefore we set it to zero.

Another important identification problem is the issue of causality: whether it is expectations causing stockholding. Serious endogeneity problems are likely to emerge both from omitted variables and simultaneity. Stockholders may form better expectations because it is in their interest, more so than for non-holders (reverse causality), or third variables such as cognitive capacity can affect stockholding (conditional on expectations) and also help forming better expectations (conditional on stockholding). It is very likely that expectations do cause stockholding as it is also necessary for the reverse causality mechanism to reflect rational behavior. We make use of the expectation-specific instruments in order to claim causality. The estimated models make use of exclusion restrictions in order to separate the effect of expectations (optimism as a personality trait, momentum of the stock market, and general uncertainty about future events). The exclusion restrictions are used to identify the causal effect of expectations, and the rest of the variables are allowed to have both a "direct" (i.e. residual) effect on stockholding and an effect through expectations.

Unobserved stochastic components are assumed to be distributed normally. All parameters are estimated, except for the risk-free return r, noise parameters σ_v^2 and ρ, and $Cov\,(u_s, u_\mu)$. These are set to pre-determined values in our main estimates (to 0.02, 0.4, 0.7 and 0, respectively).

Estimation is complicated because of nonlinearity in survey noise and the presence two survey noise components (resulting in four error terms in a three-equation system). One error component needs to be integrated out, and numerical integration is cumbersome because of the nonlinearities. The estimation therefore follows a Maximum Simulated Likelihood procedure. Technical details are laid out in Appendix B.

7 Results

We estimated six versions of the model. In model 1, no covariates or instruments are included, and the equation of s^* contains no constant. This model take the simples portfolio choice result literally, imposing $s_i^* = \alpha\left(\mu_i - r\right)/\sigma_i^2$ with possible stochastic deviation u_{si}. In model 1, heterogeneity in expectations is modeled as $\mu_i = \beta_{\mu 0} + u_{\mu i}$ and $\sigma_i^2 = \exp\left(2\beta_{\sigma 0}\right)$. Model 2 differs from model 1 by a constant allowed in the equation of s^*, in order to allow for a constant systematic deviation from the simplistic portfolio choice model. Model 3 adds instruments z_μ and z_σ to the equations of μ and σ, respectively, but allows for no constant in the equation of s^*, while model 4 allows for such a constant. Model 5 adds the demographic, education and cognitive covariates to all three equations, while Model 6 includes the full set of covariates, including the wealth variables. Models 3 through 6 use instruments z_μ and z_σ in identifying level versus dispersion of expectations and the effect of expectations on stockholding, while models 1 and 2 do not use those instruments. Models 5 and 6 allow for covariates in x_i to affect stockholding through expectations and in a residual ("direct") way as well, while models 1 through 4 would incorporate all "direct" effects into the effects of expectations to the extent that x_i are correlated with expectations. Model 6 includes the wealth variables that may control for many unobservables that affect savings and investment behavior. In fact, conditioning on wealth may control for too much, including the effects of expectations themselves: past stockholding should affect wealth exactly because of the large equity premium. Estimates of the effect of expectations on stockholding conditional on those wealth measures may therefore be biased downward. Our preferred estimates are those from Model 5. The results of models 1 through 6 are summarized in Tables 6 and 7 and Appendix Table 5. Table 7 lists the coefficients on covariates x_i from model 5, Appendix Table 5 shows those coefficients from model 6, while table 6 shows all other results.

The Sharpe ratio using subjective expectations are shown to be related to stockholding in a positive and significant way in all models. In our most preferred model (version 5), $\hat{\alpha} \approx 0.3$, corresponding to a CRRA risk aversion parameter of slightly over 3. Recall that by assumption, risk aversion here is constant. Models 1 through 4 (those without covariates in the equation of s^*) provide larger estimates of α and thus imply even milder risk aversion. Even model 6, with all the wealth variables that may control for too much, implies a risk aversion parameter of 6, well below the Mehra and Prescott estimate. The estimates show that the level of risk tolerance that links subjective beliefs to stockholding is therefore moderate, somewhere between 3 and 6. These estimates are considerably smaller than those obtained from the equity premium using historical moments of stock market returns (Mehra and Prescott, 1985; Kocherlakota, 1996). Taken as a causal estimate, this result implies that taken people's actual expectations instead of estimates from historical series, there is not much of a puzzle in their stockholding behavior. Even if one is cautious about the causal interpretation, the result lends strong support to the validity of our measures of subjective expectations.

According to our most preferred estimates (from model 5), average stock market expectations in the population represented by our sample (Americans of age 55 to 65 in 2002) are characterized by a low mean of -3 per cent and a high standard deviation at 40 per cent. These are to be compared to the yearly relative returns on the stockmarket with post-war historical mean of 7 per cent and standard deviation of 14 per cent. Heterogeneity in beliefs is substantial, with a population standard deviation of μ at 15 percentage points. Estimates of relevant heterogeneity are remarkably stable across specifications (except for the somewhat higher mean of μ in models with no constant in the equation of s^*), indicating that relevant heterogeneity is identified primarily from the two probability answers themselves.

The instruments are strong in all specifications with very similar magnitudes. We focus on the magnitudes from model 5. Positive sunny weather forecast error (as opposed to negative error) is positively related to the location of stock market expectations: those who made a positive error at each of the two sunshine forecasting occasions expect stock market returns to be two percentage points higher. The subjective probability of a recession (measured in the 1990's) is strongly negatively related to the level of stockmarket expectations. Going from a zero to one hundred per cent probability is associated with a 17 percentage points decline in expected stock market returns. Since the standard deviation of the recession answers is 0.25, the standardized magnitude is at -0.04. Recall that we interpret this effect as one of general optimism, similarly to the effect of sunshine optimism. Depressive symptoms are also negatively related to stock market expectations. One standard deviation increase in our measure of depression is associated with a four percentage point decrease in stock market expectations. Recent performance of the stock market is positively related to stock market expectations, but the magnitude is modest. It takes a five hundred points gain in the Dow Jones to result in one percentage point gain in expected yearly returns.

The fraction of fifty answers to all probability questions asked between 1992 and 2002 exhibits a strong effect on subjective stock market variance. If the fraction goes from its minimum to its maximum (zero to one), stock market uncertainty more than doubles. Since the standard deviation of the fraction of fifty-fifty answers is rather small (0.09), the standardized effect is less dramatic but still significant, at around a one tenth increase.

Demographic variables show important systematic variation in stock market expectations (see Table 7). We focus first on estimates from model 5. Single women exhibit stock market expectations that are thirteen percentage points lower and thirteen per cent more uncertain than the reference category (couples with male financial respondent). Single men also expect stock returns to be lower by six percentage points, but the differences are not significant for uncertainty. Female financial respondents in couples exhibit expectations that are also more pessimistic and uncertain than male respondents, but the differences are somewhat smaller than for single women. African Americans have significantly lower stock market expectations, by eight percentage points, while Hispanic respondents are not significantly different from the reference group (Non-Black, Non-Hispanic).

More educated people have higher expectations: one extra year of education is associated with 1.2 percentage points higher expected value of stock market returns, and the implied standardized effect is above three percentage points. Recall that higher expectations also means closer to historical returns. On the other hand, education does not seem to be related to uncertainty about stock markets (controlling for our proxy of general uncertainty). The effect of cognitive capacity on the levels of expectations not significant, with a small but statistically significant positive effect on uncertainty (again, controlling for general uncertainty).

Conditional on the wealth variables (model 6; see Table 5 in the Appendix), the relationship of demographic variables to the level of expectations is somewhat weaker while their relationship to uncertainty remains by and large the same. Wealth itself has a positive association with the level of expectations only at higher wealth levels, while it has no systematic relationship to uncertainty. Of the various asset categories mortgage has a significant positive correlation with the level (the standardized coefficient is at +0.03) and a small but negative correlation with the uncertainty of expectations.

In order to have a better understanding of the estimated direct effects of the covariates on stockholding (s^*), we compare those to the estimated coefficients from reduced-form Tobit models without expectation variables. Estimates from the reduced-form models represent "full effects" (not necessarily in a causal sense), and comparing them to our estimates of the "direct effects" reveals the role of expectations in those full effects. Table 7 shows the results from reduced-form Tobit model without expectations and wealth variables, together with the corresponding estimates from the structural model with expectations but without wealth variables (model 5). Table 5 in the Appendix shows the corresponding results from the models with the wealth variables.

Single women have a significantly lower fraction of stocks than the reference group (couples with male financial respondent), with a difference of -0.238 in terms of the non-censored asset share. Comparing that to the insignificant and positive direct effect (+0.073) reveals that the entire difference is due to expectations. The same is true for single men who start with a somewhat smaller difference. Stockholding is less in couples with female financial respondents as well but only to a modest degree. The significantly lower and more uncertain expectations of such women more than explains this difference, resulting in a positive direct "effect" on stockholding. In other words, the lower and more uncertain expectations of female financial respondents in couples does not lead to lower stockholding to the same extent as for single women.

Stockholding is significantly lower for African Americans than for the reference group (non-Black, non-Hispanic Americans). Differences in stock-market expectations seem to explain half of the observed difference in stockholding, primarily through lower level of such expectations. Stockholding is low among Hispanic Americans in a similar way, but their expectations seem to explain a very little of the difference.

Stockholding is significantly higher for more educated people: one year of education is associated with +0.074 in the fraction of stocks within financial and retirement assets (among those with uncensored fraction). Expectations explain half of this correlation. The association with our cognitive measure is weaker to begin with, and it disappears after controlling for expectations. The role of expectations there is less clear as the positive association with levels is not statistically significant and the statistically significant association with uncertainty is positive (but small).

Controlling for wealth (Table 5 in the Appendix) shows that wherever the association of demographics and stockholding remains significant conditional on wealth, the role of expectations is qualitatively similar to their role without the wealth variables. Wealth itself is strongly related to stockholding in intuitive (strong and positive) ways. Expectations do not seem to affect these relationships with the possible exception of mortgages.

Finally, Table 8 shows results from models 1 through 6 with the Kimball-Sahm-Shapiro proxy for risk tolerance. Recall that the proxy was estimated from answers to hypothetical gambles over lifetime earnings in HRS 1992 to 2002. If the proxy is denoted as a_i, this specification amounts to replacing the constant α parameter in front of the Sharpe ratio by $\alpha_i = \beta_a a_i$. The interpretation of β_α is how much a_i, the Kimball-Sahm-Shapiro proxy, captures risk tolerance in our model, $\beta_\alpha = 1$ implying a perfect measure. Note that results from models 1 and 2 (those without the instruments) are not shown because they turned out to be poorly estimated.

In our preferred specification (model 5), the coefficient estimate on the risk tolerance proxy is somewhat above 0.6. Taken this result at face value implies that the proxy overstates risk tolerance in the context of stockholding. Note however that the proxy is estimated as an expected value conditional exclusively on the hypothetical gamble questions. As Kimball, Sahm and Shapiro (2007) argued, using the proxy in regressions with other covariates is likely to lead to downward bias in the coefficient on risk tolerance because the difference between true risk tolerance and the proxy (an expected value) is unobserved and may be correlated with those covariates. The evidence in Table 8 supports this argument: the richer models produce significantly smaller coefficients on the risk tolerance proxy. In model 5, estimates of the relevant heterogeneity are identical to their baseline estimates, except for a significantly larger risk aversion. Note that the log likelihood of all models with the risk tolerance proxy is substantially lower than the log likelihood of their baseline counterpart, suggesting that information in the proxy may add less to the models than what's taken away by the noise.

8 Conclusions

Using survey data on expectations and the composition of household savings, the goal of this paper is to explain the phenomenon called the stockholding puzzle in the literature: the fact that, despite the high historical returns and relatively low risk of stock-market based assets, many American households own no such assets, and many of those who do own a little.

We develop a measurement model that is consistent with a theory of survey response and allows us to separate noise from heterogeneity in expectations that is relevant in investment decisions. We provide detailed descriptive evidence on survey noise. Survey non-response to the probability questions is strongly negatively related to stockholding, and it seems to reflect genuine ignorance. Answers contain substantial noise, as the direct test-retest evidence (the comparison of core and experimental module answers) clearly demonstrates. Adjacent answers also seem to contain some inconsistency, and rounding is prevalent, but none of these latter features is significantly related to stockholding. The measurement model is made consistent with all these features by allowing for survey noise in the expectations and rounding in the probability answers based on those expectations.

We estimate a joint model of household portfolio choice and survey answers based on the latent subjective mean and variance of stock market returns. The estimates show that the level of risk tolerance that links subjective beliefs to stockholding is moderate, lending strong support to the validity of our measures of subjective expectations. Average stockholding is low in large part because, on average, people expect the mean of future returns to be significantly lower and the variance significantly higher than what historical returns would imply.

We document substantial heterogeneity in expectations, a heterogeneity that is a strong predictor of heterogeneity in stockholding. A significant part of the deficit in stockholding of single households (especially single women), African Americans, and less educated Americans is explained by their lower expectations. These findings raise the question of why there are systematic differences in beliefs about the probability distribution of stock returns across different demographic groups who face a common and highly public history of stock prices.

A straightforward explanation is that it takes effort, intelligence and motivation to acquire knowledge of this body of evidence and use it to make savings and portfolio decisions that will raise the individual's or household's level of expected utility. Delavande, Rohwedder and Willis (2008) formalize this idea by treating the acquisition of financial knowledge as a form of human capital which allows households to obtain a higher expected rate of return on their assets, holding risk constant. One component of financial knowledge is knowledge about the probability distribution of returns in the stock market. There is an important scale economy in this investment process: while increased knowledge raises the feasible expected return per dollar, holding risk constant, the total value of the investment depends on the number of dollars to which the improved return is applied. This creates a non-convexity that helps explain household non-participation in the stock market because it does not pay a household to make a significant investment in financial knowledge unless it expects to make an investment in stock that is large enough to generate excess returns sufficient to repay the cost of the human capital investment. Treating knowledge of the distribution of stock returns as a component of human capital suggests that differentials among individuals in beliefs about expected returns and risk will be related to factors that affect the benefits and costs of acquiring knowledge about these returns. The expectation differences of various groups, as estimated by our model, are broadly consistent with such an interpretation.

This paper makes a methodological contribution by explicitly linking an economic model of the effect of subjective probability beliefs about returns in the stock market on stockholding with a model of survey response that relates these beliefs to answers to survey questions. The model allows us to distinguish the effect of heterogeneity in survey responses that reflect differences in beliefs about risk and return that affect choices in the economy from survey noise.

Our approach can be useful for analyzing the role and sources of subjective expectations outside household finances as well. We believe that effects of expectations on socially relevant behavior and survey answers need to be modeled jointly. Any such model should recognize that answering survey questions is a form of behavior in itself with its own distinct incentives and constraints. Our model is a simple but sensible attempt that may be a useful reference for further research in this direction.

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Appendices

A Estimating variance and correlation of survey noise

$$\begin{aligned} R_{i(t+1)} &= \mu_{it} + \eta_{it} \\ \tilde{R}_{i(t+1)j} &= \mu_{it} + \eta_{it} + v_{itj} \\ v_{itj} &\perp \perp \mu_{it}, \sigma_{it} \end{aligned}$$

The noise components are assumed to be normally distributed

$$\begin{aligned} \begin{bmatrix} v_{i0} \\ v_{i10} \end{bmatrix} &\sim iidN\left(0, \Sigma\right) \quad \Sigma = \begin{bmatrix} \sigma_v^2 & (1-\rho)\,\sigma_v^2 \\ (1-\rho)\,\sigma_v^2 & \sigma_v^2 \end{bmatrix} \\ E\left[v_{ij} v_{ij'}\right] &= 0 \end{aligned}$$

The goal of the exercise is to estimate moments of the noise distribution so we can calibrate those in the estimation. We are interested in :

$$\sigma_v^2 \ and \ \rho$$

Use probability answers only, and assume that they are based on the precise probability

$$p_{ij}^* = \Pr\left(\tilde{R}_{ij} > \tau_j | \mu_i, v_{ij}\right) = \Pr\left(\frac{\eta_i}{\sigma_i} > \frac{\tau_j - \mu_i - v_{ij}}{\sigma_i}\right) = \Phi\left(\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i}\right)$$

We assume that actual responses are equal to the precise probability:

$$p_{ij} = p_{ij}^*$$

We justify this by the fact that we are not going to use the individual answers themselves but their sample averages (to be more precise the sample average of various functions of the answers).

We are going to make use linearity of noise in the inverse of the probability answers:

$$\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i} = \Phi\left(p_{ij}\right)$$

A.1 Moment condition 1

Compare core and module answers to the p_0 probability question and take expectation of the squares:

$$\begin{aligned} \Phi^{-1}\left(p_{i0}\right) - \Phi^{-1}\left(p_{i0'}\right) &= \frac{v_{i0} - v_{i0'}}{\sigma_i} \\ E\left[\left\{\Phi^{-1}\left(p_{i0}\right) - \Phi^{-1}\left(p_{i0'}\right)\right\}^2\right] &= E\left[\left(\frac{v_{i0} - v_{i0'}}{\sigma_i}\right)^2\right] = E\left[\left(v_{i0} - v_{i0'}\right)^2\right] E\left[\frac{1}{\sigma_i^2}\right] \end{aligned}$$

so that

$$E\left[\left\{\Phi^{-1}\left(p_{i0}\right) - \Phi^{-1}\left(p_{i0'}\right)\right\}^2\right] = 2\sigma_v^2 E\left[\frac{1}{\sigma_i^2}\right] \tag{1}$$

The same can be derived using p_{10} and $p_{10'}$, because there again $\Phi^{-1}\left(p_{i10}\right) - \Phi^{-1}\left(p_{i10'}\right) = \frac{v_{i10} - v_{i10'}}{\sigma_i}$, so that

$$E\left[\left\{\Phi^{-1}\left(p_{i10}\right) - \Phi^{-1}\left(p_{i10'}\right)\right\}^2\right] = 2\sigma_v^2 E\left[\frac{1}{\sigma_i^2}\right] \tag{2}$$

A.2 Moment condition 2

Compare adjacent core answers and take expectation of the squares:

$$\begin{aligned}
\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i10}) &= \frac{v_{i0} - v_{i10} + 0.1}{\sigma_i} \\
E\left[\left\{\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i10})\right\}^2\right] &= E\left[\frac{(v_{i0} - v_{i10} + 0.1)^2}{\sigma_i^2}\right] \\
&= E\left[v_{i0}^2 + v_{i10}^2 + 0.01 - 2v_{i0}v_{i10} + 0.2v_{i0} - 0.2v_{i10}\right] E\left[\frac{1}{\sigma_i^2}\right]
\end{aligned}$$

so that

$$E\left[\left\{\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i10})\right\}^2\right] = \left[2(1-\rho)\sigma_v^2 + 0.01\right] E\left[\frac{1}{\sigma_i^2}\right] \tag{3}$$

Again, we can do this for the other pair of answers, which are, in this case, the module answers, with the result of

$$E\left[\left\{\Phi^{-1}(p_{i0'}) - \Phi^{-1}(p_{i10'})\right\}^2\right] = \left[2(1-\rho)\sigma_v^2 + 0.01\right] E\left[\frac{1}{\sigma_i^2}\right] \tag{4}$$

A.3 Moment condition 3

Compare answers of one of the core probability question to answers to the other probability question asked in the module, and take expectation of the squares:

$$\begin{aligned}
\left\{\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i10'})\right\} &= \frac{v_{i0} - v_{i10'} + 0.1}{\sigma_i} \\
E\left[\left\{\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i10'})\right\}^2\right] &= E\left[\frac{(v_{i0} - v_{i10'} + 0.1)^2}{\sigma_i^2}\right] \\
&= E\left[v_{i0}^2 + v_{i10'}^2 + 0.01 - 2v_{i0}v_{i10'} + 0.1v_{i0} - 0.1v_{i10'}\right] E\left[\frac{1}{\sigma_i^2}\right]
\end{aligned}$$

so that

$$E\left[\left\{\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i10'})\right\}^2\right] = \left[2\sigma_v^2 + 0.01\right] E\left[\frac{1}{\sigma_i^2}\right] \tag{5}$$

A.4 Moment condition 4

Look at the probability to giving negative mass answers

$$\begin{aligned}
\Pr\left[p_{i0} < p_{i10}\right] &= \Pr\left[\Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right) < \Phi\left(\frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right)\right] = \Pr\left[\frac{\mu_i + v_{i0}}{\sigma_i} < \frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right] \\
&= \Pr\left[v_{i0} < v_{i10} - 0.1\right] = \Pr\left[v_{i0} - v_{i10} < -0.1\right] = \Pr\left[v_{i0} - v_{i10} < -0.1\right] \\
&= \Pr\left[\frac{v_{i0} - v_{i10}}{\sqrt{2(1-\rho)\sigma_v^2}} < \frac{-0.1}{\sqrt{2(1-\rho)\sigma_v^2}}\right] = \Phi\left[\frac{-0.1}{\sqrt{2(1-\rho)\sigma_v^2}}\right]
\end{aligned}$$

so that

$$\Phi^{-1}\left[\Pr\left(p_{i0} < p_{i10}\right)\right] = \frac{-0.1}{\sqrt{2(1-\rho)\sigma_v^2}}$$

and therefore

$$\left\{\Phi^{-1}\left[\Pr\left(p_{i0} < p_{i10}\right)\right]\right\}^2 = \frac{0.01}{2\left(1-\rho\right)\sigma_v^2} \tag{6}$$

And, similarly for module responses:

$$\left\{\Phi^{-1}\left[\Pr\left(p_{i0'} < p_{i10'}\right)\right]\right\}^2 = \frac{0.01}{2\left(1-\rho\right)\sigma_v^2} \tag{7}$$

A.5 Rearranging the moment conditions

We have four types of conditions:

$$\begin{aligned}
2\sigma_v^2 E\left[\frac{1}{\sigma_i^2}\right] &= A \\
\left[2\left(1-\rho\right)\sigma_v^2 + 0.01\right] E\left[\frac{1}{\sigma_i^2}\right] &= B \\
\left[2\sigma_v^2 + 0.01\right] E\left[\frac{1}{\sigma_i^2}\right] &= C \\
\frac{0.01}{2\left(1-\rho\right)\sigma_v^2} &= D
\end{aligned}$$

We can rearrange these in order to get rid of the here ancillary moment, $E\left[1/\sigma_i^2\right]$:

$$\begin{aligned}
\frac{2\left(1-\rho\right)\sigma_v^2 + 0.01}{2\sigma_v^2} &= \frac{B}{A} \\
\frac{2\sigma_v^2 + 0.01}{2\sigma_v^2} &= \frac{C}{A} \\
\frac{0.01}{2\left(1-\rho\right)\sigma_v^2} &= D
\end{aligned}$$

From the third one, we get

$$\sigma_v^2 = \frac{0.005}{C/A - 1}$$

We can express ρ in two ways. First, using the first and second moment conditions, $\left(1-\rho\right)\sigma_v^2 + 0.005 = \frac{B}{A}\sigma_v^2$, thus $\left(1-\rho\right) + 0.005/\sigma_v^2 = \frac{B}{A}$ and so

$$\rho = 1 - \frac{B}{A} + 0.005/\sigma_v^2$$

Or alternatively from the fourth moment condition can be rewritten as $\left(1-\rho\right)\sigma_v^2 = 0.005/D$ and thus

$$\rho = 1 - \frac{0.005}{D\sigma_v^2}$$

A.6 Estimation of ρ and σ_v^2 by Minimum Distance

First we took averages of two versions of A, B, C, and D. These averages were simple means for A and C and weighted (by square root of sample size) averages for B and D. Then we estimated σ_v^2 by the formula above, and ρ in the two alternative ways.

There are two issues. The first one arises with moment conditions 1 through 3. The problem there is that $\Phi^{-1}\left(p\right)$ is not defined $p = 0$ or $p = 1$. Ad-hoc solution: replace then with $p = 0+\varepsilon$ and $p = 1-\varepsilon$, respectively.

Various values for ε are considered for robustness checks (like 0.01 or 0.005 to 0.000001). The second problem arises with moment condition 4. While $\Pr[p_{i0} < p_{i10}] = \Pr[p_{i0} \leq p_{i10}]$ under normally distributed R if the p_i are the precise probabilities indeed, in practice we have quite a few cases with $p_{i0} = p_{i10}$. Whether we count them as $p_{i0} < p_{i10}$ or $p_{i0} > p_{i10}$ (or a fraction here, the other fraction there) has a large effect on the ρ estimate using moment condition 4.

The results are the following. For $\varepsilon = 0.01$, $\sigma_v^2 = 0.14$. As $\varepsilon \to 0$, we have $\sigma_v^2 \to 0.26$. As for ρ identified from the first three moment conditions only, for $\varepsilon = 0.01$, $\rho = 0.60$. As $\varepsilon \to 0$, $\rho \to 0.67$. When the fourth moment condition is used for estimating ρ, the results are very sensitive to how we count all the $p_{i0} = p_{i10}$ responses as $p_{i0} > p_{i10}$, we have $\rho = -0.8$ to -0.5 (depending on ε), results that are clearly counterintuitive. When we count all the $p_{i0} = p_{i10}$ responses as $p_{i0} < p_{i10}$, we have $\rho = 0.55$ to 0.80 (as ε is decreased from 0.01 towards 0). These latter results are very much in line with the other ρ estimate. When we count one half of the equal answers as greater, the other half as smaller (a middle-of-the-road approach), we get $\rho = 0.3$ to 0.45 (depending on ε).

Overall, taking uncertainty in the second ρ estimates into account, we arrive to the following results:

$$\begin{aligned} \sigma_v^2 &\approx 0.14 \;\; to \;\; 0.26 \\ \rho &\approx 0.50 \;\; to \;\; 0.70 \end{aligned}$$

B Maximum Simulated Likelihood estimation

B.1 Setup

We have three observable variables: the share of stocks in the portfolio (s_i) and the two probability answers p_{i0} and p_{i10}. The likelihood has the following elements (drop t subscripts for simplicity):

$$\begin{aligned} s_i^* &= \beta_s' x_i + \alpha \frac{(\mu_i - r)}{\sigma_i^2} + u_{si} \\ \mu_i &= \beta_\mu' x_{\mu i} + u_{\mu i} \\ \log(\sigma_i) &= \beta_\sigma' x_{\sigma i} \\ s_i &= \left\{ \begin{array}{lll} 0 & if & s_i^* < 0 \\ s_i^* & if & 0 \leq s_i^* \leq 1 \\ 1 & if & s_i^* > 1 \end{array} \right\} \\ p_{i0} &\in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right) < \bar{b} \\ p_{i10} &\in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right) < \bar{b} \end{aligned}$$

where $\left[\underline{b}, \bar{b}\right]$ are (in percentage terms) $[0,5)$, $[5,15)$, $[15,25)$,..., $[95,100]$.

Stochastic components are assumed to be distributed normally with covariance matrix Ω^* :

$$\left(\begin{array}{c} u_{si} \\ u_{\mu i} \\ v_{i0} \\ v_{i10} \end{array}\right) \sim N(0, \Omega^*) \;,\; \Omega^* = \left[\begin{array}{cccc} \sigma_{us}^2 & \rho_{s\mu}\sigma_{us}\sigma_{u\mu} & 0 & 0 \\ \rho_{s\mu}\sigma_{us}\sigma_{u\mu} & \sigma_{u\mu}^2 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & \rho\sigma_v^2 \\ 0 & 0 & \rho\sigma_v^2 & \sigma_v^2 \end{array}\right]$$

By assumption, u_μ is independent of the vector (v_0, v_{10}) but may be correlated with u_s It turns out, however, that their correlation is poorly identified. Therefore we assume independence of u_s and u_μ In that case the variance-covariance matrix of unobservables simplifies to

$$\begin{pmatrix} u_{si} \\ u_{\mu i} \\ v_{i0} \\ v_{i10} \end{pmatrix} \sim N(0,\Omega)\ ,\ \Omega = \begin{bmatrix} \sigma_{us}^2 & 0 & 0 & 0 \\ 0 & \sigma_{u\mu}^2 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & \rho\sigma_v^2 \\ 0 & 0 & \rho\sigma_v^2 & \sigma_v^2 \end{bmatrix}$$

For future notation let X_i be the vector of all observables and β the vector of all regression coefficients on those:

$$\begin{aligned} X_i &= \left[x'_{si}, x'_{\mu i}, x'_{\sigma i}\right]' \\ \beta &= \left[\beta'_s, \beta'_\mu, \beta'_\sigma\right]' \end{aligned}$$

B.2 Likelihood of s

The first part of the likelihood function is about portfolio composition. The mapping from the latent s_i^* to the observed s_i is by censoring at 0 and 1 as described in the setup. The interior solution leads to a standard normal p.d.f., which we denote as f_i for further derivations. Note that, among other things, f_i is a function of the unobservable $u_{\mu i}$:

$$\begin{aligned} f\left(s_i^*|.\right) &= \frac{1}{\sigma_{us}}\phi\left[\frac{1}{\sigma_{us}}\left(\beta'_s x_{si} + \alpha\frac{\beta'_\mu x_{\mu i} + u_{\mu i} - r}{\exp\left(2\beta'_\sigma x_{\sigma i}\right)}\right)\right] \\ &\equiv f\left(\sigma_{us},\beta,\alpha;X_i;u_{\mu i}\right) \equiv f_i\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right) \end{aligned} \tag{8}$$

Each of the two corner solutions leads to a standard normal c.d.f., which we denote by F_{0i} and F_{1i}. These are also functions of the unobservable $u_{\mu i}$:

$$\begin{aligned} \Pr\left(s_i^* < 0|.\right) &= \Pr\left[\beta'_s x_i + \alpha\frac{(\mu_i - r)}{\sigma_i^2} + u_{si} < 0\right] = \Pr\left[\frac{u_{si}}{\sigma_{us}} < -\frac{1}{\sigma_{us}}\left(\beta'_s x_{si} + \alpha\frac{(\mu_i - r)}{\sigma_i^2}\right)\right] \\ &= \Phi\left[-\frac{1}{\sigma_{us}}\left(\beta'_s x_{si} + \alpha\frac{\beta'_\mu x_{\mu i} + u_{\mu i} - r}{\exp\left(2\beta'_\sigma x_{\sigma i}\right)}\right)\right] \qquad (9) \\ &\equiv F_0\left(\sigma_{us},\beta,\alpha;X_i;u_{\mu i}\right) \equiv F_{0i}\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right) \\ \Pr\left(s_i^* > 1|.\right) &= \Pr\left[\beta'_s x_i + \alpha\frac{(\mu_i - r)}{\sigma_i^2} + u_{si} > 1\right] = \Pr\left[\frac{u_{si}}{\sigma_{us}} > 1 - \frac{1}{\sigma_{us}}\left(\beta'_s x_{si} + \alpha\frac{(\mu_i - r)}{\sigma_i^2}\right)\right] \\ &= \Phi\left[\frac{1}{\sigma_{us}}\left(\beta'_s x_{si} + \alpha\frac{\beta'_\mu x_{\mu i} + u_{\mu i} - r}{\exp\left(2\beta'_\sigma x_{\sigma i}\right)}\right) - 1\right] \qquad (10) \\ &\equiv F_1\left(\sigma_{us},\beta,\alpha;X_i;u_{\mu i}\right) \equiv F_{1i}\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right) \end{aligned}$$

The conditional likelihood of s_i taking a particular value is, then

$$\begin{aligned} &\ell_s\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right) \qquad (11) \\ =\ & f_i\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right)^{1(0<s_i<1)} \times F_{0i}\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right)^{1(s_i=0)} \times F_{1i}\left(\sigma_{us},\beta,\alpha;u_{\mu i}\right)^{1(s_i=1)} \end{aligned}$$

where $1(.)$ is the indicator function (equals one if the expression is true and zero otherwise).

B.3 Likelihood of p

In order to see how things work, let's start with something simpler. The probability of p_{i0} observed in an interval would the following.

$$
\begin{aligned}
\Pr\left(p_{i0} \in \left[\underline{b}, \bar{b}\right]\right) &= \Pr\left[\underline{b} \leq \Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right) < \bar{b}\right] = \Pr\left[\underline{b} \leq \Phi\left(\frac{\beta_\mu' x_{\mu i} + u_{\mu i} + v_{i0}}{\exp\left(\beta_\sigma' x_{\sigma i}\right)}\right) < \bar{b}\right] \\
&= \Pr\left[\Phi^{-1}\left(\underline{b}\right) \leq \frac{\beta_\mu' x_{\mu i} + u_{\mu i} + v_{i0}}{\exp\left(\beta_\sigma' x_{\sigma i}\right)} < \Phi^{-1}\left(\bar{b}\right)\right] \\
&= \Pr\left[\Phi^{-1}\left(\underline{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i} \leq v_{i0} < \Phi^{-1}\left(\bar{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}\right] \\
&= \Pr\left[\frac{\Phi^{-1}\left(\underline{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v} \leq \frac{v_{i0}}{\sigma_v} < \frac{\Phi^{-1}\left(\bar{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v}\right] \\
&= \Phi\left[\frac{\Phi^{-1}\left(\bar{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v}\right] - \Phi\left[\frac{\Phi^{-1}\left(\underline{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v}\right]
\end{aligned}
$$

It would be similar for p_{i10}, only with a $+0.1$ in the numerators.

But what we are interested in are not the probabilities p_0 and p_{10} separately but their joint probability:

$$
\Pr\left(p_{i0} \in \left[\underline{b}, \bar{b}\right] \ \& \ p_{i10} \in \left[\underline{c}, \bar{c}\right]\right)
$$

Based on what we derived, this probability is the following

$$
\begin{aligned}
&\Pr\left(p_{i0} \in \left[\underline{b}, \bar{b}\right] \ \& \ p_{i10} \in \left[\underline{c}, \bar{c}\right]\right) \\
= \ &\Pr\left[\begin{array}{c}
\frac{\Phi^{-1}\left(\underline{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v} \leq \frac{v_{i0}}{\sigma_v} < \frac{\Phi^{-1}\left(\bar{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v} \\
\& \\
\frac{\Phi^{-1}\left(\underline{c}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i} + 0.1}{\sigma_v} \leq \frac{v_{i10}}{\sigma_v} < \frac{\Phi^{-1}\left(\bar{c}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i} + 0.1}{\sigma_v}
\end{array}\right] \\
= \ &\Pr\left[l_{0i} \leq \frac{v_{i0}}{\sigma_v} < h_{0i} \ \ \& \ \ l_{10i} \leq \frac{v_{i10}}{\sigma_v} < h_{10i}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
l_{0i} &= \frac{\Phi^{-1}\left(\underline{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v} \\
h_{0i} &= \frac{\Phi^{-1}\left(\bar{b}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i}}{\sigma_v} \\
l_{10i} &= \frac{\Phi^{-1}\left(\underline{c}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i} + 0.1}{\sigma_v} \\
h_{10i} &= \frac{\Phi^{-1}\left(\bar{c}\right)\exp\left(\beta_\sigma' x_{\sigma i}\right) - \beta_\mu' x_{\mu i} - u_{\mu i} + 0.1}{\sigma_v}
\end{aligned}
$$

The event defined within the bracket is a rectangle defined over the two-dimensional space spanned by the two standard normal random variables v_{i0}/σ_v and v_{i10}/σ_v, with covariance ρ. This rectangle can be constructed as the quadrant below h_0 and h_{10}, minus the quadrant below h_0 and l_{10}, minus the quadrant below l_0 and h_{10}, and because we subtracted the quadrant below l_0 and l_{10}, we have to add it back again. The joint probability is therefor equal to the sum of these four probabilities:

$$
\begin{aligned}
& \Pr\left[l_{0i} \le \frac{v_{i0}}{\sigma_v} < h_{0i} \;\;\&\;\; l_{10i} \le \frac{v_{i10}}{\sigma_v} < h_{10i}\right] \\
= \;& \Pr\left[\frac{v_{i0}}{\sigma_v} < h_{0i} \;\;\&\;\; \frac{v_{i10}}{\sigma_v} < h_{10i}\right] + \Pr\left[\frac{v_{i0}}{\sigma_v} < l_{0i} \;\;\&\;\; \frac{v_{i10}}{\sigma_v} < l_{10i}\right] \\
& - \Pr\left[\frac{v_{i0}}{\sigma_v} < h_{0i} \;\;\&\;\; \frac{v_{i10}}{\sigma_v} < l_{10i}\right] - \Pr\left[\frac{v_{i0}}{\sigma_v} < l_{0i} \;\;\&\;\; \frac{v_{i10}}{\sigma_v} < h_{10i}\right] \\
\equiv \;& Q\left(\sigma_v, \rho, \beta_\mu, \beta_\sigma, x_{\mu i}, x_{\sigma i}; u_{\mu i}\right) \equiv Q_i\left(\sigma_v, \rho, \beta_\mu, \beta_\sigma; u_{\mu i}\right)
\end{aligned} \tag{12}
$$

Each of these probabilities is equal to the bivariate standard normal c.d.f. (with covariance ρ) evaluated at the appropriate pair of scalars. Since bivariate normal c.d.f. is built into the advanced statistical packages (Stata, Matlab, etc.), there is no practical need to write out the probabilities in more detail.

B.4 Joint likelihood

By assumption, the random vector (v_{0i}, v_{10i}) is independent of everything, including u_{si}. As a result, joint log likelihood of a particular left-hand side vector (s_i, p_{0i}, p_{10i}) is the product of the likelihoods for s_i (equation (11)) and the appropriate probability rectangle Q_i (equation (12)):

$$\ell_i\left(u_{\mu i}, .\right) = \ell_s\left(\sigma_{us}, \beta, \alpha; u_{\mu i}\right) \times Q_i\left(\sigma_v, \rho, \beta_\mu, \beta_\sigma; u_{\mu i}\right)$$

Let

$$\theta = \left(\beta_s, \beta_\mu, \beta_\sigma, \alpha, \sigma_{us}, \sigma_v, \rho\right)$$

then the log likelihood is the sum is

$$\log \ell\left(\theta | X_i, u_{\mu i}\right) = \log \ell_s\left(\sigma_{us}, \beta_s, \beta_\mu, \beta_\sigma, \alpha | X_i, u_{\mu i}\right) + \log Q\left(\sigma_v, \rho, \beta_\mu, \beta_\sigma | X_i, u_{\mu i}\right)$$

This is still a function of the unobservable $u_{\mu i}$. Provided $u_{\mu i}$ is independent of the entire vector $(u_{si}, v_{0i}, v_{10i})$, we can simply integrate it out in order to get a proper likelihood.

$$\log \ell\left(\theta | X_i\right) = \int \log \ell\left(\theta | X_i, u_{\mu i}\right) dF\left(u_\mu\right)$$

We do simulated integration for simplicity and flexibility. In order to speed up the process, Halton draws are used instead of pseudo-random numbers.

Tables

Table 1. Stockholding of households: distribution in per cent. HRS 2002 and 2004, average age between 55 and 65 in 2002.

	2002	2004
Direct stockholder	31	30
Indirect stockholder	13	13
Non stockholder	56	57
All	100	100
n	3642	3423

Note. Source: HRS 2002 and 2004. Sample: households interviewed in the 2002 survey.

Table 2. Stock market expectations by stockholding status (non-missing stock market expectations answers).

	HRS 2002			HRS 2004	HRS 2006
	p0	p10	p0 – p10	p0	p0
Direct stockholder	57	45	12	57	56
Indirect stockholder	54	44	11	56	55
Non stockholder	41	34	7	48	45
All	49	39	9	53	51
Observations	3036			2998	2516

Table 3. Percentage of consistent and inconsistent answer-pairs

Relationship of p0 and p10	Per cent
p0 > p10 (positive mass)	43
p0 = p10 (zero mass)	43
p0 < p10 (negative mass)	14
All	100

*Rounded to nearest ten or 25 or 75

Table 4. Core versus module answers to same questions (non-missing answers, n=179)

	p0		p10	
	Mean	Standard Dev.	Mean	Standard Dev.
Core	52	30	30	30
Module	51	31	31	31
Difference	1	31	-1	31
Absolute Diff.	22	22	22	22
Fraction core = module	0.28		0.18	
Correlation of core and module	0.49		0.37	

Table 5. Measurement errors in stock market expectations and stock ownership. HRS 2002.

	Partial effect of stock ownership on the measurement problem	
	Controls: p0 intervals[a]	Controls: p0 intervals and demographics [b]
Prob (missing answer) [c,d]	-0.201	-0.100
[SE]	[0.011]**	[0.013]**
Prob (p0 focal at 50) [c]	0.004	0.004
[SE]	[0.007]	[0.009]
Prob (p0 focal at 0 or 100) [c]	-0.001	0.000
[SE]	[0.001]	[0.000]
Prob (p0 other round number) [c]	0.004	0.002
[SE]	[0.007]	[0.004]
Prob (p0 = p10) [c]	-0.030	-0.017
[SE]	[0.019]	[0.021]
Prob (p0 < p10) [c]	0.007	0.018
[SE]	[0.014]	[0.015]
| p0 - p0' | [e]	0.015	0.007
[SE]	[0.038]	[0.042]

Notes.
Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65.
[a] p0 interval control variables: dummies for p0 in (0,5] (5,15], …, [95,100].
[b] Demographic control variables: gender and single/couple interaction, race, education and cognitive capacity.
[c] Average partial ("marginal") effect of the stockholding dummy (direct and indirect ownership combined), from probit models.
[d] p0 controls are not applicable. First column includes no control variables; last column includes demographic controls only.
[e] OLS coefficient of the stockholding dummy (direct and indirect ownership), from linear regressions. Dependent variable measured on the [0,1] interval. Standard error estimates are robust to heteroskedasticity.
Standard errors in parentheses.
* Significant at 5%. ** Significant at 1%.

Table 6. The effect of expectations on stockholding, the effect of the instruments on expectations, and estimates of relevant heterogeneity in expectations and risk aversion.
Estimates from the joint structural model of probability answers and stockholding.

	(1)	(2)	(3)	(4)	(5)	(6)
Latent dependent variable: s*						
Alpha	0.596	0.848	0.362	0.361	0.297	0.161
	[0.073]**	[0.060]**	[0.030]**	[0.029]**	[0.040]**	[0.035]**
Latent dependent variable: mu						
Positive sunny forecast error			0.023	0.022	0.020	0.031
			[0.010]*	[0.009]*	[0.010]	[0.013]*
Prob(economic recession)			-0.212	-0.207	-0.173	-0.172
			[0.023]**	[0.023]**	[0.026]**	[0.030]**
Depressive symptoms			-0.063	-0.062	-0.036	-0.025
			[0.005]**	[0.005]**	[0.005]**	[0.006]**
Dow Jones /100			0.002	0.002	0.002	0.002
			[0.001]**	[0.001]**	[0.001]*	[0.001]**
Latent dependent variable: log(sigma)						
Fraction fifty-fifty answers			1.333	1.315	1.302	1.257
			[0.112]**	[0.112]**	[0.113]**	[0.119]**
Other right-hand side variables						
Constants in mu and sigma	YES	YES	YES	YES	YES	YES
Constant in s*		YES		YES	YES	YES
Demographic covariates in mu, sig and s*					YES	YES
Wealth covariates in mu, sig and s*						YES
Moments of relevant heterogeneity						
Relative risk aversion (constant)	1.7	1.2	2.8	2.8	3.4	6.2
Average mu in the population	0.02	-0.02	0.01	-0.03	-0.03	-0.03
Standard deviation of mu in the population	0.13	0.15	0.16	0.16	0.15	0.17
Average sigma in the population	0.38	0.39	0.39	0.40	0.40	0.40
Log likelihood	-15809	-15768	-15533	-15512	-15229	-14728
Observations	3004	3004	3004	3004	3004	3004

Notes.
Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65; non-missing answers to the core survey stock market expectation questions.
The different columns refer to different specifications of the equations for the latent variables, defined in (10)-(12) in the main text.

Column (1): no $\beta_s x$ $\gamma_\mu z_\mu$ or $\gamma_\sigma z_\sigma$; $\beta_\mu x$ and $\beta_\sigma x$ contain a constant each;
Column (2): $\beta_s x$ contains a constant; no $\gamma_\mu z_\mu$ or $\gamma_\sigma z_\sigma$; $\beta_\mu x$ and $\beta_\sigma x$ contain a constant each;
Column (3): no $\beta_s x$; $\gamma_\mu z_\mu$ and $\gamma_\sigma z_\sigma$ included; $\beta_\mu x$ and $\beta_\sigma x$ contain a constant each;
Column (4): $\beta_s x$ contains a constant; $\gamma_\mu z_\mu$ and $\gamma_\sigma z_\sigma$ included; $\beta_\mu x$ and $\beta_\sigma x$ contain a constant each;
Column (5): $\beta_s x$ contains a constant and demographics; $\gamma_\mu z_\mu$ and $\gamma_\sigma z_\sigma$ included; $\beta_\mu x$ and $\beta_\sigma x$ contain a constant and demographics each;
Column (6): $\beta_s x$ contains a constant, demographics and wealth; $\gamma_\mu z_\mu$ and $\gamma_\sigma z_\sigma$ included; $\beta_\mu x$ and $\beta_\sigma x$ contain a constant, demographics and wealth each;

Demographic covariates include gender of the respondent interacted by whether single or lives in a couple; race (African Americans and Hispanics), education in years, and the cognitive score factor of measures from 1992 to 2000. Wealth controls are five splines in the log of total net wealth as well as separate variables for the log of home value, mortgages, financial assets and individual retirement accounts.
Estimates of the coefficients on the x variables are in Table 12.
Standard errors in parentheses. * significant at 5 per cent. ** significant at 1 per cent.

Table 7. The effect of demographic covariates on expectations and their direct and overall stockholding. Estimates from the joint structural model of probability answers and stockholding (model 5) as well as from the reduced-form Tobit models of stockholding without expectations.

	Reduced-form model without expectations	Structural model (5) with expectations		
	s*	mu	ln(sigma)	s*
Single female	-0.238	-0.126	0.133	0.073
	[0.030]**	[0.018]**	[0.025]**	[0.058]
Single male	-0.176	-0.060	0.032	-0.007
	[0.039]**	[0.022]**	[0.033]	[0.062]
Female respondent in couple	-0.081	-0.093	0.108	0.156
	[0.035]*	[0.021]**	[0.030]**	[0.058]**
Black	-0.356	-0.086	0.038	-0.182
	[0.040]**	[0.020]**	[0.031]	[0.056]**
Hispanic	-0.374	-0.033	-0.043	-0.251
	[0.062]**	[0.029]	[0.044]	[0.088]**
Education	0.074	0.012	0.003	0.039
	[0.005]**	[0.003]**	[0.004]	[0.009]**
Cognitive score	0.066	0.015	0.051	0.008
	[0.015]**	[0.009]	[0.012]**	[0.023]
Constant	-0.766	-0.187	-1.264	-0.414
	[0.079]**	[0.073]**	[0.064]**	[0.115]**
Log Likelihood	-2227		-15229	
Observations	3004		3004	

Notes.
Reduced-form Tobit models have the same x variables but no expectations. Details of model (5) are after Table 10. Standard errors in parentheses. * significant at 5 per cent. ** significant at 1 per cent.

Table 8. The effect of expectations on stockholding, relevant heterogeneity in expectations, and the effect of the instruments on expectations. Estimates from the joint structural model of probability answers and stockholding with risk tolerance measured by the Kimball-Sahm-Shapiro (2007) proxy.

	(3)	(4)	(5)	(6)
Latent dependent variable: s*				
Coefficient on the risk tolerance proxy	1.414	1.354	0.634	0.184
	[0.128]**	[0.123]**	[0.101]**	[0.073]*
Other right-hand side variables				
Constants in mu and sigma	YES	YES	YES	YES
Instruments in mu and sigma	YES	YES	YES	YES
Constant in s*		YES	YES	YES
Demographic covariates in mu, sig and s*			YES	YES
Wealth covariates in mu, sig and s*				YES
Point estimates of relevant parameters				
Average relative risk aversion in population	4.8	5.0	10.7	36.9
Average mu in the population	0.01	0.00	-0.03	-0.03
Standard deviation of mu in the population	0.14	0.15	0.15	0.14
Average sigma in the population	0.39	0.39	0.40	0.40
Log likelihood	-15580	-15577	-15266	-14739
Observations	3004	3004	3004	3004

Notes: Models with entering $\alpha_i=\beta_a a_i$ for α. For other details see under Table 6.

Figures

Figure 1. Histogram of yearly log returns on the Dow Jones Industrial Average (June 1st to June 1st), 1946 to 2002. Normal density with appropriate mean and standard deviation superimposed.

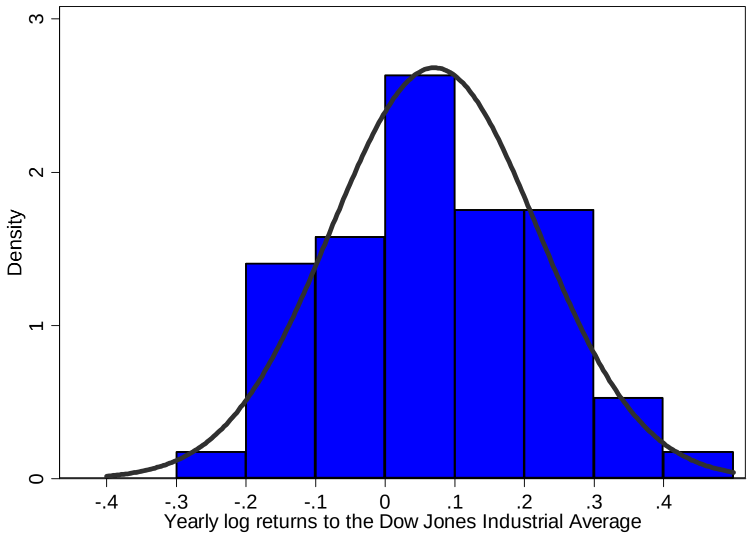
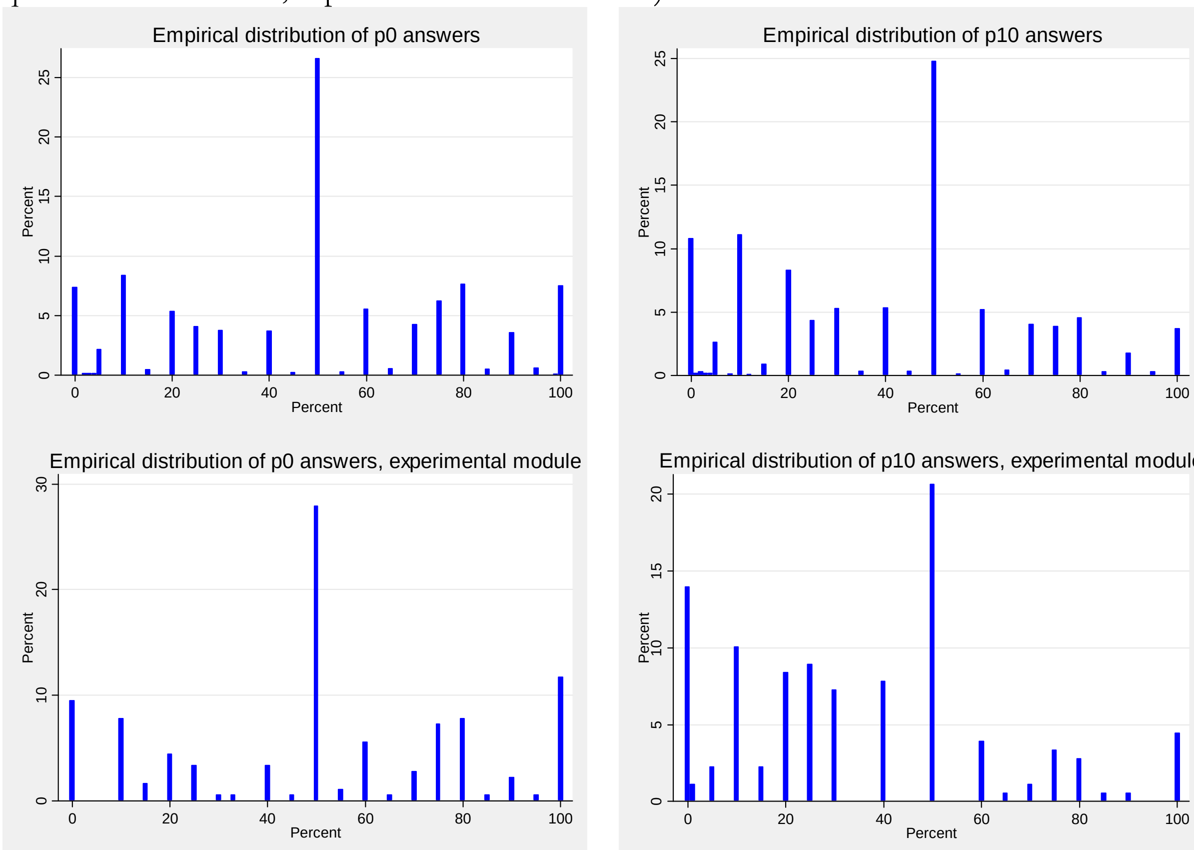
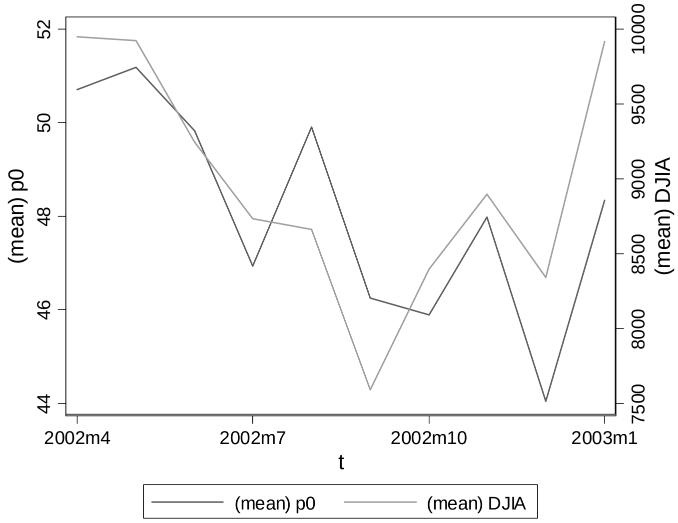


Figure 2. The distribution of reported subjective probabilities of a positive one-year return of the stock market (p_0) and the 10 per cent or larger gain (p_{10}). HRS 2002, estimation sample (core questionnaire: n=3004; experimental module: n=179)



Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65; non-missing answers to the core stock market probability questions. Experimental module respondents are a random subsample.

Figure 3. Monthly average answer to the p_0 question and the Dow Jones Industrial Average on the first day of the interview



Notes. Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65; non-missing answers to the core survey stock market expectation questions. The graph does not show the 26 (of 3004) respondents who were interviewed in 2003m2; their average p_0 answer is very low at 30 per cent., while the index at the beginning of the month was high at 10,000.

Appendix Tables

Appendix Table 1. Probability of stockholding and within financial wealth (inc. retirement accounts), in groups defined by financial wealth (zero wealth and 9 groups of positive wealth)

	zero wealth	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Direct stockholder	0.00	0.01	0.04	0.13	0.25	0.34	0.44	0.57	0.67	0.79
Indirect stockholder	0.00	0.00	0.03	0.08	0.15	0.22	0.23	0.21	0.20	0.15
Non stockholder	1.00	0.99	0.93	0.80	0.60	0.44	0.33	0.22	0.13	0.06
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Wealth $'000	0.0	0.2	1	5	13	29	62	119	237	837
Number of observations	483	317	398	377	309	418	365	361	367	364

Appendix Table 2. Fraction of various assets within financial wealth (inc. retirement accounts), in groups defined by financial wealth (zero wealth and 9 groups of positive wealth)

	zero wealth	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
stocks outside retirement accounts		0.01	0.03	0.07	0.14	0.15	0.17	0.26	0.28	0.37
stocks within retirement accounts		0.00	0.02	0.05	0.08	0.12	0.18	0.16	0.23	0.24
other assets within retirement accounts		0.00	0.02	0.05	0.12	0.19	0.22	0.22	0.21	0.17
bonds		0.00	0.01	0.01	0.00	0.00	0.01	0.01	0.02	0.06
bank accounts		0.98	0.91	0.79	0.59	0.43	0.35	0.24	0.19	0.12
cds, t-bills		0.01	0.02	0.04	0.07	0.09	0.07	0.11	0.07	0.04
sum		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Wealth $'000	0.0	0.2	1.3	5.2	14.2	32.0	62.8	119.6	235.2	830.2
Number of observations	478	312	373	366	351	349	358	350	352	353

Appendix Table 3. Predictors of measurement error.
Panel A: Probability of missing answers, the probability of p0=p10 and the probability of p0<p10.
Probit coefficient estimates.

	missing expectations		p0 = p10			p0 < p10		
	(1)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Stockholder	-0.908	-0.489	-0.221	-0.077	-0.044	-0.149	0.031	0.083
	[0.058]**	[0.066]**	[0.046]**	[0.049]	[0.053]	[0.057]**	[0.062]	[0.068]
Single male		0.627			0.130			-0.010
		[0.071]**			[0.061]*			[0.076]
Single male		0.356			0.071			-0.035
		[0.092]**			[0.078]			[0.100]
Couple, female respondent		0.532			0.153			-0.028
		[0.086]**			[0.071]*			[0.091]
Black		0.246			0.136			-0.089
		[0.068]**			[0.071]			[0.088]
Hispanic		0.633			-0.118			0.299
		[0.085]**			[0.105]			[0.117]*
Education (Grades)		-0.061			-0.008			-0.011
		[0.011]**			[0.010]			[0.013]
Cognitive test		-0.183			0.014			-0.061
		[0.034]**			[0.031]			[0.041]
p0 in [0,5)				0.839	0.831		0.164	0.144
				[0.106]**	[0.106]**		[0.116]	[0.116]
p0 in [5,15)				-0.044	-0.069		0.603	0.604
				[0.083]	[0.084]		[0.094]**	[0.095]**
p0 in [15,25)				-0.265	-0.281		0.247	0.249
				[0.105]*	[0.106]**		[0.125]*	[0.125]*
p0 in [25,35)				-0.394	-0.398		0.554	0.553
				[0.094]**	[0.095]**		[0.105]**	[0.105]**
p0 in [35,45)				-0.590	-0.596		0.490	0.504
				[0.128]**	[0.128]**		[0.137]**	[0.137]**
p0 in [55,65)				-0.683	-0.684		0.204	0.209
				[0.112]**	[0.112]**		[0.127]	[0.128]
p0 in [65,75)				-0.572	-0.552		-0.371	-0.372
				[0.118]**	[0.119]**		[0.173]*	[0.176]*
p0 in [75,85)				-0.553	-0.528		-0.387	-0.385
				[0.079]**	[0.080]**		[0.114]**	[0.115]**
p0 in [85,95)				-0.770	-0.731		-0.806	-0.798
				[0.134]**	[0.134]**		[0.251]**	[0.251]**
p0 in [95,100]				-0.400	-0.364		.	.
				[0.093]**	[0.094]**		[.]	[.]
Constant	-0.634	-0.585	-0.066	0.079	0.071	-1.019	-1.189	-1.067
	[0.030]**	[0.146]**	[0.032]*	[0.050]	[0.148]	[0.039]**	[0.065]**	[0.184]**
Observations	3642	3642	3004	3004	3004	3004	2756	2756

Notes.
Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65.
Standard errors in parentheses.
* Significant at 5%. ** Significant at 1%.

Appendix Table 3, continued.

Panel B: The probability of giving focal and other round answers. Probit coefficient estimates.

	p0 = 50			p0 = 0 or 100			p0 = other round answer		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Stockholder	-0.048	0.162	0.143	-0.109	-0.395	-0.380	0.120	0.065	0.052
	[0.049]	[0.270]	[0.302]	[0.056]	[0.197]*	[0.219]	[0.046]**	[0.105]	[0.116]
Single female			0.022			0.056			-0.020
			[0.323]			[0.252]			[0.129]
Single male			0.000			-0.420			-0.091
			0.000			[0.252]			[0.168]
Couple, female respondent			0.022			0.019			-0.012
			[0.381]			[0.290]			[0.153]
Black			0.000			-0.313			-0.103
			0.000			[0.266]			[0.137]
Hispanic			-0.080			0.090			0.576
			[0.531]			[0.477]			[0.270]*
Education (Grades)			0.060			-0.006			0.028
			[0.060]			[0.042]			[0.022]
Cognitive test			-0.102			-0.141			-0.069
			[0.182]			[0.117]			[0.065]
p0 in [0,5)		.	.		7.767	8.404		.	.
		[.]	[.]		[0.196]**	[0.572]**		[.]	[.]
p0 in [5,15)		.	.		.	.		6.867	7.538
		[.]	[.]		[.]	[.]		[0.182]**	[0.303]**
p0 in [15,25)		.	.		.	.		7.453	8.130
		[.]	[.]		[.]	[.]		[0.212]**	[0.321]**
p0 in [25,35)		.	.		.	.		.	.
		[.]	[.]		[.]	[.]		[.]	[.]
p0 in [35,45)		.	.		.	.		7.477	8.140
		[.]	[.]		[.]	[.]		[0.000]	[0.342]**
p0 in [55,65)		.	.		.	.		7.759	8.427
		[.]	[.]		[.]	[.]		[0.233]**	[0.339]**
p0 in [65,75)		.	.		.	.		7.227	7.878
		[.]	[.]		[.]	[.]		[0.213]**	[0.334]**
p0 in [75,85)		.	.		.	.		.	.
		[.]	[.]		[.]	[.]		[.]	[.]
p0 in [85,95)		.	.		.	.		7.186	7.813
		[.]	[.]		[.]	[.]		[0.222]**	[0.343]**
p0 in [95,100]		.	.		7.649	8.289		.	.
		[.]	[.]		[0.000]	[0.601]**		[.]	[.]
Constant	-0.601	2.259	1.410	-0.989	-6.011	-6.471	0.005	-6.107	-7.106
	[0.034]**	[0.171]**	[0.773]	[0.039]**	[0.188]**	[0.000]	[0.032]	[0.168]**	[0.000]
Observations	3004	616	616	3004	1288	1288	3004	1870	1870

Notes.

Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65.

Standard errors in parentheses.

* Significant at 5%. ** Significant at 1%.

Appendix Table 3, continued.
Panel C: The absolute value of the difference between core answers (to p0 and p10) and the corresponding answer in the experimental module. OLS coefficient estimates.

	p0 = 50			p0 = 0 or 100		
	(1)	(2)	(3)	(1)	(2)	(3)
Stockholder	-0.155	1.512	0.713	-0.305	-4.414	-2.264
	[3.273]	[3.809]	[4.264]	[3.254]	[3.410]	[3.702]
Single female			6.581			6.050
			[4.519]			[4.017]
Single male			-2.764			3.968
			[6.069]			[5.999]
Couple, female respondent			-1.356			3.178
			[4.584]			[4.834]
Black			-9.733			-0.129
			[4.228]*			[5.809]
Hispanic			0.644			3.910
			[11.843]			[8.467]
Education (Grades)			-0.094			-0.997
			[0.828]			[0.834]
Cognitive test			-1.414			-0.861
			[1.980]			[2.153]
p0 in [0,5)		14.244	12.812		4.798	3.541
		[8.195]	[8.290]		[8.068]	[7.997]
p0 in [5,15)		-0.001	-0.953		-5.548	-7.114
		[6.823]	[6.926]		[4.775]	[5.007]
p0 in [15,25)		8.901	4.444		-10.505	-10.637
		[10.259]	[10.901]		[6.464]	[7.111]
p0 in [25,35)		10.526	10.455		-4.224	-4.235
		[6.814]	[6.991]		[5.817]	[5.710]
p0 in [35,45)		0.917	0.719		-6.063	-4.317
		[4.961]	[5.261]		[4.464]	[4.700]
p0 in [55,65)		2.783	3.648		-4.497	-3.915
		[5.736]	[5.410]		[4.172]	[4.272]
p0 in [65,75)		12.060	14.085		8.771	10.151
		[17.692]	[19.077]		[14.448]	[13.778]
p0 in [75,85)		-0.791	1.276		7.111	8.597
		[3.593]	[3.809]		[4.657]	[5.006]
p0 in [85,95)		7.641	8.256		25.340	27.169
		[7.658]	[7.714]		[7.185]**	[7.126]**
p0 in [95,100]		8.508	8.320		15.577	16.482
		[7.808]	[7.974]		[7.326]*	[7.495]*
Constant	21.655	16.932	18.011	23.621	22.505	31.277
	[2.446]**	[3.511]**	[11.171]	[2.352]**	[3.290]**	[11.269]**
Observations	179	179	179	179	179	179
R-squared	0.00	0.05	0.09	0.00	0.16	0.19

Notes.
Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65.
Standard errors (robust to heteroskedasticity) in parentheses.
* Significant at 5%. ** Significant at 1%.

Appendix Table 4. Descriptive statistics of the variables used in the main analysis

	Mean	Std.Dev.	Min.	Max.
Fraction of stocks in financial wealth and retirement accounts	0.28	0.36	0.00	1.00
Whether stockholder (direct or indirect)	0.50	0.50	0	1
Single female	0.32	0.47	0	1
Single male	0.13	0.33	0	1
Female respondent in couple	0.17	0.38	0	1
Male respondent in couple	reference group			
Black	0.15	0.36	0	1
Hispanic	0.06	0.24	0	1
Non-Black, non-Hispanic	reference group			
Education	13.2	2.7	0.0	17.0
Cognitive score	0.08	0.91	-4.01	3.37
log(-wealth) if wealth<0	-0.51	2.04	-12.49	0.00
log(wealth) if wealth<$1k	0.14	0.88	0.00	6.88
log(wealth) if $1k<wealth<$10k	1.20	2.86	0.00	9.20
log(wealth) if $10k<wealth<$100k	4.72	4.94	0.00	11.50
log(wealth) if wealth>$100k	5.38	6.32	0.00	16.85
log(value of home)	10.01	4.32	0.00	15.48
log(value of mortgage)	5.38	5.49	0.00	14.18
log(value of financial assets)	8.56	3.68	0.00	16.36
log(value of retirement accounts)	4.88	5.42	0.00	15.10
p0	0.49	0.30	0.00	1.00
p10	0.39	0.28	0.00	1.00
Positive sunny forecast error	0.57	0.48	0.00	1.00
Prob(economic recession)	0.43	0.24	0.00	1.00
Depressive symptoms	-0.04	1.22	-1.71	6.16
Dow Jones /100	90.99	7.24	75.92	101.06
Fraction fifty-fifty answers	0.17	0.09	0.00	0.59
Risk tolerance proxy	0.19	0.12	0.04	1.36
Number of observations	3004			

Notes.
Sample: HRS 2002, singles of age 55 to 65 and financial respondents from couples with average age 55 to 65.

Appendix Table 5. Detailed results of the models with wealth controls

	Reduced-form model without expectations	Full structural model with expectations	
	s*	s*	mu	log(sigma)
Single female	-0.014	0.118	-0.106	0.146
	[0.027]	[0.043]**	[0.019]**	[0.027]**
Single male	-0.014	0.046	-0.040	0.044
	[0.035]	[0.045]	[0.023]	[0.034]
Female respondent in couple	-0.001	0.112	-0.085	0.120
	[0.030]	[0.043]**	[0.021]**	[0.030]**
Black	-0.102	-0.031	-0.068	0.058
	[0.037]**	[0.044]	[0.022]**	[0.033]
Hispanic	-0.184	-0.148	-0.018	-0.035
	[0.056]**	[0.066]*	[0.031]	[0.048]
Education	0.016	0.005	0.007	-0.001
	[0.005]**	[0.006]	[0.003]*	[0.005]
Cognitive score	0.000	-0.018	0.009	0.046
	[0.014]	[0.017]	[0.009]	[0.013]**
log(-wealth) if wealth<0	-0.056	-0.046	-0.007	0.002
	[0.017]**	[0.018]*	[0.006]	[0.009]
log(wealth) if wealth<$1k	0.014	0.009	-0.003	0.027
	[0.023]	[0.024]	[0.009]	[0.013]*
log(wealth) if $1k<wealth<$10k	-0.017	-0.018	0.000	0.004
	[0.006]**	[0.007]**	[0.003]	[0.004]
log(wealth) if $10k<wealth<$100k	0.056	0.046	0.006	-0.006
	[0.014]**	[0.015]**	[0.005]	[0.007]
log(wealth) if wealth>$100k	0.056	0.042	0.010	-0.008
	[0.012]**	[0.013]**	[0.005]*	[0.006]
log(value of home)	0.000	0.000	0.000	0.000
	[0.004]	[0.004]	[0.002]	[0.003]
log(value of mortgage)	0.013	0.006	0.006	-0.007
	[0.002]**	[0.003]*	[0.001]**	[0.002]**
log(value of financial assets)	0.061	0.058	0.001	0.012
	[0.006]**	[0.007]**	[0.003]	[0.005]*
log(value of retirement accounts)	0.030	0.029	0.000	0.007
	[0.002]**	[0.003]**	[0.002]	[0.002]**
Constant	-1.522	-1.220	-0.352	-1.237
	[0.152]**	[0.169]**	[0.098]**	[0.089]**
Log Likelihood	-1715		-14728	
Observations	3004		3004	